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Biological Feature of the Learning Process
In a Man-to-man Game

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Abstract

The uncertainty principle under finite velocity of observation propagation (VOP) is obvious in learning without desired output. We take a man-to-man game called Renja in which probable progress is excluded and the past moves are all open like in Chess, Othello and Go. The number of elementary events of these games is too enormous to count up and players cannot know the best strategy or desired output with the games. They have to make a move 'at a wild guess'. If they are allowed to make a revisional move, they still have to make a wild guess in other ways. Then we can ready learning without desired output. We asked more than 200 persons to follow the procedure that a pair of men play a game once (it is called SG) till the terminal N-th step and after that the same pair replay the game starting from each step of SG, that is, they replay from the 1-st step of SG a certain of times, replay from the 2-nd step the same number of times, replay from the 3-rd step similarly and so on till the (N-1)-th step of SG. Analysis is composed of three parts. First, one suggests that the anticipatory system is transformed in replay. If uncertainty is excluded and the transformation system is definitely described, the second analysis suggests that the jump between local minimums is forced by fluctuation which is independent of local minimum solutions, if they exist. The third part of the analysis suggests that the driving force of the jump is dependent on local minimum solutions. Indirectly, though, such contradictive results suggest that the players' brains are under the uncertainty principle with finiteness of VOP. And fluctuation, which is derived from our attempt to explain the orbit of the game definitely, tends to be interpreted as somewhat independent of minimum solutions if they exist, but it occasionally seems to be somehow dependent on biological aspects.

key words: the uncertainty principle, learning, finite velocity of observation propagation, contradiction.

1. Introduction

We focus on the biological aspects in learning. Matsuno (Matsuno, 1989) experimentally showed that the velocity of observation propagation (VOP) is finite in the flagellar movement of Actin filament and spermatozoa of starfish. The experiential law, as the law of conservation, suggests the need of a time interval for the internal measurement which measures external fluctuations and fixes the amount of it as inputs (Matsuno, 1992). If the law, which derives from the uncertainty principle, is observed in the brain or the neural network,
biological learning has to be reexamined in terms of showing the different properties from mechanical learning while we still ignore the uncertainty in describing biological learning.

Whenever one describes the interaction of neural network, one has to take the space which inevitably features metric, whether the progress is going under finite or infinite VOP. Because metric prescribes objective measurement, the interaction, even if it is in very small distance, involves the idea of globalism. As for the category theory, metric is prescribed in the form a of product \( A \times B \) induced from the definition of limit \( \lim_{x} \rightarrow D \), in which the subcategory \( D = \{ A, B, id_{A}, id_{B} \} \), \( A \) and \( B \) are objects, and \( id_{A} \) (resp. \( id_{B} \)) shows identical morphism \( id_{A} : A \rightarrow A \) (resp. \( id_{B} : B \rightarrow B \)). Limit \( X \) suggests that the 'true measurement' can be realized which entails that an agent can observe \( D \) at a moment. It implies that the velocity of VOP is infinite. So, the computation \( f \), which operates the product \( X \), and product \( X \) should be identified at a moment, even if VOP is so small that the effect of it is negligible (Gunji & Nakamura, 1993).

If VOP is finite, the identification of \( f \) and \( X \) needs finite time and another input from the neighborhood can break the identification process. Thus, the uncertainty can be realized in principle. This is supported also by the vertical scheme in the neural network (Conrad, 1984). If one calls such process biological computation, the process can proceed in spite of the mixture of the identification and the computation, while it is not programmable computation. The mixture is always possible to occur. Of course, in many cases, one can make an adequate model fit, though approximately, to obtained data. If an approximated model is adopted as a somewhat spatial interaction, measurements under infinite VOP are always included while the approximation form is complicated. Here, the problem is whether we treat the outside derived from the approximation as mere fluctuation or not in biological aspects.

