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The mod p cohomology algebras of finite groups with metacyclic Sylow p-subgroups

HIROKI SASAKI
(佐々木　洋城)

Department of Mathematics, Faculty of Education
Yamaguchi University

INTRODUCTION

The following p-groups are known as noncommutative p-groups that have cyclic maximal subgroups:

(1) $p=2$, dihedral 2-group

$$D_m = \langle\, x, y \mid x^{2^{m-1}} = y^2 = 1, yxy = x^{-1} \,\rangle,\ m \geq 3;$$

(2) $p=2$, generalized quaternion 2-group

$$Q_m = \langle\, x, y \mid x^{2^{m-2}} = y^2 = z, z^2 = 1, yxy = x^{-1} \,\rangle,\ m \geq 3;$$

(3) $p=2$, semidihedral 2-group

$$SD_m = \langle\, x, y \mid x^{2^{m-1}} = y^2 = 1, yxy = x^{-1+2^{m-2}} \,\rangle,\ m \geq 4;$$

(4)

$$M_m(p) = \langle\, x, y \mid x^{p^{m-1}} = y^p = 1, yxy^{-1} = x^{1+p^{m-2}} \,\rangle$$
$$m \geq 3 \text{ when } p > 2,\ m \geq 4 \text{ when } p = 2.$$

The mod 2 cohomology algebras of finite groups that have these 2-groups as Sylow 2-subgroups have been computed by

(i) Martino [14], 1988
(ii) Martino-Priddy [15], 1991
(iii) Asai-Sasaki [2], 1993
(iv) Sasaki [19].

The first two ones are actually concerned with the classifying spaces. The latters depend on the theory of modular representation and the cohomology varieties of modules.

On the other hand the mod p cohomology algebras of metacyclic groups have been computed as follows:

(i) Diethelm [6], 1985 for split metacyclic p-groups
(ii) Rusin [18], 1987 for metacyclic 2-groups
(iii) Huebschmann [11], 1989 for general metacyclic groups.

The purpose of this report is to calculate the mod p cohomology algebras of finite groups with metacyclic Sylow p-subgroups for an odd prime p. Our method will be again module theoretic.

From now on we let p be an odd prime. Let P be a nonabelian metacyclic p-group

$$\boxed{\langle x, y \mid x^{p^m} = 1,\ y^{p^n} = x^{p^f},\ yxy^{-1} = x^{1+p^l} \rangle}$$

where

$$0 < l < m,\ m - l \leq n,\ m - l \leq f \leq m.$$

$$\begin{tikzcd}
 & P \arrow[dl, dash] \arrow[d, dash] \\
\langle y \rangle \arrow[d, dash] & \langle x \rangle \arrow[dl, dash] \\
\langle y^{p^n} = x^{p^f} \rangle \arrow[d, dash] & \\
1 &
\end{tikzcd}$$

Let G be a finite group with a Sylow p-subgroup P and let k be a field of characteristic p.

1. Structure of G

Since the p-group P is regular, it is known

$$\boxed{H^1(G, k) \simeq H^1(N_G(P), k)}$$

and the group G has the following structure:

$$\begin{tikzcd}[column sep=small]
 & & G \arrow[dl, dash] \arrow[dd, dash] \\
 & N_G(P) \arrow[dl, dash] \arrow[dd, dash] & \\
P \arrow[dd, dash] & & O^p(G) \arrow[dl, dash] \\
 & O^p(N_G(P)) \arrow[dl, dash] & \\
I & &
\end{tikzcd}$$

where

$$I = P \cap O^p(G) = P \cap O^p(N_G(P)).$$

To investigate the structure of the Sylow normalizer $N_G(P)$ we study the automorphism group $\operatorname{Aut} P$ of P.

LEMMA 1.1. *The extension*

$$1 \longrightarrow \langle x \rangle \longrightarrow P \longrightarrow P/\langle x \rangle \longrightarrow 1$$

splits if and only if

$$m \geq f \geq n \quad \text{or} \quad m = f < n.$$

If the extension

$$1 \longrightarrow \langle x \rangle \longrightarrow P \longrightarrow P/\langle x \rangle \longrightarrow 1$$

splits, we shall say that the group P is of split type; while if the extension above does not split, we shall say that the group P is of non-split type.

