The Japanese Subcontracting System, 
Competition and Asymmetric Equilibrium

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Abstract

This paper examines two manufacturers’ competition with their own affiliated supplier in duopoly. Each manufacturer purchases intermediate goods from its own affiliated supplier. A cost reduction investment takes place before the intermediate goods are produced by each supplier. We illustrate the asymmetric equilibrium, in the sense that the intermediate prices paid from manufacturers to suppliers are different. The asymmetric equilibrium arises in the efficient environments of the cost reduction investment. Under the asymmetric equilibrium, a manufacturer setting a lower component price has a competitive advantage. We also explain two interesting results of a comparative analysis. One is that the larger the demand becomes, the less output the advantageous manufacturer produces, while the more output the disadvantageous manufacturer produces. The other is that the worse the cost condition becomes, the more output the advantageous manufacturer produces, while the less output the disadvantageous manufacturer produces.

JEL classification: L13, L25, L62

Keywords: Japanese Subcontracting System; Asymmetric Equilibrium; Competitive advantages

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1 Introduction

Heterogeneous phenomena are widely spread in our real world. Therefore, economists have been studied and considered as an important area of their research. Empirical studies also suggest that firms, which have different technologies, sizes, capacity, and strategies, and so on, compete in the same industry. The phenomenon arises even within a firm. For example, some employees are facing on strong incentives, while the others are provided with weak incentives. To our regret, such differences have been mainly taken as exogenous rather than endogenous variables in theoretical economics. So, this paper addresses this logical gap in a duopoly.

This paper deals with a competition between two manufacturers within the Japanese subcontracting system. The Japanese subcontracting system is characterized by a cooperative relationship between the manufacturer and the supplier. In addition, the supplier’s skill-and-ability plays a greatly important role in the manufacturer’s performance4.

The result of this paper in the vein of earlier contributions that have sought to explain intraindustry heterogeneity of firms in environments where firms have identical opportunity sets. For instance, Mills (1990) demonstrates that heterogeneous plant sizes can emerge in equilibrium capacity expansion of a growing industry, even though scale economies give larger plants a unit cost advantage. Salop and Stiglitz (1977) examine the consequences of imperfectly

4 See Asanuma (1985a, 1985b), Womack et al. (1990), Cusmano and Takeishi (1991), and Nishiguchi (1994) for the Japanese subcontracting system. They show the importance of a cooperative relationship between automakers and suppliers as well as of the suppliers’ skill-and-ability in the Japanese subcontracting system. In particular, the supplier’s skill-and-ability in this paper focuses on the cost reduction investment adopting prior to producing the intermediate. Each manufacturer provides its affiliated supplier with several supports to accumulate and upgrade the supplier’s skill-and-ability. We regard the manufacturer’s support as a pecuniary transfer, $F$, from the manufacturer to the supplier.
informed consumers in a ‘tourists and natives’ model, where some firms choose high prices and small scale by catering to the poorly informed consumer segment, while other firms choose low prices and large scale as they attract well-informed consumers.

Under non convexity attributed to the moral hazard between principal and agent, the best response to other firms providing strong incentives can be to provide weak incentives (Hermalin, 1994). Under the cost tradeoff between the fixed cost and the variable cost, if their technology set is insufficiently convex, heterogeneous equilibrium is attainable (Mills and Smith, 1996). They also showed that uncertainty about demand or costs favors the emergence of heterogeneous firms. From the social welfare’s standpoint, equilibria tend to have too little heterogeneity. Gal-Or (1999) considered an oligopoly market competing with differentiated but competing products. If the demand between two products is moderately correlated, asymmetric equilibrium may arise; one firm establish its own sales force, while the other has its independent sales force. He also showed that vertical separation is more likely than vertical integration when their products are highly substitutable.

This paper is summarized as follows; this paper shows that asymmetric equilibrium, in the sense that the intermediate goods (parts or component) prices paid from manufacturers to suppliers are different. The asymmetric equilibrium arises in the efficient environments of the cost reduction investment. Under the asymmetric equilibrium, a manufacturer setting a lower component price has a competitive advantage in duopoly. This paper also suggests two interesting results of a comparative analysis. One is that increase in demand makes the advantageous manufacturer, who sets the price of the intermediate to lower, reduce its Cournot equilibrium output, whereas it inspires the disadvantageous manufacturer to increase its Cournot equilibrium.