Cairnsian mutation (Cairns, et al., 1988; Hall, 1990) suggests that the mutation rate which is reduced to be independently separated from the environments depends on the environment. It looks as if the reproductive system could detect its surrounding environments. Because the idea of a system proceeding at a finite VOP leads to an idea of in-formed or generated information, it is expected that in a system with a finite VOP the information which is certainly identified \( a \ priori \) can be changed \( a \ posteriori \), which looks like a system can detect its surroundings. It entails that a system adapts to its environment. Therefore, as long as a system proceeds at a finite VOP, adaptation and/or learning is necessarily brought-forth. In other words, adaptability (Conrad, 1988) resulting from an \( a \ posteriori \) meaning-change is the most important property in an evolutionary generative system. Our main purpose is to extract the adaptability in the above context and to evaluate meaning-change in the learning process.

We adopted a man-to-man game called Renju. It is similar to Chess, Othello and Go with respect to the absence of an optimal strategy. It is also the most simple game of all four. The most specific character of learning in this game is the learning without desired output in which players, by themselves, estimate an orbit at a wild guess. In most cases, the number of solutions is too many to be countable with the same situation. Then the intra-system for anticipation cannot be identified from a given situation, and an external observer can ready the progress with the mixture of the identification of the anticipatory system and
the computation like under finite VOP (Nakamura, et al., 1992). We expect that with this game the effect of finite VOP in the players' brains is more distinct than learning with desired output if the progress of the game is decided by the interplay in the players' brains.

2. Renju

Renju (Go-bang) is a finite zero-sum game played by two players in terms of the game theory (Eigen, 1975). Like Chess, Go and Othello, the past moves are all open on the board and probable indeterminacy is excluded. Players make one move on the board in their turn. Both players and external observers can observe all of the moves made till the present time step, while in card games and mahjong each player has a different hand respectively (Matsuda, 1989). So, both of them can, in principle, count up all of the elementary events to happen.

Most of us have played Renju, Chess, Go, Othello, or other games. The reason why we did not adopt Chess, Go, or Othello in our experiments is discussed later. It is possible to know all of the elementary events at a given step, but it is too enormous to count them up. However, one experientially knows that he can play the game or make moves in any situation, by way of 'prediction'. To think about our performance of 'prediction', we will discuss it in a white with our own experience. One player may declare, for example, that the reason for one move in his turn is the precaution of the playmate's offence. In most cases, there can be other precautionary ways. Of course, he cannot know whether the move is only one or not, because he did not count all of the orbits up within a finite time. Even if one chooses the most effective move for a given situation, his own criteria of choice must not be proved in its own right nor be verified. Similarly, can one verify the criteria to judge whether the playmate's move is the most dangerous or whether the precaution should be prior to his own offence? While one cannot confirm his own 'prediction', he manages to make a move with expectation or hesitation. This is why games are amazing and interesting, for all probable indeterminacy is excluded. He does not know the optimal strategy and cannot count up all of the orbits, so he cannot help making each 'at a wild guess'. Thus, the identification of the system is destined to fail. While the 'prediction' is possible if any configuration is definite, a configuration in Renju is indefinite (Gunji & Nakamura, 1993). Thus we expect that the progress of Renju can show the characteristic variation derived from the uncertainty under finite VOP.

The discrepancy of wild guesses between players' can be exposed with the proceeding of game play. We may say that both players and observers cannot become aware of the discrepancy with respect to the meaning of a configuration and/or a move, till that move is taken. The player can a posteriori know the mistake of the wild guess in the past step. If the playmate was allowed to turn back to the past step, the player would take another move referring to the mistake. It should be remarked here that he would still make another wild guess because he knows the mistake about the belief in a unique orbit and he does not necessarily know a more hopeful move, far from the optimal move which implies the most effective move that is ideally assumed. Then, even if he makes another move from the past move, it is not until the playmate makes the next move that he knows whether his own move is more hopeful or not. Thus, the move taken at a wild guess will be revised still further by another wild guess. The experiment
was planned to extract the feature of this revision with wild guesses.

First we will explain the rules of Renju as:

1: Each player alternately makes a move, or puts one 'ishi', in his color, black or white, on a site on the board (two dimensional lattice, 15×15 or 17×17).

2: When a pieces of ishi in either player's color are arranged in a horizontal, perpendicular, or a diagonal line without an open site on the board, we call the sequence 'n-ren'. If '5-ren' is arranged, the game is over and the player wins (see Fig.1).