Suppose that the group P is of split type. Then by Lemma 1.1 we may assume

$$P = \langle x, y \mid x^{p^m} = y^{p^n} = 1,\ yxy^{-1} = x^{1+p^l} \rangle$$

where

$$0 < l < m,\ m - l \leq n.$$

The p-group P has the automorphism

$$\boxed{\sigma : \begin{cases} x \longmapsto x^r \\ y \longmapsto y \end{cases}}$$

where r is a primitive $(p-1)$th root of unity modulo p^m.

LEMMA 1.2. *Let*

$$P = \langle\, x, y \mid x^{p^m} = y^{p^n} = 1,\ yxy^{-1} = x^{1+p^l} \,\rangle$$

where

$$0 < l < m,\ m - l \le n.$$

Then

$$\boxed{\operatorname{Aut} P = R \rtimes \langle\, \sigma \,\rangle} \quad R \text{ a } p\text{-group}.$$

LEMMA 1.3. *For P above there exists $\varphi \in \operatorname{Aut} P$ such that*

$$\boxed{P \cap O^p(N_G(P)) = \langle\, {}^{\varphi}x \,\rangle}$$

Therefore the finite group G with a Sylow p-subgroup P of split type has the following structure:

$$\begin{tikzcd}[column sep=small]
 & & G \arrow[dl, dash] \arrow[dd, dash] \\
 & N_G(P) \arrow[dl, dash] \arrow[dd, dash] & \\
P \arrow[dd, dash] & & O^p(G) \arrow[dl, dash] \\
 & O^p(N_G(P)) \arrow[dl, dash] & \\
\langle\, {}^{\varphi}x \,\rangle & &
\end{tikzcd}$$

Next let us consider the non-split case. Let

$$P = \langle\, x, y \mid x^{p^m} = 1,\ y^{p^n} = x^{p^f},\ yxy^{-1} = x^{1+p^l} \,\rangle$$

where

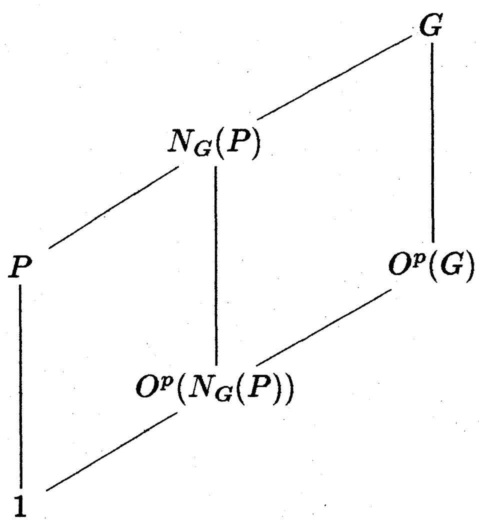
$$0 < l < m,\ m - l \le n,\ m - l \le f < m,\ f < n.$$

The situations differ according with whether $f > l$ or not.

Lemma 1.4. *Suppose* $1 \to \langle x \rangle \to P \to P/\langle x \rangle \to 1$ *does not split. Suppose that* $f > l$. *Then*

$$\boxed{\text{Aut}\, P \text{ is a } p\text{-group}}$$

Therefore the Sylow normalizer $N_G(P)$ has a normal p-complement, whence so does the finite group G:



In particular one has

Lemma 1.5. *With* P *above*

$$\boxed{\text{res} : H^*(G,k) \simeq H^*(P,k)}$$

While if $f \leq l$, then the following holds.

Lemma 1.6. *Suppose* $1 \to \langle x \rangle \to P \to P/\langle x \rangle \to 1$ *does not split. Suppose that* $f \leq l$. *Then*

(1) $\langle y \rangle \lhd P$;
(2) $1 \to \langle y \rangle \to P \to P/\langle y \rangle \to 1$ *splits.*

Thus the result of the split case can be applied to this case.

2. A transfer theorem

In this and the following two sections we shall deal with the split case. Let

$$P = \langle x, y \mid x^{p^m} = y^{p^n} = 1,\ yxy^{-1} = x^{1+p^l} \rangle$$

where

$$0 < l < m, \; m - l \leq n.$$

By Lemma 1.3 we may assume that the group G has the following structure:

$$\begin{tikzcd}
 & & & G \arrow[dl, dash] \arrow[dd, dash] \\
 & & N \arrow[dl, dash] \arrow[dd, dash] & \\
 & P \arrow[dl, dash] \arrow[dd, dash] & & O^p(G) \arrow[dl, dash] \\
\langle y \rangle \arrow[dd, dash] & & O^p(N) \arrow[dl, dash] & \\
 & \langle x \rangle \arrow[dl, dash] & & \\
1 & & &
\end{tikzcd}$$

where $N = N_G(P)$.