5 The model is similar to our model. The main difference between ours and theirs is that the technology set in our model is continuous, while their technology set has only two cases; high and low technology set.
output. The other is that the worse the cost condition becomes, the more output the advantageous manufacturer produces, while the less output the disadvantageous manufacturer does.

The rest of this paper is organized as follows. Section 2 describes the model. In section 3, it is shown that asymmetric equilibrium exists in a duopoly. Section 4 characterizes equilibria and deals with a comparative analysis. Concluding remarks are in Section 5.

2 The Model

Consider two manufacturers producing a homogeneous final product. The inverse demand function is specified as follows:

\[ p = a - b(q_i + q_j) \]  

where \( p \) is the price of the final goods, \( a \) and \( b \) are positive constants, \( q_i \) and \( q_j \) are the output of each manufacturer.

Each manufacturer purchases an intermediate for the final good from its own affiliated supplier. We assume that a procurement contract between them consists of a unit of the parts price \( w \) and a pecuniary transfer \( F \). To put it precisely, when each manufacturer \( kA(k=i,j) \) offers the procurement contract to its supplier \( kS(k=i,j) \), it proposes the contract consisting of the intermediate price \( w_k \) and the pecuniary transfer \( F_k \). For simplicity, we regard the pecuniary transfer as a lump-sum payment.

A cost reduction investment takes place before the intermediate goods are produced in this model. The investment can decrease the marginal cost of the intermediate. For a complete explanation, suppose that the investment amount is \( x_k \). Then, the marginal cost becomes \( c - x_k \). The investment costs \( tx_k^2 / 2 \) where \( t \) is a strictly positive constant. For simplicity, the cost of transforming the

\[ \text{See Nariu et al. (2009) for details.} \]
intermediate goods into the final goods is normalized to zero. We also assume that each unit of the final good requires exactly one unit of the intermediate goods.

For the conditions that the S.O.C is satisfied and all variables are non-negative, specifically, these assumptions take the following forms:

**Assumption 1.** \( bt > 1. \)

**Assumption 2.** \( \frac{2}{5} \leq \frac{a}{c} \leq \frac{5bt}{2}. \)

A more detailed description of the timing of the three-stage game is given as follows: In stage 1, each manufacturer offers a take-it-or-leave-it contract to its Keiretsu supplier. In stage 2, each supplier determines its cost-reducing investment. Then, each manufacturer chooses his output level in stage 3. The payments and the intermediate goods are transferred between them according to the initial contract between stage 2 and stage 3. We focus on sub-game perfect equilibria for this game.

3 The Analysis

In the third stage, manufacturer \( iA \) chooses an output level for the final product in order to maximize its profit given the output of a rival firm. Then, firm \( iA \)'s maximization problem is:

\[
\max \quad \pi_{iA} = (a - b(q_i + q_j) - w_i)q_i - F_i, \quad \text{w.r.t.} \quad q_i, \quad i, j = 1, 2; i \neq j.
\]

where the subscript \( A \) denote the manufacturer.

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\( ^7 \) The manufacturer offers to its supplier a procurement contract which includes the price of an intermediate good \( w_i \) and a pecuniary transfer \( F_i \).
From the first-order condition that \( \partial \pi_i / \partial q_i = a - 2bq_i - bq_j - w_i = 0 \), the reaction function is given by

\[
q_i (q_j) = (a - bq_j - w_i) / (2b)
\]

The above two reaction functions yield the equilibrium outputs as solutions to the third-stage game:

\[
q_i = \frac{(a - 2w_i + w_j)}{3} \quad (3-1)
\]
\[
q_j = \frac{(a - 2w_j + w_i)}{3} \quad (3-2)
\]

The price for the final good and the manufacturers’ payoffs are obtained by substituting Eq. (3-1) and Eq. (3-2) into Eq. (1) and Eq. (2):

\[
p = \frac{(a + w_i + w_j)}{3}, \quad i, j = 1, 2; \ i \neq j. \quad (4-1)
\]
\[
\pi_{ii} = \frac{(a - 2w_i + w_j)^2}{9b} - F_i, \quad i, j = 1, 2; \ i \neq j. \quad (4-2)
\]

What is important to note from Eq. (4-1) is that

\[
q_i \preceq q_j \iff w_j \preceq w_i
\]

Eq. (4-2) show how the marginal procurement cost \( w_i \) affect their output levels. Concisely speaking, the higher firm \( i \)’s marginal procurement cost \( w_i \) is, the more firm \( j \)’s output \( q_j \) is, and vice versa. It is also worth noting from Eq. (3-1), Eq. (3-2) and Eq. (4-1) that
\begin{align*}
q_i > 0 & \iff w_i < (a + w_j)/2 \iff p > w_i \\
q_j > 0 & \iff w_j < (a + w_i)/2 \iff p > w_j
\end{align*}

The above equations imply that the price for the final product is higher than the marginal procurement cost of each firm in order to have a positive output level.