3: When 3 (resp. 4) pieces of ishi in a color are arranged in a line with one open site, we call the sequence 'tobi-3' (resp. 'tobi-4'). Player-1, who makes his move as the odd numbered turns with black ishi, is prohibited from arranging two '3-ren' or 'tobi-3' in one move. This prohibited move is called '3-3' and he will lose the game if he makes it. He is also prohibited from arranging '6-ren' or '4-4' (see Fig.2). Player-2 who makes his moves on the even numbered turns is not prohibited. The unbalance between the two players is not important in this work.

4: The most hopeful strategy for Player-1 is to form '4-3' (see Fig.2). For Player-2 it is to form '4-3', '3-3', '4-4', or to force Player-1 to make a prohibited move.

5: '3-ren', 'tobi-3', '4-ren', and 'tobi-4' are called 'oi-te' or the offensive moves. 'Misete' and 'Ryougatari' are also offensive moves. The preparatory move to form '4-3' (or 4-4)' in the next turn is the double purposeful 'Misete' for two different '4-3' (or 4-4)'. If one player continues to make '4-ren' or 'tobi-4' in his turns and at last he makes '4-3' or '4-4', the way to win is called '4-oigatari' (Sakata, 1989).

3. Method and Results

3.1. Method

Renju, as well as Chess, Othello or Go can ready learning without desired output. A player makes a move at a wild guess and a game proceeds. He is not stimulated by desired output, the difference from it, nor the experimenter. He must correct past unsatisfactory moves still by way of other wild guesses. If he is allowed to repetitiously correct many times, he must continue to make revisional moves at a wild guess. In this aspect one can find information generation due to the indefinite arrangement of ishi. Though a given arrangement of ishi at any step looks definite, the meaning of ishi is determined by the player's predictive system.

Let a set of configuration of ishi be \( A \). The prediction or anticipation can be defined by \( \phi: A \to \text{Hom}(A,A) \). However, the prediction depends on the meaning of configuration \( b \in B \), and should be defined \( B \to \text{Hom}(B,B) \). Hence the possibility of a predictive system depends on the possibility of a commutative diagram as,

\[
\begin{array}{ccc}
A & \xrightarrow{\text{Hom}(A,A)} & A \\
\downarrow & & \\
B & \xrightarrow{\text{Hom}(B,B)} & B
\end{array}
\]

Note that \( B \) can be determined by the anticipated path or the number of steps
used in the prediction if a player estimates that the move is determined only by
the anticipatory next step, \( E \approx \text{Hom}(A, A) \). Similarly, it is possible that
\( E \approx \text{Hom}(A, \text{Hom}(A, A)) \). Finally, one is determined to fall into an infinite
regression, unless he breaks the sequence

\[ A \rightarrow \text{Hom}(A, A) \rightarrow \text{Hom}(A, \text{Hom}(A, A)) \rightarrow \cdots \]

and identifies \( S \) with the resulting sequence. Definiteness in a configuration is
originated from such breaking and entails the aspect of information generation.

If we explain the first games (later it is called 'SG') by the player's
prediction which must be formally described such as the well-defined form of
\( \text{Hom}(A, \text{Hom}(A, A)) \), it is formed with fluctuation appearing in \( \text{Hom}(A, \text{Hom}(A, A)) \). The
explanation of the revision can also induce another fluctuation. It should be
interpreted as merely an error with mechanical learning, however, it can give
rise to a specific feature if it comes from our intention to describe learning
without desired output. Then we planned an experiment to make obvious the
revisional move which is performed still at another wild guess.

We asked some pairs of persons to take the following procedure;

1: Each pair of men play Renju once till the end. We call this game the
'Sample Game (SG)'. It ends in the \( N \)-th step.

2: The same pair replay the game \( K \) times from each \( i \)-th situation that is
the same as in SG \((K \geq 1, i \leq i(N-1))\). That is, after SG, they replay the game from
the \( 1 \)-st step of SG \( K \) times, replay the game from the \( 2 \)-nd step of SG \( K \) times,
replay from the \( 3 \)-rd step \( K \) times \( \cdots \) and replay from the \( (N-1) \)-th step \( K \) times.