One may compute $H^*(G,k)$ by means of the spectral sequence

$$H^s(G/O^p(G), H^t(O^p(G),k)) \Longrightarrow H^{s+t}(G,k)$$

associated with

$$1 \longrightarrow O^p(G) \longrightarrow G \longrightarrow \langle y \rangle \longrightarrow 1.$$

However we adapt another way. We shall prove

THEOREM 2.1. *Let G be a finite group with the split metacyclic Sylow p-subgroup P. Then*

$$\boxed{\mathrm{res} : H^*(G,k) \simeq H^*(N_G(P),k)}$$

LEMMA 2.2. *Let $Z = \langle x^{p^{m-1}} \rangle$. Then*

$$\boxed{\mathrm{res} : H^*(G,k) \simeq H^*(N_G(Z),k)}$$

Proof. Let $L = N_G(Z)$. Since $P \leq L$, it is enough to show

$$\mathrm{res}_P \, \mathrm{cor}^G(\lambda) = \mathrm{res}_P(\lambda) \quad \forall \, \lambda \in H^n(L,k).$$

By Mackey formula

$$\mathrm{res}_P \,\mathrm{cor}^G(\lambda) = \sum_{g\in[P\backslash G/L]} \mathrm{cor}^P \,\mathrm{res}_{{}^gL\cap P} \,\mathrm{con}^g(\lambda).$$

Let us show

$$g \notin L \Longrightarrow Z \cap ({}^gL \cap P) = 1.$$

Now suppose $Z \leq {}^gL \cap P$. Then we have $Z^g \leq L$ so that

$$\exists\, a \in L \quad \text{such that} \quad Z^{ga} \leq P.$$

On the other hand, since $Z \leq O^p(G)$

$$Z^{ga} \leq O^p(G).$$

Thus we see that

$$Z^{ga} \leq P \cap O^p(G) = \langle\, x \,\rangle.$$

Namely we have

$$Z^{ga} = Z \quad \text{and} \quad g \in N_G(Z) = L$$

as desired.

Therefore if $g \notin L$, then it follow for $\zeta \in H^n({}^gL \cap P, k)$ that

$$\begin{aligned} \mathrm{cor}^P(\zeta) &= \mathrm{cor}^P \,\mathrm{cor}^{({}^gL\cap P)\times Z}(\zeta) \\ &= 0. \end{aligned}$$

$$\begin{tikzcd}
Z \times ({}^gL \cap P) \arrow[d, dash] & \exists\, \theta \arrow[d, "\mathrm{res}"] & p\theta = 0 \\
{}^gL \cap P & \zeta \arrow[r, equal] & \zeta \arrow[u, "\mathrm{cor}"']
\end{tikzcd}$$

□

LEMMA 2.3.

$$\boxed{\mathrm{res} : H^*(N_G(Z), k) \simeq H^*(N_G(P), k)}$$

Proof. This follows from the fact that the normalizer $N_G(Z)$ has the following structure:

$$\begin{tikzcd}
 & N_G(Z) \arrow[d, dash] \\
N_G(P) \arrow[ur, dash] \arrow[d, dash] & O_{p'}(N_G(Z)) \\
O_{p'}(N_G(P)) \arrow[ur, dash] &
\end{tikzcd}$$

□

3. $H^*(P,k)$ OF P OF SPLIT TYPE

We compute $H^*(P,k)$ by a module theoretic method as in Okuyama-Sasaki [17]. Let

$$a_1 = y-1, \quad b_1 = x-1.$$

Then clearly we have

$$\Omega^1(k) = \langle a_1, b_1 \rangle_{kP}.$$

To compute $\Omega^i(k)$ we let

$$u = 1 + x + \cdots + x^{p^l}.$$

First we assume

$$\langle y \rangle \text{ acts on } \langle x \rangle \text{ faithfully.}$$

Let us define some elements in $kP \oplus kP$:

$$\begin{cases} a_{2i} = ((y-1)^{p^n-1}, 0) \\ b_{2i} = ((x-1)^i, -(u^i y - 1)) \end{cases} \qquad 1 \le i \le p$$

$$\begin{cases} a_{2i+1} = (y-1, 0) \\ b_{2i+1} = ((x-1)^{i+1}, -(u^i y - 1)^{p^n-1}(x-1)) \end{cases} \qquad 1 \le i \le p-1$$

and

$$c_{2p} = (0, (x-1)^{p^m-1}).$$

Syzygies of the trivial kP-module k are calculated as follows.