We turn then to the second stage game. In stage two, supplier \( iS \) makes an investment for reducing its marginal cost before the intermediate goods are produced. Therefore, supplier \( iS \) chooses the investment level \( x_i \) in order to maximize its profit. Supplier \( iS \)'s maximization problem is

\[
Max \; \pi_{iS} = \frac{(w_i - c + x_r)(a - 2w_i + w_j)}{3b} - \frac{\tau_i^2}{2} + F_i, \quad \text{w.r.t.} \; x_i \tag{5}
\]

This yields the Cournot-Nash equilibrium investment level as solutions to the second-stage game:

\[
x_i(w_i, w_j) = \frac{(a - 2w_i + w_j)}{3bt} \tag{6-1}
\]

The second-order condition is given by

\[
\frac{\partial^2 \pi_{iS}}{\partial w_i} = t > 0.
\]

The marginal cost and the payoff for supplier \( iS \) are obtained by substituting Eq. (6-1) into supplier \( iS \)'s marginal cost function and Eq. (5):

\[
c_i(w_i, w_j) = \frac{(3bct - a + 2w_i - w_j)}{3bt} \tag{3-6-2}
\]
From Eq. (6-1), the investment level $x_i$ of firm $i$ is affected by firm $j$'s investment level $x_j$ positively, and vice versa. Concisely speaking, Eq. (6-1) implies that the higher the intermediate procurement price $w_i$ is, the less supplier $i$'s investment amount $x_i$ is, while the more supplier $j$'s is, and vice versa.

We now turn to the first stage game. Manufacturer $iA$ chooses intermediate goods price $w_i$ and pecuniary transfer $F_i$ to maximize its own profit given two constraint conditions that its supplier $i$'s profit and intermediate goods price $w_i$ are nonnegative. Manufacturer $iA$’s maximization problem is

$$\pi_{iA} = \frac{(a - 2w_i + w_j)^2}{9b} - F_i, \text{ w.r.t. } w_i \text{ and } F_i$$

subject to

$$\pi_{iS} = \frac{(a - 2w_i - w_j)(a - 2w_i + w_j + 6bt(w_i - c))}{18b^2t} + F_i \geq 0, \text{ and } w_i \geq 0$$

Note that the first constraint condition is binding. Therefore, the Eq. (7) can be reduced as follows:

$$\pi_{iA} = \frac{(a - 2w_i + w_j)^2}{9b} + \frac{(a - 2w_i + w_j)(a - 2w_i + w_j + 6bt(w_i - c))}{18b^2t}, \text{ w.r.t. } w_i$$

subject to

$$w_i \geq 0$$

The first-order conditions\textsuperscript{8} is given by

\textsuperscript{8} The second-order condition $\frac{\partial^2 \pi_{iA}}{\partial w_i^2} = \frac{-4(bt - 1)w_i}{9b^2t} < 0$ is satisfied by the Assumption 1 ($bt > 1$).
\[
\frac{\partial \pi_{i\Delta}}{\partial w_i} = \frac{\{6bt - a(bt + 2) - 4(bt - 1)w_j - (bt + 2)w_j\}}{9b^2t} \leq 0,
\]

\[
\begin{pmatrix}
\frac{\partial \pi_{i\Delta}}{\partial w_i}
\end{pmatrix} w_i = 0, \text{ and } w_i \geq 0
\]

It is worth noting that \(\frac{\partial \pi_{i\Delta}}{\partial w_i} = \frac{\{6bt - a(bt + 2) - (bt + 2)w_j\}}{9b^2t}\) if \(w_i = 0\). Therefore, the reaction functions can be rewritten as:

\[
w_i(w_j) = \frac{\{6bt - a(bt + 2) - (bt + 2)w_j\}}{4(bt - 1)} \text{ if } 6bt - a(bt + 2) \geq (bt + 2)w_j \quad (8-1)
\]

\[
w_i(w_j) = 0 \quad \text{if } 6bt - a(bt + 2) < (bt + 2)w_j \quad (8-2)
\]

It is also worth noting that if \(6bt - a(bt + 2) < 0\), Eq. (8-2) should be satisfied for given \(w_j \geq 0\). Therefore, the case corresponds to a corner solution:

\[
w_i^O = w_j^O = 0 \quad (9)
\]

where the superscript \(O\) denotes corner solution.