See Fig.3. 'The \( i \)-th situation' consists of \( i \) pieces of 'ishi'. In the
\((i+1)\)-th step, the player in his turn makes a move referring to the \( i \)-th
situation. We call the replay from the \( i \)-th situation of SG 'i-Replay' and we
call the experiment with \( i \)-Replay repeated \( K \) times, '\( k \)-instant-i-Replay'.

Of course, in Replay, they can make another move from SG if they wish. As
mentioned above, the known orbits cannot always be the indication for the more
hopeful move so long as the player still makes a move at a wild guess in Replay.

To analyze Replay, we will define four measures for the orbit of Replay.
Partly because we experimenters do not know the optimal path for any situation
and partly because there exist numberless local solutions in a potential of
optimization, we do not measure the distance of the move from the optimal path
nor local solutions. We attend to offensive moves. We suppose that players wish
to win. Players cannot win without offensive moves though offensive moves do not
always cause them to win. So offensive moves are interpreted as necessary
conditions. In Replay, the same move as in SG is interpreted as meaningless
since the uncertain aspect from the wild guess in SG is replaced by another move
and we regard that the player does not 'learn' if he makes the same move. In \( i \)-
Replay, if the orbit reflects the uncertainty with the \( i \)-th situation, the
earlier move is more meaningful. Then the move in the \((i+j)\)-th step in \( i \)-Replay
should be estimated in inverse proportion to \( j \).

**Measure 1. Advantage**

\[ A(i, k) = \frac{\sum_{j=1}^{K} E(j)C(j)\delta(i, j)/m - E(j)C(j)\epsilon(i, j)/m}{m} \]

where \( E(j) = 1 \), when the move is offensive,
\( = 0 \), otherwise,
\[ C(j) = 1, \text{ when the move is different from the move in SG,} \]
\[ = 0, \text{ when the move is the same as in SG.} \]
\[ \delta(i,j) = (i+1) \mod 2, \]
\[ \epsilon(i,j) = (i+j) \mod 2, \]
\[ m = (j-1) \mod 2 + 1, \]

in the \((i+j)\)-th step in the \(k\)-instant-i-Replay, in which \(\%\) is the operation for the quotient. The symbol \(J\) represents the estimated length of the orbit in \(k\)-instant-i-Replay. Then \(A(i,k)>0\) shows that Player-1 is more offensive or gains an advantage to Player-2 till \(k\)-instant-i-Replay, and \(A(i,k)<0\) shows that Player-2 gains an advantage.

**Measure 2. Fraction of information**

\[ F(i,k) = \frac{\sum_{j=1}^{\delta(i,j)} E(j) C'(j)}{j}, \]

where \(C'(j) = 1, \text{ when the move is different from the move in SG and from the move in } p\)-instant-i-Replay \((1s<p<k)\), and \(C'(j) = 0, \text{ when the move is the same in SG or in } p\)-instant-i-Replay \((1s<p<k)\), in the \((i+j)\)-th step in the \(k\)-instant-i-Replay. \(E\) and \(J\) are the same as in Measure Advantage.

**Measure 3. Fraction of information for players**

\[ F_1(i,k) = \frac{\sum_{j=1}^{\delta(i,j)} E(j) C'(j)}{j}, \]
\[ F_2(i,k) = \frac{\sum_{j=1}^{\epsilon(i,j)} E(j) C'(j)}{j}, \]

where \(\delta(i,j)\) and \(\epsilon(i,j)\) are the same as shown in Measure 1, and \(E(j)\) and \(C'(j)\) are the same as in Measure 2. \(F_1\) (resp. \(F_2\)) indicates how offensive Player-1 (resp. Player-2) is, or how different the move is in \(k\)-instant-i-Replay.

**Measure 4. The amount of information for players**

\[ I_x(i) = \sum_{k=1}^{K} F_x(i,k), \]

where \(x=1,2\). \(K\) shows how many times the Replay was repeated.

\(T_p\) shows the step in which \(I_x\) is peaked, and \(T_0\) shows the step in which \(I_x=0\) \((x=1,2)\). \(Z_x(i)\) shows the summation of \(I_x\) from the step \(i\) to the nearest \(T_x\). \(S(T_p)\) shows the number of offensive moves from the step \(T_p\) to the nearest \(T_0\) in SG.

