WU-HEP-09-01
KUNS-2202
YITP-09-29

Non-Abelian Discrete Flavor Symmetries from Magnetized/Intersecting Brane Models

Hiroyuki Abe[1,*], Kang-Sin Choi[2,†], Tatsuo Kobayashi[2,‡]
and Hiroshi Ohki[2,3,§]

[1] *Department of Physics, Waseda University, Tokyo 169-8555, Japan*
[2] *Department of Physics, Kyoto University, Kyoto 606-8502, Japan*
[3] *Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

Abstract

We study non-abelian discrete flavor symmetries, which can appear in magnetized brane models. For example, D_4, $\Delta(27)$ and $\Delta(54)$ can appear and matter fields with several representations can appear. We also study the orbifold background, where non-abelian flavor symmetries are broken in a certain way.

[*]email: abe@waseda.jp
[†]email: kschoi@gauge.scphys.kyoto-u.ac.jp
[‡]email: kobayash@gauge.scphys.kyoto-u.ac.jp
[§]email: ohki@scphys.kyoto-u.ac.jp

1 Introduction

It is one of important issues to study the origin of the quark/lepton flavor structure; why there are three generations, why the hierarchy of quark/lepton masses and mixing angles appear, etc. Non-abelian discrete symmetries are interesting ideas to address the above flavor issue.

It is plausible that such non-abelian discrete flavor symmetries are originated from extra dimensional theories, because non-abelian symmetries are symmetries of geometrical solids. Indeed, in Ref. [1, 2, 3] it has been shown that certain types of non-abelian discrete flavor symmetries such as D_4 and $\Delta(54)$ can appear in four-dimensional effective field theories derived from heterotic string theory with orbifold background. (See also [4].) In those analyses, the important ingredients to derive the non-abelian discrete flavor symmetry are geometrical symmetries of the compact space and stringy coupling selection rules. Thus, stringy non-abelian discrete flavor symmetries are, in general, larger than geometrical symmetries of the compact space.

It is important to extend such an analysis on heterotic orbifold models to other types of string models. In this paper, we study which types of non-abelian flavor symmetries can appear from magnetized/intersecting brane models. Magnetized D-brane models and intersecting D-brane models are T-duals of each other [5].[1] Selection rules for allowed couplings in these models have been studied [7, 8, 9, 10]. Furthermore, three-point and higher order couplings have been computed explicitly [11, 9, 12, 10, 13].[2] Using these results, we study the flavor structures, which can appear in four-dimensional effective field theory derived from magnetized/intersecting brane models. For concreteness, we study the flavor structure in magnetized brane models as well as magnetized orbifold models. Then, we show several non-abelian discrete flavor symmetries can appear in magnetized brane models and they include D_4, $\Delta(27)$ and $\Delta(54)$, although $\Delta(27)$ is not realized in heterotic orbifold models.[3] Furthermore, most of their representations can appear in magnetized brane models, while certain representations appear in heterotic orbifold models. We would obtain the same results in intersecting D-brane models, because of the T-duality between magnetized and intersecting D-brane models.

This paper is organized as follows. In section 2, we review on magnetized brane models, in particular their zero-modes. In section 3, we study three-point and higher order couplings and their selection rules. In section 4, we study non-abelian discrete flavor symmetries, which can appear in magnetized brane models with non-vanishing Wilson lines. Such analysis is extended to the models with vanishing Wilson lines in section 5 and enhancement of symmetries are shown. In section 6, we discuss the flavor symmetries on the orbifold background. Section 7 is devoted to conclusion and discussion.

[1]See for a review [6] and references therein.

[2]See for three-point and higher order couplings and their selection rules in heterotic orbifold models [14, 15, 1, 16].

[3]Indeed, these flavor symmetries are interesting for phenomenological model building. See e.g. [17, 18, 19].

2 Magnetized brane models

We start with $\mathcal{N}=1$ ten-dimensional $U(N)$ super Yang-Mills theory. We consider the background $R^{3,1}\times(T^2)^3$, whose coordinates are denoted by x_μ ($\mu=0,\cdots,3$) for the uncompact space $R^{3,1}$ and y_m ($m=4,\cdots,9$) for the compact space $(T^2)^3$. The Lagrangian is given by

$$\mathcal{L} \;=\; -\frac{1}{4g^2}\mathrm{Tr}\left(F^{MN}F_{MN}\right)+\frac{i}{2g^2}\mathrm{Tr}\left(\bar{\lambda}\Gamma^M D_M\lambda\right),$$

where $M,N=0,\cdots,9$. Here, λ denotes gaugino fields, Γ^M is the gamma matrix for ten-dimensions and the covariant derivative D_M is given as

$$D_M\lambda \;=\; \partial_M\lambda - i[A_M,\lambda], \tag{1}$$

where A_M is the vector field. Furthermore, the field strength F_{MN} is given by

$$F_{MN} \;=\; \partial_M A_N - \partial_N A_M - i[A_M,A_N]. \tag{2}$$

The gaugino fields λ and the vector fields A_m corresponding to the compact directions are decomposed as

$$\begin{aligned}\lambda(x,y) &= \sum_n \chi_n(x)\otimes\psi_n(y),\\ A_m(x,y) &= \sum_n \varphi_{n,m}(x)\otimes\phi_{n,m}(y).\end{aligned}$$

We factorize the six-torus into two-tori $(T^2)^3$, each of which is specified by the complex structure τ_d and the area $A_d=(2\pi R_d)^2\,\mathrm{Im}\tau_d$ where $d=1,2,3$. We introduce the following form of the magnetic flux,

$$F_{z^d\bar{z}^d}=\frac{2\pi}{\mathrm{Im}\tau_d}\begin{pmatrix} m_1^{(d)}\,{}_{N_1} & & \\ & \ddots & \\ & & m_n^{(d)}\,{}_{N_n}\end{pmatrix},\quad d=1,2,3, \tag{3}$$

where ${}_{N_a}$ are the unit matrices of rank N_a, $m_i^{(d)}$ are integers and we use the complex coordinates z^d. This background breaks the gauge symmetry $U(N)\to\prod_{a=1}^n U(N_a)$ where $N=\sum_{a=1}^n N_a$.

By introducing magnetic fluxes, we can realize four-dimensional chiral theory. Let us focus on a submatrix consisting of two blocks,

$$F_{z^d\bar{z}^d,ab}=\frac{2\pi}{\mathrm{Im}\tau_d}\begin{pmatrix} m_a^{(d)}\,{}_{N_a} & 0\\ 0 & m_b^{(d)}\,{}_{N_b}\end{pmatrix}. \tag{4}$$

Then, the corresponding internal components $\psi_n(z)$ of gaugino fields $\lambda(x,z)$ also have the following form

$$\psi_n(z)=\begin{pmatrix}\psi_n^{aa}(z) & \psi_n^{ab}(z)\\ \psi_n^{ba}(z) & \psi_n^{bb}(z)\end{pmatrix}. \tag{5}$$

The off-diagonal components of zero-modes transform as bifundamental representations $\psi^{ab} \sim (\mathbf{N_a}, \overline{\mathbf{N}}_\mathbf{b})$, $\psi^{ba} \sim (\overline{\mathbf{N}}_\mathbf{a}, \mathbf{N_b})$ under $SU(N_a) \times SU(N_b)$, where we omit the subscript 0 corresponding to the zero-modes, $n = 0$. For a fixed four-dimensional chirality, either ψ^{ab} or ψ^{ba} appears as zero-modes with normalizable wavefunctions, since the ten-dimensional chirality of λ is fixed. Which zero-modes appear, ψ^{ab} or ψ^{ba}, depends on the sign of the relative magnetic flux $M^{(d)} \equiv m_a^{(d)} - m_b^{(d)}$. Furthermore, the internal part $\psi(z)$ is decomposed as a product of the d-th T^2 part, i.e. $\psi_{(d)}(z^d)$, and each of them is two-component spinor.