LEMMA 3.1.

$$\begin{aligned} \Omega^{2i}(k) &= \langle a_{2i}, b_{2i} \rangle_{kP} \\ \Omega^{2i+1}(k) &= \langle a_{2i+1}, b_{2i+1} \rangle_{kP} \end{aligned} \qquad 1 \le i \le p-1$$

and

$$\Omega^{2p}(k) = \langle a_{2p}, b_{2p}, c_{2p} \rangle_{kP}.$$

Let us define kP-homomorphisms as follows:

$$\widehat{\beta}_1 : \Omega^1(k) \longrightarrow k \; ; \; \begin{cases} a_1 \longmapsto 1 \\ b_1 \longmapsto 0 \end{cases}$$

$$\widehat{\beta}_2 : \Omega^2(k) \longrightarrow k \; ; \; \begin{cases} a_2 \longmapsto 1 \\ b_2 \longmapsto 0 \end{cases}$$

$$\widehat{\alpha}_{2i-1} : \Omega^{2i-1}(k) \longrightarrow k \; ; \; \begin{cases} a_{2i-1} \longmapsto 0 \\ b_{2i-1} \longmapsto 1 \end{cases} \qquad 1 \le i \le p$$

$$\widehat{\tau}_{2p} : \Omega^{2p}(k) \longrightarrow k \; ; \; \begin{cases} a_{2p} \longmapsto 0 \\ b_{2p} \longmapsto 0 \\ c_{2p} \longmapsto 1. \end{cases}$$

These define cohomology elements

$$\beta_1 \in H^1(P,k),\ \beta_2 \in H^2(P,k),\ \alpha_{2i-1} \in H^{2i-1}(P,k),\ 1 \leq i \leq p,\ \tau_{2p} \in H^{2p}(P,k)$$

which have the following properties.

LEMMA 3.2. *(1) β_1 and α_1 in $H^1(P,k)$ are the duals y^* and x^* of the elements y and x, respectively, regarding $H^1(P,k)$ as* Hom(P,k).

(2) $\beta_2 = \inf(\bar{\beta})$, where $0 \neq \bar{\beta} \in H^2(P/\langle x \rangle, k)$.

(3) τ_{2p} is not a zero-divisor.

(4) The tensor product $L_{\beta_2} \otimes L_{\tau_{2p}}$ of the Carlson modules of β_2 and τ_{2p} is projective. Namely the elements β_2 and τ_{2p} form a homogeneous system of parameters of $H^*(P,k)$.

By Lemma 3.2 (4) and Lemma 3.2 of Okuyama-Sasaki [17] we have

LEMMA 3.3. *For n, $n \geq 2p+1$*

$$\boxed{H^n(P,k) = H^{n-2}(P,k)\beta_2 + H^{n-2p}(P,k)\tau_{2p}}$$

Applying the theory of Benson-Carlson [5] Section 9, we obtain a dimension formula.

LEMMA 3.4.

$$\boxed{\dim H^{n+2p}(P,k) = \dim H^n(P,k) + 2} \quad n \geq 0.$$

Using these informations we can calculate $H^*(P,k)$. A similar method can be applied to the case that $\langle y \rangle$ does not act faithfully on $\langle x \rangle$.

THEOREM 3.5 (DIETHELM). *Let*

$$P = \langle x, y \mid x^{p^m} = y^{p^n} = 1,\ yxy^{-1} = x^{1+p^l} \rangle$$

where

$$0 < l < m,\ m - l \leq n.$$

(1) If $m - l = n$, then

$$\boxed{H^*(P,k) = k[\beta_1, \beta_2, \alpha_1, \alpha_3, \ldots, \alpha_{2p-1}, \tau_{2p}]}$$

$$\alpha_{2i-1}\alpha_{2j-1} = 0, \quad 1 \leq i,j \leq p;$$

$$\beta_2\alpha_{2i-1} = 0, \quad 1 \leq i \leq p-1.$$

(2) *If* $m - l < n$, *then*

$$\boxed{H^*(P,k) = k[\beta_1, \beta_2, \alpha_1, \tau_2]}$$

Remark. Since the prime p is odd, the squares of the homogeneous cohomology elements of odd degree vanish.