### 3.2. Results.

The experiments were done by university or high school students, in total more than 200 persons. Each player was familiar with Renju almost as well as the playmate. Some did not play the game and some did not understand the procedure.

We can see the obvious correlation between the sequence of the move in SG and the advantage \(A\) in Replay at \(k=1\). For example, the progress of SG in the example in Fig.3 can be represented as 'nnnnnnnn211nn121n1n' where 1 represents that Player-1 made an offensive move in the step, 2 represents Player-2 did and n represents the move is not offensive. The game
(SG) was terminated by '4-3' of the Player-1, when \(N=23\), at which Player-1 took '4-3'. In the 9-th step, Player-1 took the initiative firstly and in the 13-th, and 21-th step he turned the tables, and in the 12-th and 18-th step Player-2 turned the tables in reverse. The sign of \(A(i,1)\) is represented as

\[ '000-+---+++----+00' \]

through \(i(1\leq i \leq 22)\) with \(J=6\). This is shown in Fig. 4-a with an interrupted line. It should be remarked that \(A(i,1)<0\) when \(i=5, 9\) and 17 and that \(A(i,1)>0\) when \(i=8\) and 14. In this example, we can say that if in the \(t\)-th step either player took the initiative firstly or turned the tables in SG, in \(i\)-instant-(\(t-\tau\))-Replay the other player took the advantage. We can see the same correlation in Fig.4-b-e.

We could obtain, in total, 90 examples for the first initiative or the turning of the tables in SG. We introduce the notation \(t\) for the steps of the taking of the first initiative of the turn of the tables, and call \(k\)-instant-(\(t-\tau\))-Replay '\(\tau\)-rollback' if the \(t\)-th move in SG is represented as '+' and \(A(t-\tau, k)\leq 0\) is satisfied (resp. it is '-' and \(A(t-\tau, k)\geq 0\)). As for 91.1% of the data, \(\tau\)-rollback was successful when \(k=1\) and \(\tau=4\). The success explicitly shows that players somehow 'learn' as SG progresses.

One example of the change of the fraction of information \(F\) for \(k\) is shown in Fig.5. The obvious feature of the variation is as follows:

1: For \(k\), \(F\) does not seem to change remarkably, when \(i=1\) or 2.
2: \(F\) is rapidly saturated to become zero for several steps \(i\) before \(N=1\).
3: Further when \(i\) occurs earlier, \(F\) seems to be saturated once, dumped later and saturated again.
4: As SG proceeds (i.e. \(i\) increases), \(F\) tends to decrease with \(k\) more rapidly.

A remarkable feature is obvious in Fig. 5-c, in which \(K=20\) and \(N=20\). When \(i=14\), \(F\) keeps fluctuating, and peaks emerge several times. Especially at \(k=19\), \(F\) reaches the maximum value in 14-Replay. \(F\) is defined such as to be zero if the orbit of \(k\)-instant-\(i\)-Replay is the same as the SG orbit or that of \(p\)-instant-\(i\)-Replay (\(15p<k\)). Then the maximum value shows that the orbit of 19-instant-14-Replay is different from them.

Fig.6 shows an example of the variation of \(F_1\) and \(F_2\) through \(i\) for one SG. The upper figure in Fig.7 shows the correlation between \(Z(x(T_r))\) and \(|T_r-T_o|\) and the lower one shows the correlation between \(Z(x(T_r))\) and \(St(T_r)\) with both of the players(x=1,2).

4. Discussion

Here we make two different suppositions. 1: A player's move at the \(i\)-th step is a result of interplay in his brain receiving the (\(i-1\))-th situation and the past games as inputs. 2: An external observer can observe the players' brains at a moment and describe the form of the interaction in the brain or the anticipatory system. Supposition 1 cannot be excluded, whatever the results suggest, or we cannot say that players play the game. Supposition 2 is just convenient for description with limit.