With an appropriate gauge fixing, the zero-modes on each d-th T^2 are written as [9]

$$\psi_d^{j,M^{(d)}}(z^d) = N_{M^{(d)}}\, e^{i\pi M^{(d)} z^d \mathrm{Im}\ z^d/(\mathrm{Im}\ \tau_d)}\, \vartheta \begin{bmatrix} j/M^{(d)} \\ 0 \end{bmatrix} (M^{(d)} z^d, \tau_d M^{(d)}), \tag{6}$$

for $j = 1, \ldots, |M^{(d)}|$, where the normalization factor N_M is obtained as

$$N_M = \left(\frac{2\mathrm{Im}\tau_d |M|}{A_d^2} \right)^{1/4}, \tag{7}$$

and $\vartheta \begin{bmatrix} j/M^{(d)} \\ 0 \end{bmatrix} (M^{(d)} z^d, \tau_d M^{(d)})$ denotes the Jacobi theta function

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu, \tau) = \sum_{n=-\infty}^{\infty} \exp\left[\pi i (n+a)^2 \tau + 2\pi i (n+a)(\nu + b) \right]. \tag{8}$$

We have the $|M^{(d)}|$ zero-modes labelled by the index j. Note that the wavefunction for $j = k + M^{(d)}$ is identical to one for $j = k$. The total number of zero-modes is the product, $\prod_d |M^{(d)}|$ and their wavefunctions are also given as the product, $\prod_d \psi_{(d)}^{j_d, M^{(d)}}$. Furthermore, their flavor structure is also understood as a direct product of the d-th T^2 sector. Thus, we concentrate on the d-th T^2 part and hereafter we omit the subscript d. In addition, the relative magnetic flux M is more important than the magnetic fluxes themselves, m_a and m_b, from the viewpoint of the flavor structure. Hence, we examine relative magnetic fluxes without mentioning the magnetic fluxes themselves, m_a and m_b.

We can have Wilson lines, $\zeta \equiv \zeta_r + \tau \zeta_i$, whose effect is just a translation of each wavefunction [9]

$$\psi^{j,M}(z) \to \psi^{j,M}(z + \zeta), \tag{9}$$

for all of j.

3 Coupling selection rule

We study order L couplings including the three point couplings $L = 3$ in four-dimensional effective theory, i.e.,

$$Y_{i_1 \ldots i_{L_\chi} i_{L_\chi+1} \cdots i_L} \chi^{i_1}(x) \cdots \chi^{i_{L_\chi}}(x) \phi^{i_{L_\chi+1}}(x) \ldots \phi^{i_L}(x), \tag{10}$$

with $L = L_\chi + L_\phi$, where χ and ϕ collectively represent four-dimensional components of fermions and bosons, respectively. In particular, the selection rule for allowed couplings is important. The three-point couplings can appear from the dimensional reduction of ten-dimensional super-Yang–Mills theory and higher order coupling terms can be read off from the effective Lagrangian of the Dirac–Born–Infeld action with supersymmetrization. The internal component of bosonic and fermionic wavefunctions is the same [9]. Thus, the couplings are determined by the wavefunction overlap in the extra dimensions,

$$Y_{i_1 i_2 \ldots i_L} = g_L^{10} \int_{T^6} d^6 z \prod_{d=1}^{3} \psi_d^{i_1, M_1}(z) \psi_d^{i_2, M_2}(z) \ldots \psi_d^{i_L, M_L}(z), \tag{11}$$

where g_L^{10} denotes the coupling in ten dimensions. Here, as mentioned in the previous section, we concentrate on the two-dimensional T^2 part of the overlap integral of wavefunctions,

$$y_{i_1 i_2 \ldots i_L} = \int_{T^2} d^2 z \; \psi^{i_1, M_1}(z) \psi^{i_2, M_2}(z) \ldots \psi^{i_L, M_L}(z), \tag{12}$$

where we have omitted the subscript d, again.

For example, we calculate the three-point couplings,

$$y_{i_1 i_2 \bar{i}_3} = \int d^2 z \; \psi^{i_1, M_1}(z) \psi^{i_2, M_2}(z) \left(\psi^{i_3, M_3}(z) \right)^* . \tag{13}$$

For the moment, we consider the case with vanishing Wilson lines. The gauge invariance requires that $M_1 + M_2 = M_3$ and that the wave function $\left(\psi^{i_3, M_3}(z) \right)^*$ but not $\psi^{i_3, M_3}(z)$ appears in the allowed three-point couplings. If these are not satisfied, there is not corresponding operators in the ten dimensions, i.e. $g_3^{10} = 0$. The results are obtained as [9]

$$y_{i_1 i_2 \bar{i}_3} = \sum_{m \in Z_{M_3}} \delta_{i_1 + i_2 + M_1 m, i_3} \; \vartheta \begin{bmatrix} \frac{M_2 i_1 - M_1 i_2 + M_1 M_2 m}{M_1 M_2 M_3} \\ 0 \end{bmatrix} (0, \tau M_1 M_2 M_3), \tag{14}$$

where the numbers in the Kronecker delta is defined modulo M_3. Indeed, the Kronecker delta part leads to the selection rule for allowed couplings as

$$i_1 + i_2 - i_3 = M_3 l - M_1 m, \quad m \in Z_{M_3}, \; l \in Z_{M_1}. \tag{15}$$

When $\gcd(M_1, M_2, M_3) = 1$, every combination (i_1, i_2, i_3) satisfies this constraint (15) because of Euclidean algorithm. On the other hand, when $\gcd(M_1, M_2, M_3) = g$, the above constraint becomes

$$i_1 + i_2 - i_3 = 0 \qquad (\bmod\ g). \tag{16}$$

This implies that we can define Z_g charges from i_k for zero-modes and the allowed couplings are controlled by such Z_g symmetry. Indeed, each quantum number i_k corresponds to quantized momentum defined with the M_i modulo structure. When $\gcd(M_1, M_2, M_3) =$

g, the modulo structure becomes Z_g and the conservation law of these discrete momenta corresponds to a requirement due to the Z_g invariance.

Let us consider higher order couplings. In [10], it has been shown that higher order couplings can be decomposed as productions of three-point couplings. For example, we consider the four-point coupling,

$$y_{i_1 i_2 i_3 \bar{i}_4} = \int d^2z \; \psi^{i_1,M_1}(z)\psi^{i_2,M_2}(z)\psi^{i_3,M_3}(z)\left(\psi^{i_4,M_4}(z)\right)^* . \tag{17}$$

This four-point coupling can be decomposed as

$$y_{i_1 i_2 i_3 \bar{i}_4} = \sum_{s\in Z_M} y_{i_1 i_2 \bar{s}} \; y_{s i_3 \bar{i}_4}, \tag{18}$$

where

$$\begin{aligned} y_{i_1 i_2 \bar{s}} &= \int d^2z \; \psi^{i_1,M_1}(z)\psi^{i_2,M_2}(z)\left(\psi^{s,M}(z)\right)^*, \\ y_{s i_3 \bar{i}_4} &= \int d^2z \; \psi^{s,M}(z)\psi^{i_3,M_3}(z)\left(\psi^{i_4,M_4}(z)\right)^*, \end{aligned} \tag{19}$$

with $M = M_1 + M_2 = M_4 - M_3$. Here, $\psi^{s,M}(z)$ denotes the s-th zero-mode of Dirac equation with the relative magnetic flux M, and these modes correspond to intermediate states in the above decomposition. Each of $y_{i_1 i_2 \bar{s}}$ and $y_{s i_3 \bar{i}_4}$ is obtained as eq. (14). That is, the coupling selection rule is controlled by the Z_g invariance (15), i.e. the conservation law of discrete momenta, and its modulo structure is determined by $\gcd(M_1, M_2, M_3, M_4) = g$.