Let us recall that the group P has the automorphism

$$\sigma : \begin{cases} x \longmapsto x^r \\ y \longmapsto y \end{cases}.$$

LEMMA 3.6. *Let* r' *be an inverse of* r. *The automorphism* σ *above acts on the cohomology algebra* $H^*(P,k)$ *as follows.*

(1) *When* $m - l = n$

$$^{\sigma}\alpha_{2i-1} = (r')^i \alpha_{2i-1}, \quad {}^{\sigma}\beta_1 = \beta_1,$$
$$^{\sigma}\beta_2 = \beta_2 \quad {}^{\sigma}\tau_{2p} = r'\tau_{2p}.$$

(2) *When* $m - l < n$

$$^{\sigma}\alpha_1 = r'\alpha_1, \quad {}^{\sigma}\beta_1 = \beta_1,$$
$$^{\sigma}\beta_2 = \beta_2 \quad {}^{\sigma}\tau_2 = r'\tau_2.$$

4. $H^*(G,k)$ WITH P OF SPLIT TYPE

By Theorem 2.1 we may assume that the Sylow p-subgroup P is normal in G and the group G has the following structure:

$$\begin{tikzcd}
 & G \arrow[dl, dash] \arrow[dd, dash, "\triangledown" description] \\
\langle s \rangle \arrow[dd, dash] & \\
 & P \arrow[dl, dash] \\
1 &
\end{tikzcd}$$

where

$$^{s}x = x^t \ (\exists\, t \in \boldsymbol{Z}), \quad {}^{s}y = y, \quad C_{\langle s \rangle}(P) = 1.$$

This group is metacyclic so that one can obtain $H^*(G,k)$ from Huebschmann [11]. However, since the element s acts on $H^*(P,k)$ as a scalar multiplication, we can easily compute the stable elements.

THEOREM 4.1. *Let G be a finite group with a split metacyclic Sylow p-subgroup*

$$P = \langle\, x, y \mid x^{p^m} = y^{p^n} = 1,\ yxy^{-1} = x^{1+p^{m-n}} \,\rangle$$

where

$$m > n.$$

Let $e = |\, N_G(P) : C_G(P) \,|_{p'}$. Then there exist

$$\beta_j \in H^j(G,k),\ j = 1,2,\ \zeta_{2ei-1} \in H^{2ei-1}(G,k),\ 1 \le i \le p,\ \rho_{2ep} \in H^{2ep}(G,k)$$

with

$$\boxed{H^*(G,k) = k[\beta_1, \beta_2, \zeta_{2e-1}, \zeta_{4e-1}, \cdots, \zeta_{2ep-1}, \rho_{2ep}]}$$

$$\zeta_{2ei-1}\zeta_{2ej-1} = 0,\ 1 \le i,j \le p;$$

$$\beta_2\zeta_{2ei-1} = 0,\ 1 \le i \le p-1.$$

THEOREM 4.2. *Let G be a finite group with a split metacyclic Sylow p-subgroup*

$$P = \langle\, x, y \mid x^{p^m} = y^{p^n} = 1,\ yxy^{-1} = x^{1+p^l} \,\rangle$$

where

$$0 < l < m,\ m - l < n.$$

Let $e = |\, N_G(P) : C_G(P) \,|_{p'}$. Then

$$\boxed{H^*(G,k) = k[\beta_1, \beta_2, \zeta_{2e-1}, \rho_{2e}]}$$

5. $H^*(P,k)$ OF P OF NON-SPLIT TYPE

Finally let us consider the cohomology algebra of

$$P = \langle\, x, y \mid x^{p^m} = 1,\ y^{p^n} = x^{p^f},\ yxy^{-1} = x^{1+p^l} \,\rangle$$

where

$$0 < l < f < m,\ m - l < n,\ m - l \le f.$$

$$\begin{tikzcd}
 & & P \arrow[d, no head] \\
\langle y \rangle \arrow[urr, no head] \arrow[d, no head] & & \langle x \rangle \arrow[dll, no head] \\
\langle y^{p^n} = x^{p^f} \rangle \arrow[d, no head] & & \\
1 & &
\end{tikzcd}$$

Let us note by Lemma 1.5 that the finite group G with P as a Sylow p-subgroup has the mod p cohomology algebra isomorphic with that of P.