The analysis is composed of three parts. First, we take the correlation between the turning of the tables in SG and the measurement Advantage at \(k=1\) and \(\tau=4\) for revisional moves. When the turning of tables occurs at the \(t\)-th step in one player's turn, \((t-\tau)\)-Replay starts from the playmate's turn if \(\tau\) is even. In
many times, rollback is not in time to form a Renju rule when \( r=2 \), and when \( r \) is very large (\( r \geq 6 \)) the replay orbit may be utterly far from the SG orbit. Other offensive moves can be overlooked in SG since the players make moves at a wild guess. We can narrowly say that \( r \)-rollback (\( r=4 \)) is the most probable if rollback can occur (Nakamura, et al., 1992). For all the playmate is destined to be less advantageous in the direction of SG, in many cases he can turn the tables in Replay. Then, the high rate of the success resulting in \( r \)-rollback (\( r=4 \)) implies that the anticipatory system is obviously transformed from SG to Replay. Secondly, we take the variants of the orbits by \( k \) times trial for each \( i \)-Replay. The gradient of the variation reflects the number of the elementary events for the \( i \)-th situation. We emphasize that a peak can appear after it is almost saturated when \( i \) is smaller than \( N \). Saturation occurs when the orbit in Replays follow the same as SG, the past trials involving SG, or the players abandon the game. It suggests the absence of newly emergent input. Since the number of orbits is finite, one has to take subsets of the orbits not at existing intersections and give a value to the subsets. We can say that given information is acquired through learning when the estimated values of the Replay orbits show monotonic increase (or decrease). If saturation is accompanied with the absence of new input, one can interpret that the peak occurs with the jump to another local minimum induced by fluctuation. We take the correlation among \( Zx(T_x), |T_x-T_0| \), and \( St(T_x) \). \( Zx \) is added when the orbit in \( i \)-Replay is a different form in SG. Then, the correlation suggests that the orbit in \( i \)-Replay always tends to keep away from the past orbits and to offend with the different moves.

The high rate of the success of \( r \)-rollback implies the transformation system under Supposition 1 and 2. The peak after the saturation suggests the difficulty in describing the transformation of a system a priori because we external observers cannot deny the emergence of other possible systems. Still, if the peak appears after transitory saturation repeating \( i \)-Replay 'enough' \( k \) times, it can be interpreted as a jump from a local minimum to another one in a given potential if it exists. One shall think that it is caused by fluctuation which is independent of the fitness landscapes of the potential. That is why we assume Supposition 1. The temporary saturation will show the fadeout of new input. The correlation among \( Zx(T_x), |T_x-T_0| \), and \( St(T_x) \) will show the tendency that the orbit of Replay avoids the past orbits and other offensive moves are found. Thus, players intend to break down past stalemate situations. This is supported by the fact that either player is necessarily less advantageous and is just stalemate in the saturated orbit. If he tries to win, he somehow attempts to break it down. So, we may say that the driving force to avoid the stalemate situation is always active in revisional moves. Thus, the driving force is dependent on the past orbits since they tend to be avoided.

Any symbolic data cannot directly induce contradiction with Supposition 2. Our way is indirect. First, the success of \( r \)-rollback suggests the existence of a hierarchical system which transforms the anticipatory system. The transformation will be derived from the players' experiences with respect to the past orbits. Secondly, the variation of the fraction of information through \( k \) suggests that there are some local minimum solutions in the landscape for the transformation system from the viewpoint of mechanical learning. The transitory saturation suggests the fadeout of new input. It gives rise to an idea that the driving force for the jump to another local minimum results from fluctuation.
which is independent of the orbits in the local minimum solutions. Finally, the correlation among $Z(t)$, $|T_s - T_0|$, and $S(t)$ suggests that players tend to break down stalemated situations. The orbit of a local minimum solution is always less advantageous for either player and the driving force to avoid it cannot be independent of orbits of local minimum solutions. Thus, the first and second analysis show that the driving force of the jump is fluctuation and it is independent of orbits of local minimum solutions as long as one sticks to the viewpoint of mechanical learning or assumes both Supposition 1 and 2. On the other hand, the third analysis suggests that the driving force is not independent. Thus, the interpretation of the two is contradictory.