If, on the other hand, \(6bt - a(bt + 2) \geq 0\), the reaction functions are obtained from Eq. (3-8-1), respectively, as follows:

(1) \(w_i(w_j = 0) = \frac{6bt - a(bt + 2)}{4(bt - 1)} > 0\) and

(2) \(\frac{w_i}{w_j} = -\frac{(bt + 2)}{4(bt - 1)} < 0\), \(\frac{w_i}{w_j}[-<>]-1\) iff \(bt[<>]2\).

[Figure 2 here]

Figure 2 shows the reaction curves in \(1 < bt < 2\) that support the various equilibria.
Note that the intersection of two reaction curves is a symmetric equilibrium but it is unstable. Two points, which corresponds to \( \{0, \frac{6bct-a(bt+2)}{4(bt-1)}\} \) and \( \{\frac{6bct-a(bt+2)}{4(bt-1)}, 0\} \), of the vertical and the horizontal axis support asymmetric equilibria.

If \( bt>2 \), the equilibrium intermediate goods price is, therefore, the intersection of two reaction curves as follows:

\[
 w_i^* = w_j^* = \frac{6bct - a(bt + 2)}{5bt - 2}.
\]

Figure 3-3 shows the reaction curves in \( bt>2 \) that support a symmetric equilibrium. Note that the equilibrium of \( \frac{6bct-a(bt+2)}{(5bt-2)} \) is stable.

Let us then see the case that \( bt=2 \). We know that two reaction curves are completely identical. Therefore, multi equilibria occur in this case.

We turn to the case that \( 1<bt<2 \). If \( w_i(w_j)=0, w_j \geq \frac{6bct-a(bt+2)}{(bt+2)} \) should be satisfied. This lead to two asymmetric equilibria in the sense that each manufacturer set its intermediate goods price to be different:

\[
\begin{align*}
 w_i^{**} &= 0 \\
 w_j^{**} &= \frac{6bct - a(bt + 2)}{4(bt - 1)}, \quad i, j = 1, 2; \quad i \neq j.
\end{align*}
\]

We also obtain another equilibrium that two reaction curves, \( w_i(w_j) \) and \( w_j(w_i) \), intersect at \( w_i = w_j = \frac{6btc-a(bt+2)}{(5bt+2)} \).

Putting what we mentioned above together, when \( 1<bt<2 \), there exist three

\[9 \text{ It is worth noting that if } bt<2, \frac{6btc-a(bt+2)}{4(bt+1)} \geq \frac{6btc-a(bt+2)}{(bt+2)}.\]
Cournot-Nash equilibria.

\[
\begin{align*}
    w_i^* = w_j^* &= \frac{6bt - a(bt + 2)}{5(bt - 2)} \\
    (w_i^{**}, w_j^{**}) &= \left( 0, \frac{6bt - a(bt + 2)}{4(bt - 1)} \right) \\
    (w_i^{**}, w_j^{**}) &= \left( \frac{6bt - a(bt + 2)}{4(bt - 1)}, 0 \right)
\end{align*}
\] (10-1) (10-2) (10-3)

However, it is worth noting that the symmetric equilibrium described by Eq. (3-10-1) is unstable in the interval that \(1 < bt < 2\).

**Proposition 1:** Given Assumption 1 and Assumption 2,
1. If \(6bt - a(bt + 2) < 0\), there exists a corner solution, \(w_i^0 = w_j^0 = 0\).
2. If \(6bt - a(bt + 2) \geq 0\),
   (a) There exists a symmetric equilibrium \(w_i^* = w_j^* = \frac{6bt - a(bt + 2)}{(5bt - 2)}\) when \(bt > 2\).
   (b) There exist multi equilibria when \(bt = 2\).
   (c) When \(1 < bt < 2\), there exist two stable asymmetric equilibria,
      \[(w_i^{**}, w_j^{**}) = \left( \frac{6bt - a(bt + 2)}{4(bt - 1)}, 0 \right) \text{ and } (w_i^{**}, w_j^{**}) = \left( 0, \frac{6bt - a(bt + 2)}{4(bt - 1)} \right)\].