Similarly, higher order couplings are decomposed as products of three-point couplings [10]. Therefore, the above analysis is generalized to generic order L couplings. That is, the coupling selection rule is given as the Z_g invariance and its modulo structure is determined by $\gcd(M_1, \cdots, M_L) = g$.

So far, we have considered the model with vanishing Wilson lines. Non-vanishing Wilson lines do not affect the coupling selection rule due to the Z_g invariance, but change values of couplings $y_{i_1 i_2 \bar{i}_3}$. For example, when we introduce Wilson lines ζ_k for $\psi^{i_k,M_k}(z)$, the three-point coupling (14) becomes

$$\begin{aligned} y_{i_1 i_2 \bar{i}_3} &= \sum_{m\in \mathbf{Z}_{M_3}} \delta_{i_1+i_2+M_1 m, i_3} e^{i\pi(\sum_{k=1}^{3} M_k \zeta_k \mathrm{Im}\zeta_k)/\mathrm{Im}\tau} \\ &\quad \times \vartheta \begin{bmatrix} \frac{M_2 i_1 - M_1 i_2 + M_1 M_2 m}{M_1 M_2 M_3} \\ 0 \end{bmatrix} (M_2 M_3(\zeta_2 - \zeta_3), \tau M_1 M_2 M_3), \end{aligned} \tag{20}$$

where Wilson lines must satisfy $\zeta_3 M_3 = \zeta_1 M_1 + \zeta_2 M_2$. Similarly, higher order couplings with non-vanishing Wilson lines can be obtained.

4 Non-abelian flavor symmetries

Here we study non-abelian flavor symmetries, by using the analysis on the coupling selection rule in the previous section.

4.1 Generic case

First we study generic case with non-vanishing Wilson lines. We consider the model with zero-modes ψ^{i_k,M_k} for $k=1,\cdots,L$. We denote $\gcd(M_1,\cdots,M_L)=g$. As studied in the previous section, these modes have Z_g charges and their couplings are controlled by the Z_g invariance. For simplicity, suppose that $M_1=g$. Then, there are g zero-modes of ψ^{i_1,M_1}. The above Z_g transformation acts on $\psi^{i_1,g}$ as $Z\psi^{i_1,g}$, where

$$Z=\begin{pmatrix} 1 & & & & \\ & \rho & & & \\ & & \rho^2 & & \\ & & & \ddots & \\ & & & & \rho^{g-1} \end{pmatrix}, \tag{21}$$

and $\rho=e^{2\pi i/g}$.

In addition to this Z_g symmetry, the effective theory has another symmetry. That is, the effective theory must be invariant under cyclic permutations

$$\psi^{i_1,g}\to\psi^{i_1+n,g}, \tag{22}$$

with a universal integer n for i_1. That is nothing but a change of ordering and also has a geometrical meaning as a discrete shift of the origin, $z=0\to z=-\frac{n}{g}$. This symmetry also generates another Z_g symmetry, which we denote by $Z_g^{(C)}$ and its generator is represented as

$$C=\begin{pmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ & & & & \ddots & \\ 1 & 0 & 0 & & \cdots & 0 \end{pmatrix}, \tag{23}$$

on $\psi^{i_1,g}$. That is, the above permutation (22) is represented as $C^n\psi^{i_1,g}$. These generators, Z and C, do not commute each other, i.e.,

$$CZ=\rho ZC. \tag{24}$$

Then, the flavor symmetry corresponds to the closed algebra including Z and C. Diagonal matrices in this closed algebra are written as $Z^n(Z')^m$, where Z' is the generator of another Z'_g and written as

$$Z'=\begin{pmatrix} \rho & & \\ & \ddots & \\ & & \rho \end{pmatrix}, \tag{25}$$

on $\psi^{i_1,g}$. Hence, these would generate the non-abelian flavor symmetry $(Z_g\times Z'_g)\rtimes Z_g^{(C)}$, since $Z_g\times Z'_g$ is a normal subgroup. These discrte flavor groups would include g^3 elements totally.

Let us study actions of Z and C on other zero-modes, ψ^{i_k,M_k}, with $M_k = gn_k$, where n_k is an integer. First, the generator C acts as

$$\psi^{i,gn_k} \to \psi^{i+n_k,gn_k}, \tag{26}$$

because the above discrete shift of the origin $z = 0 \to z = -\frac{n}{g}$ can be written as $z = 0 \to z = -\frac{nn_k}{gn_k}$ for these zero-modes. Thus, the generator C is represented as the same as (23) on the basis

$$\begin{pmatrix} \psi^{p,gn_k} \\ \psi^{p+n_k,gn_k} \\ \vdots \\ \psi^{p+(g-1)n_k,gn_k} \end{pmatrix}, \tag{27}$$

where p is an integer. Note that ψ^{p+gn_k,gn_k} is identical to ψ^{p,gn_k}. Furthermore, the generator Z is represented on this basis (27) as

$$Z = \rho^p \begin{pmatrix} 1 & & & & \\ & \rho^{n_k} & & & \\ & & \rho^{2n_k} & & \\ & & & \ddots & \\ & & & & \rho^{(g-1)n_k} \end{pmatrix}. \tag{28}$$

Thus, the zero-modes ψ^{i_k,gn_k} include n_k g-plet representations of the symmetry $(Z_g \times Z'_g) \rtimes Z_g^{(C)}$ and some of them may be reducible g-plet representations. For example, when we consider the zero-modes corresponding to $n_k = g$, i.e. $M_k = g^2$, the generator Z is represented as $\rho^p{}_g$ on the above g-plet (27), where ${}_g$ is the $(g \times g)$ unit matrix. In such a case, the generator C can also be diagonalized. Then, these zero-modes correspond to g singlets of $(Z_g \times Z'_g) \rtimes Z_g^{(C)}$ including trivial and non-trivial singlets.

As illustrating examples, we consider the models with $g = 2, 3$ in the next subsections and study more concretely about non-abelian discrete flavor symmetries.

4.2 $g = 2$ model

Here we consider the model with $g = 2$, that is, all of relative magnetic fluxes M_k are even. Its flavor symmetry is given as the closed algebra of Z_2, Z'_2 and $Z_2^{(C)}$, and all of these elements are written as

$$\pm\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \pm\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \pm\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \pm\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{29}$$

That is, the flavor symmetry is D_4. The zero-modes with the relative magnetic flux $M = 2$,

$$\begin{pmatrix} \psi^{0,2} \\ \psi^{1,2} \end{pmatrix}, \tag{30}$$

M	Representation of D_4
2	$\mathbf{2}$
4	$\mathbf{1}_{++},\ \mathbf{1}_{+-},\ \mathbf{1}_{-+},\ \mathbf{1}_{--}$
6	$3\times\mathbf{2}$

Table 1: D_4 representations of zero-modes in the model with $g=2$.

correspond to the doublet representation $\mathbf{2}$ of D_4. This result is the same as the non-abelian flavor symmetry appearing from heterotic orbifold models with S^1/Z_2, where twisted modes on two fixed points of S^1/Z_2 correspond to the D_4 doublet [1, 2].