Of course, one may construct the higher hierarchical system for the transformation system as the first one in order to explain obtained data. One may also construct a system with delayed memory. But, we are unwilling to construct it to make it fit to the obtained data. Our aim is to point out that the jump will occur with something more than mere fluctuation such as thermal noise. We are willing to suggest that Supposition 2 is virtual. We will shortly mention why we do not adopt the two ideas, a higher hierarchical system and a system with delayed memory. However highly the hierarchical system is constructed, we must estimate all of the orbits at the highest hierarchy to construct a definite learning system. Again, we have to take subsets of a set of orbits not at existing intersections and give values to the subsets since Renju is a finite game. So, we will think about the driving force for the jump from one subset to another without newly emergent input as well as the first transformation system. Then it is enough here to take the first hierarchical one. One can also construct a system with delayed memory to explain the peak without fluctuation. If so, the players can accept new input to escape from the saturated orbits. But, we must think that the players are so circuitous that they pretend not to know of the more hopeful moves in Replay whether the players are conscious of them or not. If we suppose that players abruptly remember a specific past orbit, of course we cannot help including a term of fluctuation into the description. Though it is definitely possible to construct a transformation system with delayed memory, keeping a clue in store does not match with our own experience with playing a game, especially our intention to win a game.

Indirectly, though, we can gain contradictory results between the analysis above. If Supposition 1 is incorrect, we cannot say that players play the game. Then we will be willing to point out that Supposition 2 is no more than virtual. We say that an external observer can occasionally meet the necessity for a higher hierarchical system or a memory store since he intends to explain with spatial interaction in the brain.

Review the reason why Renju

We here reexplain the reason why we adopted Renju with the analysis above. If the probable process is included in the progression of a game like a card game or mahjong, then in our experiment we external observers cannot restrict the acceptance of newly emergent inputs through repetitional games. Then, the change of $F$ may be explained merely with the change of input. In Renju, players will search for the most hopeful move and the orbit will be saturated through repetition. The new input for the players seems to disappear after saturation.
The rules of Chess, Othello and Go also omit the probable progress but they are more complicated. It should be remarked that either player can put 'ishi' on any open site on the board and a slight change of move triggers a remarkable change in the progress of Renju. On the other hand, in Chess, one can move a chessman only to certain sites following the rules and the turning of the tables is not frequent. A player's 'learning' is more obvious in Renju with τ-rollback (τ=4). Moreover, each chess piece is discriminate in its own role. In Renju, a player's 'ishi' is the same as others and data collection is more simple.

The knowledge of formulas will influences the progress in the other games. In Othello, the number of options for the next move is always small and the formulas can be known. The progress of Go is also influenced by the knowledge about the formulas. Most people do not know the formulas of Renju and the number of options for the next move is always much more. Players often fail to notice the more hopeful move. It may be found in Replay and can be shown as τ-rollback or the variation of F. Of course, the further more hopeful one may be unknown after a few times of Replay. Then 'discovery' is more possible in Renju. Further, the most practical reason is that it ends more quickly than Chess, Othello and Go. (In most cases, it ends at most in 40 steps and in about 10 minutes.)

5. Conclusion

To make obvious the effect of finite VOP, we take a learning experiment without desired output. In Renju, players will make moves at a wild guess since they cannot count up all of the elementary events and they cannot know desired output. It shows that the mixture between the identification and the computation of the anticipatory system will be found once one tries to formally describe the system and that it results that the effect of finite VOP in the players' brains. Players play the game till the terminal (N-th) step, and they are allowed to replay it from the i-th situation (i is (N-1)). Since, in replay, players are not stimulated by desired output, the difference from it, nor experiment, they must still make revisional moves at a wild guess in other ways.

