P-WAVE $\Lambda N\Sigma N$ COUPLING AND THE SPIN-ORBIT SPLITTING OF $^{9}_\Lambda$Be

Y. FUJIWARA$^1$, M. KOHNO$^2$, Y. SUZUKI$^3$

$^1$ Department of physics, Kyoto University, Kyoto 606-8502, Japan
$^2$ Physics Division, Kyushu Dental College, Kitakyushu 803-8580, Japan
$^3$ Department of Physics, and Graduate School of Science and Technology, Niigata University Niigata 950-2181, Japan

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We reexamine the spin-orbit splitting of $^{9}_\Lambda$Be excited states in terms of the $SU_6$ quark-model baryon-baryon interaction. The previous folding procedure to generate the $\Lambda\Omega$ spin-orbit potential from the quark-model $\Lambda N\Sigma N$ resonating-group kernel predicted three to five times larger values for $\Delta E_{ls} = E_{\Omega}(3/2^+) - E_{\Omega}(5/2^+)$ in the model FSS and fss2. This time, we calculate $\Lambda\Omega$ LS Born kernel, starting from the LS components of the nuclear-matter $G$-matrix for the $\Lambda$ hyperon. This framework makes it possible to take full account of an important $P$-wave $\Lambda N\Sigma N$ coupling through the antisymmetric $LS^{(-)}$ force involved in the Fermi-Breit interaction. We find that the experimental value, $\Delta E_{ls}^{exp} = 43 \pm 5$ keV, is reproduced by the quark-model $G$-matrix $LS$ interaction with a Fermi-momentum around $k_F = 1.0$ fm$^{-1}$, when the model FSS is used in the energy-independent renormalized RGM formalism. On the other hand, the model fss2 gives too large splitting of almost 200 keV, owing to the uncanceled contribution of the scalar-meson exchange $LS$ components.

Keywords: Quark-model baryon-baryon interaction; spin-orbit splitting of $\Lambda$ hypernuclei


1. Introduction

In view of rich experimental data accumulated for the light $\Lambda$-hypernuclei, $^{1,2}$ it is important to examine if various models of the fundamental hyperon-nucleon ($YN$) interactions can reproduce these experimental data or not. For few-body systems, this program is most reliably carried out by detailed Faddeev calculations for the hypertriton ($^3_\Lambda$H), $^3_\Lambda$H and $^4_\Lambda$He, using some versions of the Nijmegen models $^5$ and Jülich potentials. $^6$ The knowledge of the $\Lambda N$ interaction learned from these calculations, however, is mainly about the central part of the interaction and features of the $\Lambda N\Sigma N$ coupling of the $^3S_1 + ^3D_1$ state due to the one-pion exchange tensor force. For the $p$-shell $\Lambda$-hypernuclei, some kinds of models inevitably need to be assumed so far, to connect properties of the $\Lambda$-hypernuclei and the under-
lying $YN$ interactions. For example, the small spin-orbit ($\ell s$) splitting commonly observed in many of the light $\Lambda$-hypernuclei is typically manifested in the excited states of $^9\Lambda$Be, for which a simple $\Lambda + \alpha + \alpha$ three-cluster model is usually employed with appropriate $\Lambda\alpha$ and $\alpha\alpha$ potentials. In the framework of this model, the origin of the $\ell s$ splitting for the $5/2^+$ and $3/2^+$ excited states is the spin-orbit potential between $\Lambda$ and one of the $\alpha$ clusters, which is known to be very small due to the strong cancellation between the symmetric ($LS$) and antisymmetric ($LS^{(-)}$) $LS$ forces of the $\Lambda N$ interaction.

In our previous study of the $^9\Lambda$Be spectrum, we have carried out the $\Lambda\alpha\alpha$ three-cluster Faddeev calculation, trying to reproduce the very small $\ell s$ splitting of the $5/2^+$ and $3/2^+$ excited states, $\Delta E_{\ell s}^{exp} = 43 \pm 5$ keV, experimentally observed. As a first step, Ref. 8 directly used the quark-model (QM) $\Lambda N$ $LS$ resonating-group kernel (RGM kernel) to generate the $\Lambda\alpha$ $LS$ potential by a simple procedure of the $\alpha$-cluster folding. In this approach, the QM $\Lambda N$ $LS$ interaction of FSS or fss2 predicts 3 to 5 times larger values for $\Delta E_{\ell s}$, which is not much improved in comparison with the results of Nijmegen simulated potentials. It was pointed out in Ref. 8 that a further reduction is possible in the model FSS, if one can properly take into account the short-range correlation of the $P$-wave $\Lambda N - \Sigma N$ coupling by the $LS^{(-)}$ force. This was conjectured through the analysis of the Scheerbaum factors for the single-particle (s.p.) spin-orbit potentials, calculated in the $G$-matrix formalism.

2. Calculational Procedure

Following the above suggestion, we here generate $\Lambda\alpha$ $LS$ Born kernel from the $LS$ component of the nuclear-matter $G$-matrix for the $\Lambda$ hyperon. Our calculation consists of the following three steps.