Next, we consider the zero-modes corresponding to the relative magnetic flux $M=4$, $\psi^{i,4}$ $(0=0,1,2,3)$. As discussed in the previous subsection, in order to represent C, it may be convenient to decompose them into the g-plets (27)

$$\begin{pmatrix}\psi^{0,4}\\ \psi^{2,4}\end{pmatrix}, \qquad \begin{pmatrix}\psi^{1,4}\\ \psi^{3,4}\end{pmatrix}. \tag{31}$$

However, they are reducible representations as follows. Note that both $\psi^{0,4}$ and $\psi^{2,4}$ have even Z_2 charges, and that both $\psi^{1,4}$ and $\psi^{3,4}$ have odd Z_2 charges. That is, the generator Z is represented in the form $\pm\mathbf{1}_2$, where $\mathbf{1}_2$ is the 2×2 identity matrix. Thus, the generator C can be diagonalized and such a diagonalizing basis is obtained as

$$\begin{aligned}\mathbf{1}_{++}:\ (\psi^{0,4}+\psi^{2,4}), \qquad \mathbf{1}_{+-}:\ (\psi^{0,4}-\psi^{2,4}),\\ \mathbf{1}_{-+}:\ (\psi^{1,4}+\psi^{3,4}), \qquad \mathbf{1}_{--}:\ (\psi^{1,4}-\psi^{3,4}),\end{aligned} \tag{32}$$

up to normalization factors. Obviously, these correspond to four D_4 singlets, $\mathbf{1}_{++}$, $\mathbf{1}_{+-}$, $\mathbf{1}_{-+}$ and $\mathbf{1}_{--}$. The first subscript of two denotes Z_2 charges for Z and the second one denotes Z_2 charges for C. Hence, all of irreducible representations of D_4 appear from $\psi^{i,2}$ and $\psi^{i,4}$. New representations can not appear in zero-modes $\psi^{i,M}$ with $M>4$. For example, we consider zero-modes corresponding to $M=6$, i.e. $\psi^{i,6}$. They can be decomposed as

$$|\psi^6\rangle_1=\begin{pmatrix}\psi^{0,6}\\ \psi^{3,6}\end{pmatrix}, \quad |\psi^6\rangle_2=\begin{pmatrix}\psi^{2,6}\\ \psi^{5,6}\end{pmatrix}, \quad |\psi^6\rangle_3=\begin{pmatrix}\psi^{4,6}\\ \psi^{1,6}\end{pmatrix}. \tag{33}$$

Each of $|\psi^6\rangle_i$ with $i=1,2,3$ is nothing but the D_4 doublet. That is, we have three D_4 doublets in $\psi^{i,6}$. The above representations appear repeatedly in $\psi^{i,M}$ with larger M. These results are shown in Table 1.

4.3 $g=3$ model

Here we consider the model with $g=3$, where all of relative magnetic fluxes are equal to $M_k=3n_k$. Its flavor symmetry is given as $(Z_3\times Z_3)\rtimes Z_3$, that is, $\Delta(27)$ [18]. This flavor symmetry is different from the flavor symmetry appearing from heterotic orbifold models with T^2/Z_3. Later, we will explain what makes this difference.

The zero-modes corresponding to the relative magnetic flux $M = 3$,

$$|\psi^3\rangle_1 = \begin{pmatrix} \psi^{0,3} \\ \psi^{1,3} \\ \psi^{2,3} \end{pmatrix}, \tag{34}$$

correspond to the triplet representation $\mathbf{3}$ of $\Delta(27)$. Next, we consider the zero-modes corresponding to the relative magnetic flux $M = 6$, i.e. $\psi^{i,6}$. Again, it may be convenient to decompose them into the g-plets (27)

$$|\psi^6\rangle_1 = \begin{pmatrix} \psi^{0,6} \\ \psi^{2,6} \\ \psi^{4,6} \end{pmatrix}, \quad |\psi^6\rangle_2 = \begin{pmatrix} \psi^{3,6} \\ \psi^{5,6} \\ \psi^{1,6} \end{pmatrix}. \tag{35}$$

The generator C is represented in the same way for $|\psi^3\rangle_1$ and $|\psi^6\rangle_i$ $(i = 1, 2)$. On the other hand, the representation of the generator Z for $|\psi^6\rangle_i$ $(i = 1, 2)$ is the complex conjugate to one for $|\psi^3\rangle_1$. Thus, both $|\psi^6\rangle_i$ $(i = 1, 2)$ correspond to $\bar{\mathbf{3}}$ representations of $\Delta(27)$.

Moreover, let us consider the zero-modes with the relative magnetic flux $M = 9$, i.e. $\psi^{i,9}$. Then, we decompose them into the g-plets (27)

$$|\psi^9\rangle_1 = \begin{pmatrix} \psi^{0,9} \\ \psi^{3,9} \\ \psi^{6,9} \end{pmatrix}, \quad |\psi^9\rangle_\omega = \begin{pmatrix} \psi^{1,9} \\ \psi^{4,9} \\ \psi^{7,9} \end{pmatrix}, \quad |\psi^9\rangle_{\omega^2} = \begin{pmatrix} \psi^{2,9} \\ \psi^{5,9} \\ \psi^{8,9} \end{pmatrix}, \tag{36}$$

where $\omega = e^{2\pi i/3}$. These (reducible) triplets $|\psi^9\rangle_{\omega^n}$ have Z_3 charges, n and are decomposed into nine singlets,

$$\mathbf{1}_{\omega^n,\omega^m} : \psi^{n,9} + \omega^m \psi^{n+3m,9} + \omega^{2m} \psi^{n+6m,9}, \tag{37}$$

up to normalization factors, where n and m are Z_3 charges for Z and C, respectively. In zero-modes with $M > 9$, new representations do not appear, but the above representations appear repeatedly. These results as well as zero-modes with $M > 9$ are shown in Table 2. Similar analysis can be carried out in other models with $g > 3$.

We comment on symmetries in subsectors. Suppose that our model has zero-modes ψ^{i_k,M_k} for $k = 1, \cdots, L$ with $\gcd(M_1, \cdots, M_L) = g$ and they are separated into two classes, ψ^{i_l,M_l} $(l = 1, \cdots, L_1)$ and ψ^{i_m,M_m} $(m = L_1, \cdots, L)$, where $\gcd(M_1, \cdots, M_{L_1}) = g_1$, $\gcd(M_{L_1}, \cdots, M_L) = g_2$ and $\gcd(g_1, g_2) = g$. Coupling terms including only the first class of fields ψ^{i_l,M_l} $(l = 1, \cdots, L_1)$ in the four-dimensional effective theory have the symmetry $(Z_{g_1} \times Z_{g_1}) \rtimes Z_{g_1}$, where g_1 would be larger than g. However, such a symmetry is broken by terms including the second class of fields. Thus, we would have a larger symmetry at least at tree level for the subsectors. Such larger symmetries in the subsectors would be interesting for model building.

M	Representation of $\Delta(27)$
3	$\mathbf{3}$
6	$2\times\bar{\mathbf{3}}$
9	$\mathbf{1}_1,\ \mathbf{1}_2,\ \mathbf{1}_3,\ \mathbf{1}_4,\ \mathbf{1}_5,\ \mathbf{1}_6,\ \mathbf{1}_7,\ \mathbf{1}_8,\ \mathbf{1}_9$
12	$4\times\mathbf{3}$
15	$5\times\bar{\mathbf{3}}$
18	$2\times\{\mathbf{1}_1,\ \mathbf{1}_2,\ \mathbf{1}_3,\ \mathbf{1}_4,\ \mathbf{1}_5,\ \mathbf{1}_6,\ \mathbf{1}_7,\ \mathbf{1}_8,\ \mathbf{1}_9\}$

Table 2: $\Delta(27)$ representations of zero-modes in the model with $g=3$.