The regions that support asymmetric equilibria, \(1 < bt < 2\), means the efficient environments of the investment. To put it concisely, as \(b\) is the slope of demand curve and \(t\) is the efficiency parameter of the investment, the asymmetric equilibria are likely to happen when the slope of demand is gentle or the efficiency parameter of the investment is small.

4 Comparative Analysis in Equilibria

In the previous section, we illustrated that the asymmetric equilibria arise in
1<bt<2. To begin with, we analyze comparative statics in equibria.

4.1 Corner Solution
As we explained in Proposition 1, if \(6bct - a(bt + 2) < 0\), the equilibrium intermediate goods prices are \(w_i^0 = w_j^0 = 0\). Substituting them into Eq. (4-1), Eq. (4-2), Eq. (4-3), Eq. (6-1), Eq. (6-2), and Eq. (6-3), we obtain equilibrium investment amount, equilibrium marginal cost, equilibrium output amount, equilibrium final goods price, equilibrium payoff, and equilibrium pecuniary transfer, respectively

\[
x_i^0 = x_j^0 = a/3bt \quad (11-1)
\]
\[
c_i^0 = c_j^0 = (3bct - a)/3bt \quad (11-2)
\]
\[
q_i^0 = q_j^0 = a/3b \quad (11-3)
\]
\[
p^0 = a/3b \quad (11-4)
\]
\[
\pi_{iA}^0 = \pi_{jA}^0 = \frac{a\{(2bt+1)a - 6bct\}}{18b^2t} \quad (11-5)
\]
\[
F_i^0 = F_j^0 = \frac{a(6bct - a)}{18b^2t} \quad (11-6)
\]

Now, we easily check that the optimal values of all variables obtained above are non-negative under Assumption 1, Assumption 2, and the condition for the corner solution which is \(6bct < a(bt+2)\). From Assumption 2, in fact, it can be proved to be \(c_i^0 = c_j^0 > 0\). Note that \((bt+2) < (2bt+1)\). Then, it is obvious that \(\pi_{iA}^0 = \pi_{jA}^0 > 0\) from the condition that \(6bct < a(bt+2)\).

We now turn to the comparative statics. The parameter \(a\) shifts demand up and down. Increase in a demand \((a)\) induces suppliers to make more investments. It also causes the price of the final goods to increase. More investments will induce manufacturers to produce more outputs and to gain more payoffs. Suppose that the parameter of cost \((c)\) shifts upward. The rise in the parameter of cost \((c)\) will reduce their payoffs as a result of increments of
their pecuniary transfer. However, note that it does not directly affect equilibrium investment level, equilibrium output level, and price of final good.

4.2 Symmetric Equilibrium

Under the conditions that \( \text{bt} > (\text{bt}+2) \) and \( \text{bt} > 2 \), Cournot equilibrium intermediate goods price is

\[
\hat{w}_i = \hat{w}_j = \frac{6\text{bt} - \text{a}(\text{bt} + 2)}{5\text{bt} - 2}
\]  

Substituting it into Eq. (4-1), Eq. (4-2), Eq. (4-3), Eq. (6-1), Eq. (6-2), and Eq. (6-3), we have equilibrium investment level, equilibrium marginal production costs, equilibrium output amount, equilibrium final goods price, equilibrium payoff, and equilibrium pecuniary transfer, respectively

\[
x_i^* = x_j^* = \frac{2(a - 2c)}{5\text{bt} - 2}
\]  

\[
c_i^* = c_j^* = \frac{5\text{bt} - 2a}{5\text{bt} - 2}
\]  

\[
q_i^* = q_j^* = \frac{2t(a - 2c)}{5\text{bt} - 2}
\]  

\[
p_i^* = p_j^* = \frac{(bt - 2)a + 4\text{bt}c}{5\text{bt} - 2}
\]  

\[
\pi_i^* = \pi_j^* = \frac{2t(bt - 1)(a - c)^2}{(5\text{bt} - 2)^2}
\]  

\[
\pi_{iA}^* = \pi_{jA}^* = \frac{2t(bt + 1)(a - c)^2}{(5\text{bt} - 2)^2}
\]

Now, we can easily check that the optimal values of all variables obtained above are non-negative under Assumption 1 and Assumption 2. Let us see the
effect of the demand parameter (a) on all the optimal variables. Increasing the demand parameter (a) will reduce the intermediate goods price and the marginal cost. If there is an increased demand for the final goods, each manufacturer will increase its output in order to acquire more profits. It induces each supplier to increase the cost reduction investment. The incremental investment results in decreasing in the marginal cost and the intermediate goods price. From the fact that \( c_i^* - w_i^* = \frac{bt(a - c)}{(5bt - 2)} > 0 \), each manufacturer sets the intermediate goods price to be lower than the marginal cost. Increasing the demand parameter (a) will increase output amount, investment level, and payoff.