1. Solve the $G$-matrix equation for the $\Lambda$-hyperon in symmetric nuclear matter with an appropriate Fermi momentum $k_F$ and determine the s.p. potentials for $N, \Lambda$ and $\Sigma$.\textsuperscript{9,10}

2. The $LS$ components of the $\Lambda N G$-matrices with definite momenta $K$ and starting energies $\omega$ are converted to the $\Lambda\alpha$ Born kernel by the folding procedure recently developed for the $\Lambda\alpha$ system.\textsuperscript{11}

3. Solve $\Lambda\alpha\alpha$ three-cluster system in the Faddeev formalism for composite particles.\textsuperscript{8}

We generate $\Lambda\alpha$ $LS$ Born kernel from our QM baryon-baryon interactions, FSS and fss2. For the $(0s)^2\alpha$-cluster folding, a new method developed in Ref. 11 is used to derive the direct and knock-on terms of the interaction Born kernel from the $\Lambda N G$-matrix, with explicit treatments of the nonlocality and the center-of-mass motion between $\Lambda$ and $\alpha$. The $G$-matrix calculations are carried out by assuming a constant value of the Fermi momentum, $k_F = 1.07, 1.20$, and $1.35$ fm$^{-1}$ (the normal saturation density $\rho_0$), since the local density approximation does not seem to work in light nuclear systems. The $G$-matrix equation is solved for the energy-
Table 1. The Scherbaum factor $S_A$ for symmetric nuclear matter and the $\ell s$ splitting of the $^9\text{Be}$ excited states predicted by the quark-model G-matrix $\Lambda\alpha LS$ Born kernel. In the last column, “$\Lambda N$ Born” implies the previous results, in which the $\Lambda N$ single-channel RGM kernel is used for the $S_A$ calculation and the $\alpha$-cluster folding.

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<th>$k_F$ (fm$^{-1}$)</th>
<th>$\Delta E_{\ell s}$ (keV)</th>
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<tr>
<td>Faddeev</td>
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3. Results and Discussion

Table 1 shows the $\ell s$ splitting of the $^9\text{Be}$ excited states, predicted by the $\Lambda\alpha\alpha$ Faddeev calculations, using the QM $G$-matrix $\Lambda\alpha LS$ Born kernel. The Scherbaum factor $S_A$ is also listed to indicate the strength of the spin-orbit potentials of the $\Lambda$ hyperon in symmetric nuclear matter. The Fermi momenta $k_F = 1.07$, 1.20, and 1.35 fm$^{-1}$ correspond to the densities $\rho = 0.5\rho_0$, 0.7$\rho_0$, and $\rho_0$, respectively, with $\rho_0 = 0.17$ fm$^{-3}$ being the normal saturation density. The final values for the $\ell s$ splitting of the $5/2^+$ and $3/2^+$ excited states are $\Delta E_{\ell s} = 55 - 114$ keV for FSS and 206 - 223 keV for fss2, depending on the $k_F$ values in the range of 1.07 - 1.35 fm$^{-1}$. A smaller $k_F$ value gives a smaller $\ell s$ splitting. If we compare these results with the experimental value $\Delta E_{\ell s}^{exp} = 43 \pm 5$ keV, we find that the model FSS can reproduce the experimental value if the $k_F$ value around 1.02 fm$^{-1}$ is used. We find the strong cancellation between the $LS$ and $LS(\ell s)$ forces taking place in the QM Fermi-Breit interaction by the $P$-wave $\Lambda N-\Sigma N$ coupling in the $^1P_1-^3P_1$ state, when the $G$-matrix equation is solved especially in low-density nuclear matter. This is most prominently exhibited in the model FSS. The spin-orbit contribution from the effective-meson exchange potentials in fss2 does not lead to the sufficiently small $\ell s$ splitting of the $\Lambda$ hyperon, since the scalar-meson exchange $LS$ force contains only the ordinary $LS$ and does not produce the $LS(\ell s)$ force.

4. Summary

We have carried out $\Lambda\alpha\alpha$ Faddeev calculations by employing the $\Lambda\alpha LS$ Born kernel generated from the $LS$ components of the nuclear-matter $G$-matrix for the $\Lambda$...
hyperon. One of our $SU_6$ QM baryon-baryon interactions, FSS, can reproduce the very small $\ell s$ splitting of $^9\Lambda$Be excited states, $\Delta E_{\ell s}^{\exp} = 43 \pm 5$, when an appropriate $k_F$ value corresponding to almost half of the normal saturation density is employed in the $G$-matrix calculation. The explicit value of $k_F$ depends on the model construction even within the framework of the $\Lambda\alpha\alpha$ cluster model; $k_F = 1.02$ fm$^{-1}$ for the model FSS, when the energy-independent renormalized RGM kernels are used for the $\alpha\alpha$ RGM kernel and for the QM baryon-baryon interaction. On the other hand, the model fss2 gives too large splitting of almost 200 keV, which is traced back to the un-cancelled contribution of the scalar-meson exchange $LS$ components.

An essential ingredient of the present formalism is to take into account an important $P$-wave $\Lambda N - \Sigma N$ coupling through the antisymmetric $LS$ force involved in the Fermi-Breit interaction. The present results indicate that the spin-orbit contribution from the effective meson-exchange potentials in fss2 needs to be improved to reproduce the small spin-orbit interaction of the $\Lambda$ hyperon in the nuclear medium. A new model for the $\Lambda N$ interaction with consistent central and $LS$ components is strongly desired.

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