5 Models without Wilson lines

In the previous section, we have considered the models with non-vanishing Wilson lines. Here, we study the models without Wilson lines. In this case, flavor symmetries are enhanced.

When Wilson lines are vanishing, all of zero-modes ψ^{0,M_k} have the peak at the same point in the extra dimensions. In the intersecting D-brane picture, this corresponds to the D-brane configuration, that all of D-branes intersect (at least) at a single point on T^2. This model has the Z_2 rotation symmetry around such a point. Here, we denote its generator as P. In general, this acts as

$$P:\ \psi^{i,M}\rightarrow\psi^{M-i,M}. \tag{38}$$

As in the previous section, we consider the models with $g=2,3$ as illustrating models.

5.1 $g=2$ model

First, we consider the zero-modes with $M=2$, $\psi^{i,2}$, which correspond to the D_4 doublet. For them, the generator P acts as the identity. That implies that the flavor symmetry is enhanced as $D_4\times Z_2$ and $\psi^{i,2}$ correspond to $\mathbf{2}_+$, where the subscript denotes the Z_2 charge for P.[4]

We consider the zero-modes with $M=4$, $\psi^{i,4}$, which are decomposed as the four D_4 singlets, $\mathbf{1}_{++}$, $\mathbf{1}_{+-}$, $\mathbf{1}_{-+}$ and $\mathbf{1}_{--}$ as (32). They have definite Z_2 charges for P and are represented as

$$\begin{aligned}&\mathbf{1}_{+++}:\ (\psi^{0,4}+\psi^{2,4}),\qquad \mathbf{1}_{+-+}:\ (\psi^{0,4}-\psi^{2,4}),\\ &\mathbf{1}_{-++}:\ (\psi^{1,4}+\psi^{3,4}),\qquad \mathbf{1}_{---}:\ (\psi^{1,4}-\psi^{3,4}),\end{aligned} \tag{39}$$

where the third sign in the subscripts denotes Z_2 charges for P.

Now, let us consider the zero-modes with $M=6$, $\psi^{i,6}$, which are decomposed as three D_4 doublets (33). The doublet $|\psi^6\rangle_1$ has the even Z_2 charges for P. However, other

[4] Although this is just an enhancement by the factor Z_2, such an enhanced flavor symmetry $D_4\times Z_2$ would be important to phenomenological model building. See e.g. [17].

M	Representation of $D_4 \times Z_2$
2	$\mathbf{2}_+$
4	$\mathbf{1}_{+++},\ \mathbf{1}_{+-+},\ \mathbf{1}_{-++},\ \mathbf{1}_{---}$
6	$2 \times \mathbf{2}_+,\ \mathbf{2}_-$
8	$\mathbf{1}_{+++},\ \mathbf{1}_{+-+},\ \mathbf{1}_{+++},\ \mathbf{1}_{+--},\ \mathbf{1}_{-++},\ \mathbf{1}_{-+-},\ \mathbf{1}_{---},\ \mathbf{1}_{--+}$
10	$3 \times \mathbf{2}_+,\ 2 \times \mathbf{2}_-$

Table 3: $D_4 \times Z_2$ representations of zero-modes in the model with $g = 2$.

doublets $|\psi^6\rangle_2$ and $|\psi^6\rangle_3$ transform each other under P. Thus, we take linear combinations of these two doublets as

$$|\psi^6\rangle_\pm \equiv |\psi^6\rangle_2 \pm |\psi^6\rangle_3 = \begin{pmatrix} \psi^{2,6} \\ \psi^{5,6} \end{pmatrix} \pm \begin{pmatrix} \psi^{4,6} \\ \psi^{1,6} \end{pmatrix}, \tag{40}$$

where $\pm$ also means Z_2 charge of P. As a result, these zero-modes $\psi^{i,6}$ are decomposed as two $\mathbf{2}_+$ and one $\mathbf{2}_-$.

We can repeat these analysis for larger M. For example, zero-modes with $M = 8$, $\psi^{i,8}$, are decomposed as

$$\{\mathbf{1}_{+++},\ \mathbf{1}_{+-+},\ \mathbf{1}_{+++},\ \mathbf{1}_{+--},\ \mathbf{1}_{-++},\ \mathbf{1}_{-+-},\ \mathbf{1}_{---},\ \mathbf{1}_{--+}\}, \tag{41}$$

and zero-modes with $M = 10$, $\psi^{i,10}$, are decomposed as three $\mathbf{2}_+$ and two $\mathbf{2}_-$. These results are shown in Table 3.

5.2 $g = 3$ model

Here, we study the model with $g = 3$. First, we consider the zero-modes with $M = 3$, $\psi^{i,3}$. They correspond to a triplet of $\Delta(27)$ with non-vanishing Wilson lines. At any rate, the generators, Z, C and P, acts on $\psi^{i,3}$ as

$$Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{42}$$

Their closed algebra is $\Delta(54)$. Thus, the zero-modes $\psi^{i,3}$ correspond to the triplet of $\Delta(54)$, $\mathbf{3}_1$. This is the same as the flavor symmetry, which appears in heterotic orbifold models with T^2/Z_3 [2]. Three fixed points on the orbifold T^2/Z_3 have the geometrical permutation symmetry S_3. Such symmetry is enhanced in magnetized brane models, only when Wilson lines are vanishing. Indeed, the closed algebra of generators C and P is S_3.

Similarly, we can consider the zero-modes with $M = 6$, $\psi^{i,6}$. We decompose them as (35). The generators, C and P, act on $|\psi^6\rangle_i$ $(i = 1, 2)$ in the same way as $\psi^{i,3}$, but the representation of the generator Z for $|\psi^6\rangle_i$ $(i = 1, 2)$ is the complex conjugate to one for

$|\psi^3\rangle_1$. Thus, both $|\psi^6\rangle_i$ correspond to $\bar{\mathbf{3}}_1$ representations of $\Delta(54)$. Recall that $|\psi^6\rangle_i$ are $\bar{\mathbf{3}}$ representations of $\Delta(27)$.

Next, let us consider the zero-modes with $M = 9$, $\psi^{i,9}$. Recall that they correspond to nine singlets of $\Delta(27)$ as (37). The following linear combination,

$$\psi^{0,9} + \psi^{3,9} + \psi^{6,9}, \tag{43}$$

is still a singlet under $\Delta(54)$, which is a trivial singlet $\mathbf{1}_1$. However, the others in linear combinations (37) transform each other under P. Then, they correspond to four doublets of $\Delta(54)$,

$$\begin{aligned}
&\mathbf{2}_1 : \begin{pmatrix} \psi^{0,9} + \omega\psi^{3,9} + \omega^2\psi^{6,9} \\ \psi^{0,9} + \omega^2\psi^{3,9} + \omega\psi^{6,9} \end{pmatrix}, \qquad \mathbf{2}_2 : \begin{pmatrix} \psi^{1,9} + \psi^{4,9} + \psi^{7,9} \\ \psi^{2,9} + \psi^{5,9} + \psi^{8,9} \end{pmatrix}, \\
&\mathbf{2}_3 : \begin{pmatrix} \psi^{1,9} + \omega\psi^{4,9} + \omega^2\psi^{7,9} \\ \psi^{8,9} + \omega^2\psi^{5,9} + \omega^2\psi^{2,9} \end{pmatrix}, \qquad \mathbf{2}_4 : \begin{pmatrix} \psi^{1,9} + \omega^2\psi^{4,9} + \omega\psi^{7,9} \\ \psi^{8,9} + \omega^2\psi^{5,9} + \omega\psi^{2,9} \end{pmatrix}.
\end{aligned} \tag{44}$$