Next, suppose that the parameter of cost (c) shifts upward. It is also obvious that increase in cost parameter (c) will decrease output amount, investment level, and payoff. Furthermore, increasing the demand parameter (a) and the cost parameter (c) will rise up the final goods price.

4.3 Asymmetric Equilibrium

Under the conditions that \( 6bct > a(bt+2) \) and \( 1 < bt < 2 \), there exist two stable asymmetric equilibria. Substituting it into Eq. (4-1), Eq. (4-2), Eq. (4-3), Eq. (6-1), Eq. (6-2), and Eq. (6-3), In equilibrium, the intermediate goods price, the investment levels, the marginal production costs, the output amounts, the final goods price, manufacturers’ payoffs, and pecuniary transfers is given by

\[
\begin{align*}
w_i^{**} &= \frac{6bct - a(bt + 2)}{4(bt - 1)} \\
w_j^{**} &= 0 \\
x_i^{**} &= \frac{a - 2c}{2(bt - 1)} \\
x_j^{**} &= \frac{2bct - a(2 - bt)}{4bt(bt - 1)}
\end{align*}
\]
\[
c_i^{**} = \frac{2bct - a}{2(bt - 1)}
\]
(13-5)

\[
c_j^{**} = \frac{a(2 - bt) - 2bct(3 - 2bt)}{4bt(bt - 1)}
\]
(13-6)

\[
q_i^{**} = \frac{t(a - 2c)}{2(bt - 1)}
\]
(13-7)

\[
q_j^{**} = \frac{2bct - a(2 - bt)}{4b(bt - 1)}
\]
(13-8)

\[
p^{**} = \frac{2bct - a(2 - bt)}{4(bt - 1)}
\]
(13-9)

\[
\pi_{iA}^{**} = \frac{(a - 2c)^2}{8(bt - 1)}
\]
(13-10)

\[
\pi_{jA}^{**} = \frac{\{2bct - (2 - bt)a\} \{2bt(5 - 2bt)c - (2 - bt)(2bt + 1)a\}}{32b^2t(bt - 1)^2}
\]
(13-11)

\[
F_i^{**} = \frac{t(bt + 1)(a - 2c)^2}{8(bt - 1)^2}
\]
(13-12)

\[
F_j^{**} = \frac{\{2bct - (2 - bt)a\} \{2bt(4bt - 5) + (2 - bt)a\}}{32b^2t(bt - 1)^2}
\]
(13-13)

Now, we can easily check that all optimal variables obtained above are non-negative under Assumption 1, Assumption 2, and the condition for asymmetric equilibria, \(6bct > a(bt+2)\). When manufacturer \(jA\) sets the intermediate goods price to be zero and manufacturer \(iA\) sets it to be positive, the characteristics of the asymmetric equilibrium can be described as follows.

**Proposition 2:** Given \(1 < bt < 2\), \(6bct > a(bt+2)\), Assumption 1, and Assumption 2, the asymmetric equilibria are characterized by;

1. \(x_j^{**} > x_i^{**}\)
2. \(q_j^{**} > q_i^{**}\)

\(^{10}\) See Appendix for a detail.
3. $c_i^{**} > w_i^{**}$ and $c_j^{**} > w_j^{**} = 0$

4. $\pi_{jA}^{**} > \pi_{iA}^{**}$

**Proof**

Under the conditions that $6btc > (bt+2)a$ and $1 < bt < 2$, we have

$$x_j^{**} - x_i^{**} = \frac{6btc - (bt + 2)a}{4bt(bt-1)} > 0 \quad (14-1)$$

$$q_j^{**} - q_i^{**} = \frac{6btc - (bt + 2)a}{4b(bt-1)} > 0 \quad (14-2)$$

$$c_i^{**} - w_i^{**} = \frac{(bt(a-2c) + a)}{4bt(bt-1)} > 0 \quad (14-3)$$

Furthermore, under $a > 2c$, note that

$$2bt(3-2bt)c - (2 + bt - 2bt)c = (6btc - (bt+2)a) + 2b^2t^2(a-2c) > 0.$$ 

Then, we have

$$\pi_{jA}^{**} - \pi_{iA}^{**} = \frac{6btc - (bt + 2)a}{32b^2t^2(bt-1)^2} (2bt(3-2bt)c - (2 + bt - 2bt)c) > 0 \quad (14-4)$$

In the end, manufacturer $jA$ who sets the intermediate goods price to be zero enjoys more profit, output, and investment than those of manufacturer $iA$ who sets the intermediate goods price to be positive $12$.