Similarly to the split case, we can compute $H^*(P,k)$ by calculating $\Omega^i(k)$, $i = 1,\dots,2p$.

Let

$$a_1 = y-1, \quad b_1 = x-1.$$

Then clearly

$$\Omega^1(k) = \langle a_1, b_1 \rangle_{kP}.$$

To compute $\Omega^i(k)$ we let

$$u = 1 + x + \cdots + x^{p^l}.$$

First we assume

$$m - l = f.$$

Let us define some elements in $kP \oplus kP$:

$$\begin{cases} a_{2i} = ((y-1)^{p^n-i}, -(x-1)^{p^f-1}) \\ b_{2i} = (x-1, -(uy-1)^i) \end{cases} \qquad 1 \le i \le p$$

$$\begin{cases} a_{2i+1} = ((y-1)^{i+1}, -(x-1)^{p^f-1}(u^{-ip^f}y-1)) \\ b_{2i+1} = (x-1, -(uy-1)^{p^n-i}) \end{cases} \qquad 1 \le i \le p-1$$

and

$$c_{2p} = (0, (x-1)^{p^m-1}(y-1)^{p-1}).$$

LEMMA 5.1.

$$\begin{aligned} \Omega^{2i}(k) &= \langle a_{2i}, b_{2i} \rangle_{kP} \\ \Omega^{2i+1}(k) &= \langle a_{2i+1}, b_{2i+1} \rangle_{kP} \end{aligned} \qquad 1 \le i \le p-1$$

and

$$\Omega^{2p}(k) = \langle a_{2p}, b_{2p}, c_{2p} \rangle_{kP}.$$

Let us define kP-homomorphisms as follows:

$$\widehat{\alpha}_1 : \Omega^1(k) \longrightarrow k \ ; \ \begin{cases} a_1 \longmapsto 0 \\ b_1 \longmapsto 1 \end{cases}$$

$$\widehat{\beta}_{2i-1} : \Omega^{2i-1}(k) \longrightarrow k \ ; \ \begin{cases} a_{2i-1} \longmapsto 1 \\ b_{2i-1} \longmapsto 0 \end{cases} \qquad 1 \le i \le p$$

$$\widehat{\beta}_2 : \Omega^2(k) \longrightarrow k \ ; \ \begin{cases} a_2 \longmapsto 1 \\ b_2 \longmapsto 0 \end{cases}$$

$$\widehat{\tau}_{2p} : \Omega^{2p}(k) \longrightarrow k \ ; \ \begin{cases} a_{2p} \longmapsto 0 \\ b_{2p} \longmapsto 0 \\ c_{2p} \longmapsto 1 \end{cases}$$

LEMMA 5.2. (1) $\beta_1 = y^*$ *and* $\alpha_1 = x^*$.
(2) $\beta_2 = \inf(\bar{\beta})$, *where* $0 \neq \bar{\beta} \in H^2(P/\langle x \rangle, k)$.
(3) τ_{2p} *is not a zero-divisor.*
(4) $L_{\beta_2} \otimes L_{\tau_{2p}}$ *is projective. In particular*

$$H^n(P,k) = H^{n-2}(P,k)\beta_2 + H^{n-2p}(P,k)\tau_{2p}.$$

(5)

$$\dim H^{n+2p}(P,k) = \dim H^n(P,k) + 2.$$

Using these information we can calculate the cohomology algebra $H^*(P,k)$. When $m - l < f$, similarly we can compute the cohomology algebra too.

THEOREM 5.3 (HUEBSCHMANN). *Let*

$$P = \langle\, x, y \mid x^{p^m} = 1,\ y^{p^n} = x^{p^f},\ yxy^{-1} = x^{1+p^l} \,\rangle$$

where

$$0 < l < f < m,\ m - l < n,\ m - l \leq f.$$

(1) *If* $m - l = f$, *then*

$$\boxed{H^*(P,k) = k[\alpha_1, \beta_1, \beta_2, \beta_3, \dots, \beta_{2p-1}, \tau_{2p}]}$$
$$\beta_{2i-1}\beta_{2j-1} = 0, \quad 1 \leq i, j \leq p;$$
$$\beta_2\beta_{2i-1} = 0, \quad 1 \leq i \leq p-1.$$

(2) *If* $m - l < f$, *then*

$$\boxed{H^*(P,k) = k[\,\alpha_1, \beta_1, \beta_2, \tau_2\,]}$$

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