Now, let us consider the zero-modes with $M = 12$, $\psi^{i,12}$. We decompose them into g-plets (27)

$$\begin{aligned}
|\psi^{12}\rangle_1 &= \begin{pmatrix} \psi^{0,12} \\ \psi^{4,12} \\ \psi^{8,12} \end{pmatrix}, \qquad |\psi^{12}\rangle_2 = \begin{pmatrix} \psi^{6,12} \\ \psi^{10,12} \\ \psi^{2,12} \end{pmatrix}, \\
|\psi^{12}\rangle_3 &= \begin{pmatrix} \psi^{3,12} \\ \psi^{7,12} \\ \psi^{11,12} \end{pmatrix}, \qquad |\psi^{12}\rangle_4 = \begin{pmatrix} \psi^{9,12} \\ \psi^{1,12} \\ \psi^{5,12} \end{pmatrix}.
\end{aligned} \tag{45}$$

They correspond to four triplets of $\Delta(27)$. Representations of the generators, Z, C and P, on $|\psi^{12}\rangle_1$ and $|\psi^{12}\rangle_2$ are the same as those on $\psi^{i,3}$ like Eq. (42). Thus, they correspond to $\mathbf{3}_1$. On the other hand, $|\psi^{12}\rangle_3$ and $|\psi^{12}\rangle_4$ transform each other under P. Hence, we take the following linear combinations,

$$|\psi^{12}\rangle_\pm = \begin{pmatrix} \psi^{3,12} \pm \psi^{9,12} \\ \psi^{7,12} \pm \psi^{1,12} \\ \psi^{11,12} \pm \psi^{5,12} \end{pmatrix}. \tag{46}$$

Then, representations of Z, C and P on $|\psi^{12}\rangle_+$ are the same as (42), and $|\psi^{12}\rangle_+$ corresponds to $\mathbf{3}_1$. On the other hand, representations of Z and C on $|\psi^{12}\rangle_-$ are the same as (42), but the generator P is represented on $|\psi^{12}\rangle_-$ as

$$P = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}. \tag{47}$$

That is, $|\psi^{12}\rangle_-$ corresponds to another triplet of $\Delta(54)$, i.e. $\mathbf{3}_2$. Furthermore, the zero-modes with $M = 15$, $\psi^{i,15}$ correspond to

$$3 \times \bar{\mathbf{3}}_1, \quad 2 \times \bar{\mathbf{3}}_2, \tag{48}$$

M	Representation of $\Delta(54)$
3	$\mathbf{3}_1$
6	$2 \times \bar{\mathbf{3}}_1$
9	$\mathbf{1}_1,\ \mathbf{2}_1,\ \mathbf{2}_2,\ \mathbf{2}_3,\ \mathbf{2}_4$
12	$3 \times \mathbf{3}_1,\ \mathbf{3}_2$
15	$3 \times \bar{\mathbf{3}}_1,\ 2 \times \bar{\mathbf{3}}_2$
18	$2 \times \{\mathbf{1}_1,\ \mathbf{2}_1,\ \mathbf{2}_2,\ \mathbf{2}_3,\ \mathbf{2}_4\}$

Table 4: $\Delta(54)$ representations of zero-modes in the model with $g = 3$.

and the zero-modes with $M = 18$, $\psi^{i,18}$ correspond to

$$2 \times \{\mathbf{1}_1,\ \mathbf{2}_1,\ \mathbf{2}_2,\ \mathbf{2}_3,\ \mathbf{2}_4\}. \tag{49}$$

These results are shown in Table 4. Irreducible representations of $\Delta(54)$ are two triplets $\mathbf{3}_1$, $\mathbf{3}_2$, their conjugates $\bar{\mathbf{3}}_1$ $\bar{\mathbf{3}}_2$, four doublets $\mathbf{2}_1$, $\mathbf{2}_2$, $\mathbf{2}_3$, $\mathbf{2}_4$, trivial singlet $\mathbf{1}$ and non-trivial singlet $\mathbf{1}_2$. All of them except the non-trivial singlet $\mathbf{1}_2$ can appear in this model.

Similar analysis can be carried out in other models with $g > 3$. In generic case, the Z and P satisfy

$$PZ = Z^{-1}P, \tag{50}$$

and the closed algebra of C and P is D_g. Thus, the flavor symmetry, which is generated by Z, C and P, would be written as $D_g \ltimes (Z_g \times Z_g)$. Note that $S_3 \sim D_3$ and $\Delta(54)$ is $D_3 \ltimes (Z_3 \times Z_3)$.

6 Orbifold models

We have found that several non-abelian discrete flavor symmetries like D_4, $\Delta(27)$ and $\Delta(54)$ can appear. However, these exact symmetries may be rather large to explain realistic mass matrices of quarks and leptons. Their breaking would be preferable. Such symmetry breaking can happen within the framework of four-dimensional effective field theory, that is, scalar fields with non-trivial representations are assumed to develop their vacuum expectation values. On the other hand, a certain type of symmetry breaking can happen on the orbifold background, which is called magnetized orbifold models [20, 21]. Here, we discuss the flavor structure in magnetized orbifold models.

The orbifold T^2/Z_2 is constructed by dividing T^2 by the Z_2 projection $z \to -z$. Furthermore, on such an orbifold, we require periodic or anti-periodic boundary condition for matter fields as well as gauge fields,

$$\psi(-z) = \pm\, \psi(z). \tag{51}$$

Since such boundary conditions are consistent in models with vanishing Wilson lines, we consider the case without Wilson lines. Indeed, zero-mode wavefunctions in models without Wilson lines satisfy the following relation,

$$\psi^{j,M}(-z) = \psi^{M-j,M}(z). \tag{52}$$

Thus, even and odd zero-modes are obtained as their linear combinations,

$$\psi^{j}_{\pm}(z) = \psi^{j,M}(z) \pm \psi^{M-j,M}(z), \tag{53}$$

up to a normalization factor. Which modes among even and odd modes are selected depends on how to embed the Z_2 orbifold projection into the gauge space, that is, model dependent. At any rate, either even or odd zero-modes are projected out for each kind of matter fields[5]. Note that the Z_2 orbifold parity of $\psi^j_{\pm}(z)$ is the same as the Z_2 charge of P. Thus, through the orbifold projection zero-modes with either even or odd Z_2 charge of P survive for each kind of matter fields.

Let us consider examples. First we study the model with $g = 2$. This model has the non-abelian flavor symmetry $D_4 \times Z_2$. The zero-modes with $M = 2$, $\psi^{i,2}$, correspond to $\mathbf{2}_+$ of $D_4 \times Z_2$. When we require the periodic boundary condition, they survive. On the other hand, they are projected out for the anti-periodic boundary condition. Similarly, the zero-modes with $M = 4$, $\psi^{i,4}$, correspond to $\mathbf{1}_{+++}$, $\mathbf{1}_{+-+}$, $\mathbf{1}_{-++}$ and $\mathbf{1}_{---}$, where the third subscript denotes the Z_2 charge of P. Thus, the zero-modes corresponding to $\mathbf{1}_{+++}$, $\mathbf{1}_{+-+}$ and $\mathbf{1}_{-++}$ survive for the periodic boundary condition, while only $\mathbf{1}_{---}$ survives for the anti-periodic boundary condition. Similarly, we can identify which modes can survive through the Z_2 orbifold projection. The number of matter fields are reduced through the Z_2 orbifold projection. However, four-dimensional effective field theory after orbifolding has the flavor symmetry $D_4 \times Z_2$. The reason why the flavor symmetry $D_4 \times Z_2$ remains unbroken is that the flavor symmetry is the direct product between D_4 and Z_2.