Let us see the effect of some parameters on the asymmetric equilibria values. Suppose that the parameter of demand ($a$) shifts upward. Manufacturer $iA$ with a positive intermediate goods price will increase its output amount through setting the intermediate goods price down. Increasing the output amount induces its supplier $iS$ to make more aggressively in cost reduction investment. More investment not only will result in the marginal production cost to come

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11 It is obvious that $c_j^{**} > w_j^{**} = 0$.

12 Note that $c_j^{**} < c_i^{**}$. 
down but also that manufacturer $iA$’s payoff to increase. Unlike manufacturer $iA$’s positive response to demand parameter’s increase, manufacturer $jA$ with a zero intermediate goods price reduces its Cournot equilibrium output amount. Note that manufacturer $iA$ and manufacturer $jA$ are in Cournot competition by changing their intermediate goods prices. When demand increases, manufacturer $jA$ can not decrease its intermediate goods price down because it already set a zero. Another important thing is that increase in demand parameter ($a$) decreases the price for final goods.

Secondly, let us see the effect of cost condition ($c$) on all variables. Suppose that cost condition ($c$) shifts upward. It will increase the intermediate goods price for manufacturer $iA$. Therefore, it will decrease Cournot equilibrium output amount for manufacturer $iA$. It will induce its supplier $iS$ not only to decrease investment level but also to increase marginal production cost. In the end, the payoff for manufacturer $iA$ will decrease. Unlike decreasing Cournot equilibrium output as manufacturer $iA$’s response to worse cost condition, manufacturer $jA$ with a zero intermediate goods price will increase its Cournot equilibrium output level because it takes advantage position in Cournot competition with the rival firm $iA$. Increasing the output amount induces its supplier $jS$ to make more aggressively in cost reduction investment. More investment will result in not only that the marginal production cost will come down but also that manufacturer $jA$’s payoff will increase. It is also interesting to have the positive effect of cost condition ($c$) on output of manufacturer $jA$. However, note that increasing the cost condition ($c$) will reduce the total output amounts and will increase the final goods price.

Hybrid cars have various advantages over conventional vehicles, such as fuel efficiency, low cost per a mile, and environmental benefits. However, the hybrid cars also have disadvantage over the conventional automobiles. From the manufacturer’s point of view, it takes high cost for manufacturer to produce the hybrid cars. The same can be said for the luxury cars, such as Lexus which is the most expensive among the Japanese automobiles. In this case, our model
proposes that the advantageous firm produces more hybrid cars and more luxury cars than the disadvantageous firm produces.

Let us see a more detailed economic phenomenon. From 2005 to 2008, Toyota’s market strategy is clearly different from Honda’s market strategy in the domestic passenger car market. Totally, the market size in the domestic market has been decreasing, especially plunged in 2008 by the global financial crisis caused by a collapse of the US sub-prime mortgage and the reversal of the housing boom. In the middle of decreasing the market size, the market share of the normal passenger car\(^{13}\) included the hybrid car, the luxury car, and no light car for Toyota has been increasing, while the market share of the light car for Toyota has been decreasing rapidly from 2005 to 2008. The reverse can be said for Honda. Figure 4 and Figure 5 sufficiently support the above phenomenon.

![Figure 4 and 5 here](image)

5 Concluding Remarks

This paper analyzed Cournot competition between homogeneous manufacturers with their own Keiretsu supplier. This paper dealt with a model in which a cost reduction investment took place before the intermediate goods was produced. We illustrated that asymmetric equilibrium arose in the sense that two manufacturers set their intermediate goods prices to be differently. It had been illustrated that the existence of asymmetric equilibrium by using trade-off relationship between fixed cost and variable cost (Mills and Smith, 1996). Their technology choice is selected one of the alternatives. However, our asymmetric equilibrium is generated in the continuous technology set. They obtained this result with insufficiently convex technology set and random variable, while we achieved it with sufficiently convex component price set and continuous variable.