The success of τ-rollback suggests that the anticipatory system itself is transformed among the first game (SG) to Replay. When replay is repeated several times, players will make more hopeful moves and in most cases the orbit will be saturated. We can occasionally see deviation after temporal saturation as the peak of F. Saturation of the orbit accompanies the absence of newly emergent input and one can interpret it such that deviation is the jump from a local minimum to another local minimum, and that the driving force of the jump is fluctuation. Fluctuation must be independent of the local minimum solutions in its own right. But the correlation among \(Z(x(T_0), T_n - T_0)\) and \(S(t(T_0))\) suggests the intention to break down past stalemates situations, which implies information generation or decision change (Matsuno, & Lu, 1989). The less advantageous player will attempt to break down the stalemate situation in the saturated orbit even though he cannot in most cases. Then the driving force of the jump, itself, depends on the local minimum solutions. Such contradictory results suggest the effect of finite VOP or the uncertainty principle. We can also say that under finite VOP we often introduce fluctuation since we intend to explain it reductively with a definite description. In the experiment, fluctuation can occasionally seem to play some positive role a posteriori.
Matsuno designed the experiments to extract the generated information \textit{a posteriori} in the brain as the natural language processor (Matsuno \& Lu, 1989). The meaning change between the first reading and the second trial shows that subsequent disambiguation in the first reading is surely referred to in the second reading \textit{a posteriori}. In the \textit{Renju} experiment, the correlation among $\text{Z}(T_r)$, $|T_r-T_0|$ and $\text{S}(T_r)$ also shows the reference to past orbits in revisional moves. Matsuno also designed the experiments to estimate the manageable average number of different meanings per word with reading incomplete sentences and concluded non-programmability of computation proceeding in the brain (Matsuno \& Lu, 1991; Matsuno, 1992). \textit{Renju} can readily almost numberless orbits in most situations and more hopeful orbits can be found in revisional moves. The enormous number of orbits is expected to impose upon players the hard burden of finding more hopeful moves as well as incomplete sentences in the lexical experiment. As mentioned above, the criteria of 'hopeful' is quite ambiguous since players cannot count up all of the elementary events, and the \textit{Renju} experiment can ready the circumstance in which meaning change can occur in any time. If we define the way to cut the infinite regression sequence $\text{A} \rightarrow \text{Hom}(A,A) \rightarrow \text{Hom}(A,\text{Hom}(A,A)) \rightarrow \ldots \ldots$ and take mechanical learning with a finite hierarchy to explain the variation of orbits, we will be unable to focus on the posterior meaning change. In \textit{Renju} experiment we will confront the contradictory results above, since the revisional moves can reflect the posterior meaning change.

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\textbf{References}


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Fig. 1: a, b and c show '5-ren' for Player-1 respectively in the horizontal, perpendicular and diagonal direction. Player-1 wins in these games. In d, Player-2 arranges '5-ren' in the horizontal direction.
Fig. 2: a shows respectively '3-ren', 'tobi-3', '4-ren' and 'tobi-4' for Player-1. b shows '4-3' for Player-1. The marked 'ishi' shows the current move and it forms '4-ren' and '3-ren' at once. In c, the above situation shows that '3-3' for Player-2 is formed. The low situations show respectively, that '6-ren' and '4-4' for Player-1 are arranged. If Player-1 takes the marked move in the left formation in d, it is called 'Misete'. In the right formation, it is called 'Ryougatai'. 
Fig. 3: One example of SG and the Replay (Player-1 took '4-3' in the 23-th step). The white letter against the black circle shows Player-1's move. The number is the order of the step. The black letter is Player-2's move. They start i-Replay from the i-th situation of SG(1≤i≤N-1).
Fig. 4: Five examples for the change of $A(i,1)$ through $i$ for one SG and the Replays. The real line, broken line and interrupted line show, respectively, the values for $J=2$, $J=4$ and $J=6$. Suppose that Player-1 took the initiative or turned the table in the $t_1$-th step and Player-2 did so in the $t_2$-th step. The black stars are plotted at $i=t_1-4$. The arrow shows that $A(t_1-4,1)<0$. The white stars are plotted at $i=t_2-4$ and $A(t_2-4,1)>0$. 
Fig. 5: Three examples for the change of $F(i,k)$ through $k$ for each $i$-Replay. In a $K=10$, in b $K=10$ and in c $K=20$. 
Fig. 6: Two examples of the change of $F_1(i,1)$ and $F_2(i,1)$ through $i$ (The upper figure is in the same SG and Replay as in Fig. 3 or Fig. 4-1. The lower one is in the same as in Fig. 4-v). $T_s$ shows the peaked step with $F_1$ or $F_2$. $T_s$ for $F_1$ are 5, 8, 10, 12, 14, 16 and 20. $T_s$ for $F_1$ are 17 and 21.
Fig. 7: The upper figure shows the correlation between $Z_x$ and $|T_p - T_o|$. The lower one shows the correlation between $Z_x$ and $St(T_p)$. More than 120 points are plotted.