Next, let us consider the model with $g = 3$. This model has the flavor symmetry $\Delta(54)$. The zero-modes with $M = 3$, $\psi^{i,3}$, correspond to $\mathbf{3}_1$ of $\Delta(54)$. However, the eigenstates of Z_2 are $\psi^{0,3}$ and $\psi^{1,3} \pm \psi^{2,3}$. Hence, when we project out Z_2 even or odd modes, the triplet structure is broken, that is, the flavor symmetry $\Delta(54)$ is completely broken. However, such symmetry breaking is non-trivial, because the original theory has the $\Delta(54)$ symmetry and we project out certain modes from such a theory.[6]

Orbifold models with larger g, $g > 3$ have a similar structure on flavor symmetries. The original theory before orbifolding has a large non-abelian flavor symmetry. By orbifolding, certain matter fields are projected out and the flavor symmetry is broken although some symmetries like abelian discrete symmetries remain unbroken. However, there remains a footprint of the larger flavor symmetry in four-dimensional effective theory, that is, coupling terms are constrained.

As an illustrating example, let us consider explicitly the model with three zero-modes, which have relative magnetic fluxes, $(M_1, M_2, M_3) = (4, 4, 8)$, that is, $g = 4$. The genera-

[5]Within the framework of intersecting D-brane models, analogous results have been obtained by considering D6-branes wrapping rigid 3-cycles [22].

[6]This type of flavor symmetry breaking has been proposed in not magnetized brane models, but orbifold models [23, 24, 25].

i,j,k	L_i	R_j	H_k
0	$\psi^{0,4}$	$\psi^{0,4}$	$\psi^{0,8}$
1	$\frac{1}{\sqrt{2}}\left(\psi^{1,4}+\psi^{3,4}\right)$	$\frac{1}{\sqrt{2}}\left(\psi^{1,4}+\psi^{3,4}\right)$	$\frac{1}{\sqrt{2}}\left(\psi^{1,8}+\psi^{7,8}\right)$
2	$\psi^{2,4}$	$\psi^{2,4}$	$\frac{1}{\sqrt{2}}\left(\psi^{2,8}+\psi^{6,8}\right)$
3	-	-	$\frac{1}{\sqrt{2}}\left(\psi^{3,8}+\psi^{5,8}\right)$
4	-	-	$\psi^{4,8}$

Table 5: Wavefunctions in the orbifold model.

tors, Z, C and P, are represented on the zero-modes with $M_1 = 4$ as

$$Z = \begin{pmatrix} 1 & & & \\ & i & & \\ & & -1 & \\ & & & -i \end{pmatrix}, \quad C = \begin{pmatrix} & 1 & & \\ & & 1 & \\ & & & 1 \\ 1 & & & \end{pmatrix}, \quad P = \begin{pmatrix} 1 & & & \\ & & & 1 \\ & & 1 & \\ & 1 & & \end{pmatrix}. \tag{54}$$

Obviously, we find $[P,Z] \neq 0$ and $[C,P] \neq 0$. Thus, eigenstates of P are not eigenstates for Z or C. Since eigenstates with $P = 1$ or $P = -1$ are projected out by orbifolding, the flavor symmetry is broken. However, one can find that $[P,Z^2] = [P,C^2] = 0$. The symmetry generated by Z^2, C^2 and P remains unbroken after orbifolding. Thus, the flavor symmetry is reduced to $Z_2 \times Z_2 \times Z_2$. The first two Z_2 factors are originally subgroups of $Z_4 \ltimes (Z_4 \times Z_4)$ generated by Z and C algebra and they are abelian groups.

For concreteness, let us consider the following Z_2 boundary conditions,

$$\psi^{i_1,M_1}(-z) = \psi^{i_1,M_1}(z), \quad \psi^{i_2,M_2}(-z) = \psi^{i_2,M_2}(z), \quad \psi^{i_3,M_3}(-z) = \psi^{i_3,M_3}(z), \tag{55}$$

for three types of zero-modes. Then, we assign the first and second modes with left-handed and right-handed fermions, L_i and R_j, while the third is assigned with Higgs fields H_k. There are three Z_2 even modes for $M_1 = M_2 = 4$, that is, the three generation model [20, 21], while there are five Z_2 even modes for $M_3 = 8$. Their wavefunctions are shown in Table 5.

After orbifold projection, Yukawa couplings $Y_{ijk}L_iR_jH_k$ in this model are given by [21]

$$Y_{ijk}H_k = \begin{pmatrix} y_aH_0 + y_eH_4 & y_fH_3 + y_bH_1 & y_cH_2 \\ y_fH_3 + y_bH_1 & \frac{1}{\sqrt{2}}(y_a+y_e)H_2 + y_c(H_0+H_4) & y_bH_3 + y_dH_1 \\ y_cH_2 & y_bH_3 + y_dH_1 & y_eH_0 + y_aH_4 \end{pmatrix}. \tag{56}$$

Here, Yukawa coupling strengths, $y_a, y_b, \cdots, y_f$, are written as functions of moduli and they are, in general, different from each other.

We can take the basis of L_i, R_j, H_k as eigenstates of Z^2 and C^2. Such a basis is shown in Table 6. Thus, if this effective theory has only $Z_2 \times Z_2 \times Z_2$ symmetry, the following

L_i	Z^2	C^2	R_j	Z^2	C^2	H_k	Z^2	C^2
$\frac{1}{\sqrt{2}}(L^0+L^2)$	1	1	$\frac{1}{\sqrt{2}}(R^0+R^2)$	1	1	$\frac{1}{\sqrt{2}}(H^0+H^4)$	1	1
$\frac{1}{\sqrt{2}}(L^0-L^2)$	1	–1	$\frac{1}{\sqrt{2}}(R^0-R^2)$	1	–1	$\frac{1}{\sqrt{2}}(H^0-H^4)$	1	–1
L_1	–1	1	R_1	–1	1	$\frac{1}{\sqrt{2}}(H^1+H^3)$	–1	1
–	–	–	–	–	–	$\frac{1}{\sqrt{2}}(H^1-H^3)$	–1	–1
–	–	–	–	–	–	H_2	1	1

Table 6: Eigenstates of Z^2 and C^2

couplings would be allowed,

$$Y_{ijk}H_k = \begin{pmatrix} y_1H_0+y_2H_2+y_3H_4 & y_4H_1+y_5H_3 & y_6H_0+y_7H_2+y_8H_4 \\ y_4'H_1+y_5'H^3 & y_9(H_0+H_4)+y_{10}H_2 & y_5'H_1+y_4'H_3 \\ y_8H_0+y_7H_2+y_6H_4 & y_5H_1+y_4H_3 & y_3H_0+y_2H_2+y_1H_4 \end{pmatrix} \quad (57)$$

where coupling strengths like y_1,y_2, etc. are independent parameters. For example, the $Z_2 \times Z_2 \times Z_2$ symmetry allows non-vanishing couplings of y_2, y_6 and y_8. However, these couplings are forbidden by the symmetry $Z_4 \ltimes (Z_4 \times Z_4)$ and such couplings do not appear in Eq. (56). Thus, Yukawa couplings derived from orbifolding are constrained more compared with the model, which has only the $Z_2 \times Z_2 \times Z_2$ flavor symmetry.

Similarly, other orbifold models have more constraints at least at tree level compared with unbroken symmetry as a footprint of larger flavor symmetries before orbifolding. Such a structure would be useful for phenomenological applications.