\(^{13}\) The Japan Automobile Dealer Association classifies the passenger car into two categories: normal passenger car and light passenger car.
The affiliated relationship between a manufacturer and a supplier has been regarded as a hybrid organization between vertical integration and market. It has played a greatly important role in the rapid growth periods of the Japanese economy. So, this paper focused on a theoretical model linked with Keiretsu procurement. In the model, we illustrated that asymmetric equilibrium arose in the sense that the intermediate goods prices paid from manufacturers to suppliers was different. It seems to explain the performance differences between Japanese automakers in the domestic market. We also explained two interesting results to comparative static analysis. One was that the larger the demand becomes, the less output amount the advantageous manufacturer produces, while the more output amount the disadvantageous manufacturer produces. The other was that the worse the cost condition becomes, the more output the advantageous manufacturer produces, while the less output the disadvantageous manufacturer produces.

Appendix

It will be explained that all parameters of asymmetric equilibrium are non-negative, hereinafter, under the conditions that (1) $1<bt$, (2) $2<a/c$, (3) $6bct>(bt+2)a$, and (4) $bt<2$. It will be proved that $a/c<2bt$, to begin with, under the conditions (1) and (3). $0<2bt(bt-1)$ is satisfied under the condition (1). Let's add $6bt$ to the equation and divide it into $(bt+2)$. Rearranging it, we obtain $6bt/(bt+2)<2bt$. Under the condition (3), therefore, this leads

\[
\frac{a}{c} < \frac{6bct}{(bt+2)} < 2bt
\]  

(A-1)

Secondly, it will be proved that $a/c<2bt/(2-bt)$ is satisfied, under the conditions (1), (4), and (A-1). $1<bt<2$ leads to $(2-bt)<1$. Rearranging and Multiplying it by $2bt$, we obtain $2bt<2bt/(2-bt)$. From Assumption 1 and $2bt<2bt/(2-bt)$, it is proved to be
\[
\frac{a}{c} < 2bct < \frac{2bt}{(2-bt)}
\]  
(A-2)

Thirdly, it will be proved that \(a/c > 2bt(3-2bt)/(2-bt)\) is satisfied, under the conditions (1), (2). Suppose that \(x = bt\) and \(f(x) = a/c\). Then, this leads to \(f(x) = 2x(3-2x)/(2-x)\). Differentiating it with respect to \(x\), this is easily seen to be

\[
\frac{\partial f(x)}{\partial x} = \frac{4(x-1)(x-3)}{(2-x)^2}.
\]

The function \(f(x)\) is decreasing function in the interval between \(1 < x < 2\). Therefore, the value of the function \(f(x)\) has 2, when \(x = 1\). From the condition (2), therefore, it is obvious

\[
\frac{a}{c} > \frac{2bt(3-2bt)}{(2-bt)}
\]  
(A-3)

Lastly, it will be proved that all parameters, Eq. (13), of asymmetric equilibria are non-negative on the basis of these results. It is manifest that the component prices \((w_i**, w_j**; w_i** > 0 = w_j**)\) are non-negative from the condition (3). Cost reduction investments \((x_i**, x_j**; x_j** > x_i** > 0)\) is non-negative from the conditions (1), (2), and Eq. (3-14-1). Marginal costs are non-negative from the conditions Eq. (A-1) and (A-2). Quantities \((q_i**, q_j**; q_j** > q_i** > 0)\) are apparent from the conditions (1), (2), and Eq. (14-2). The price of final good is non-negative under the condition Eq. (A-2). Manufacturers’ payoffs \(\pi_{jA}** > \pi_{iA}** > 0\) are non-negative from the conditions (1), (2), and Eq. (14-1). It is obviously proved that all variables are non-negative.
Reference
Erin, Anderson (1985), “The Salesperson as outside Agent or Employee: A


Figure 1. Feasible Domain

\[ k = \frac{a}{c} \]

\[ k(= \frac{a}{c}) = \frac{3bt}{2} \]

\[ k(= \frac{a}{c}) = \frac{6bt}{(bt+2)} \]
Figure 2. Reaction Function (bt<2)
Figure 3. Reaction Function (bt>2)

\[
\frac{6bct - (bt + 2)a}{(bt + 2)}
\]

\[
\frac{6bct - (bt + 2)a}{4(bt - 1)}
\]

\[
\frac{6bct - (bt + 2)a}{5bt - 2}
\]

\[
\frac{6bct - (bt + 2)a}{bt + 2}
\]
Figure 4. Market Shares for the Passenger Car except the Light Car

Source: Japan Automobile Dealers Association

Figure 5. Market Shares for the Light Car

Source: Japan Automobile Dealers Association