7 Conclusion and discussion

We have studied the non-abelian flavor symmetries, which can appear in magnetized brane models. We have found that D_4, $\Delta(27)$ and other $Z_g \ltimes (Z_g \times Z_g)$ flavor symmetries can appear from magnetized brane models with non-vanishing Wilson lines. Matter fields with several representations of these discrete flavor symmetries can appear. When we consider vanishing Wilson lines, these flavor symmetries are enhanced like $D_4 \times Z_2$, $\Delta(54)$, etc. These results are interesting to apply for model building of realistic quark/lepton mass matrices. We have also discussed the flavor symmetry breaking on the orbifold background.

Since intersecting D-brane models are T-duals of magnetized brane models, we would obtain the same results in intersecting D-brane models.

It is important to study anomalies of non-abelian flavor symmetries. If string theory leads to anomaly-free effective low-energy theories including discrete symmetries, anomalies of discrete symmetries must be canceled by the Green-Schwarz mechanism. Those discrete anomalies were studied within the framework of heterotic orbifold models in [26], and it was shown that discrete anomalies can be canceled by the Green-Schwarz mechanism. Furthermore, important relations of discrete anomalies with U(1) anomalies and others were found. (See also [27].) It is important to extend such an analysis to magnetized/intersecting brane models.

Acknowledgement

K.-S. C., T. K. and H. O. are supported in part by the Grant-in-Aid for Scientific Research No. 20·08326, No. 20540266 and No. 21·897 from the Ministry of Education, Culture, Sports, Science and Technology of Japan. T. K. is also supported in part by the Grant-in-Aid for the Global COE Program "The Next Generation of Physics, Spun from Universality and Emergence" from the Ministry of Education, Culture,Sports, Science and Technology of Japan.

References

[1] T. Kobayashi, S. Raby and R. J. Zhang, Nucl. Phys. B **704**, 3 (2005) [arXiv:hep-ph/0409098].

[2] T. Kobayashi, H. P. Nilles, F. Ploger, S. Raby and M. Ratz, Nucl. Phys. B **768**, 135 (2007) [arXiv:hep-ph/0611020].

[3] P. Ko, T. Kobayashi, J. h. Park and S. Raby, Phys. Rev. D **76**, 035005 (2007) [Erratum-ibid. D **76**, 059901 (2007)] [arXiv:0704.2807 [hep-ph]].

[4] G. Altarelli, F. Feruglio and Y. Lin, Nucl. Phys. B **775**, 31 (2007) [arXiv:hep-ph/0610165].

[5] C. Bachas, arXiv:hep-th/9503030; M. Berkooz, M. R. Douglas and R. G. Leigh, Nucl. Phys. B **480**, 265 (1996) [arXiv:hep-th/9606139]; R. Blumenhagen, L. Goerlich, B. Kors and D. Lust, JHEP **0010**, 006 (2000) [arXiv:hep-th/0007024]; C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, Phys. Lett. B **489**, 223 (2000) [arXiv:hep-th/0007090]; G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan and A. M. Uranga, J. Math. Phys. **42**, 3103 (2001) [arXiv:hep-th/0011073]; JHEP **0102**, 047 (2001) [arXiv:hep-ph/0011132]; R. Blumenhagen, B. Kors and D. Lust, JHEP **0102**, 030 (2001) [arXiv:hep-th/0012156]; M. Cvetic, G. Shiu and A. M. Uranga, Phys. Rev. Lett. **87**, 201801 (2001) [arXiv:hep-th/0107143]; Nucl. Phys. B **615**, 3 (2001) [arXiv:hep-th/0107166].

[6] R. Blumenhagen, M. Cvetic, P. Langacker and G. Shiu, Ann. Rev. Nucl. Part. Sci. **55**, 71 (2005) [arXiv:hep-th/0502005]; R. Blumenhagen, B. Kors, D. Lust and S. Stieberger, Phys. Rept. **445**, 1 (2007) [arXiv:hep-th/0610327].

[7] D. Cremades, L. E. Ibanez and F. Marchesano, JHEP **0307** (2003) 038 [arXiv:hep-th/0302105].

[8] T. Higaki, N. Kitazawa, T. Kobayashi and K. j. Takahashi, Phys. Rev. D **72** (2005) 086003 [arXiv:hep-th/0504019].

[9] D. Cremades, L. E. Ibanez and F. Marchesano, JHEP **0405**, 079 (2004) [arXiv:hep-th/0404229].

[10] H. Abe, K. S. Choi, T. Kobayashi and H. Ohki, arXiv:0903.3800 [hep-th].

[11] M. Cvetic and I. Papadimitriou, Phys. Rev. D **68** (2003) 046001 [Erratum-ibid. D **70** (2004) 029903] [arXiv:hep-th/0303083]; S. A. Abel and A. W. Owen, Nucl. Phys. B **663** (2003) 197 [arXiv:hep-th/0303124]; V. Braun, Y. H. He and B. A. Ovrut, JHEP **0604** (2006) 019 [arXiv:hep-th/0601204]; S. A. Abel and A. W. Owen, Nucl. Phys. B **682** (2004) 183 [arXiv:hep-th/0310257]; C. M. Chen, T. Li, V. E. Mayes and D. V. Nanopoulos, Phys. Rev. D **78**, 105015 (2008) [arXiv:0807.4216 [hep-th]]; I. Antoniadis, A. Kumar and B. Panda, arXiv:0904.0910 [hep-th].

[12] P. Di Vecchia, A. Liccardo, R. Marotta and F. Pezzella, arXiv:0810.5509 [hep-th].

[13] R. Russo and S. Sciuto, JHEP **0704**, 030 (2007) [arXiv:hep-th/0701292]; D. Duo, R. Russo and S. Sciuto, JHEP **0712**, 042 (2007) [arXiv:0709.1805 [hep-th]].

[14] S. Hamidi and C. Vafa, Nucl. Phys. B **279**, 465 (1987); L. J. Dixon, D. Friedan, E. J. Martinec and S. H. Shenker, Nucl. Phys. B **282**, 13 (1987); J. J. Atick, L. J. Dixon, P. A. Griffin and D. Nemeschansky, Nucl. Phys. B **298** (1988) 1; T. T. Burwick, R. K. Kaiser and H. F. Muller, Nucl. Phys. B **355** (1991) 689; S. Stieberger, D. Jungnickel, J. Lauer and M. Spalinski, Mod. Phys. Lett. A **7**, 3059 (1992) [arXiv:hep-th/9204037]; K. S. Choi and T. Kobayashi, Nucl. Phys. B **797** (2008) 295 [arXiv:0711.4894 [hep-th]].

[15] T. Kobayashi and N. Ohtsubo, Int. J. Mod. Phys. A **9**, 87 (1994).

[16] W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, Nucl. Phys. B **785**, 149 (2007) [arXiv:hep-th/0606187]; O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange and A. Wingerter, Phys. Rev. D **77**, 046013 (2008) [arXiv:0708.2691 [hep-th]].

[17] W. Grimus and L. Lavoura, Phys. Lett. B **572**, 189 (2003); W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura and M. Tanimoto, JHEP **0407**, 078 (2004); A. Blum, R. N. Mohapatra and W. Rodejohann, Phys. Rev. D **76**, 053003 (2007); A. Blum, C. Hagedorn and M. Lindner, Phys. Rev. D **77**, 076004 (2008) [arXiv:0709.3450 [hep-ph]]; W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura, H. Sawanaka and M. Tanimoto, Nucl. Phys. B **713**, 151 (2005). H. Ishimori, T. Kobayashi, H. Ohki, Y. Omura, R. Takahashi and M. Tanimoto, Phys. Lett. B **662**, 178 (2008) [arXiv:0802.2310 [hep-ph]]; Phys. Rev. D **77**, 115005 (2008) [arXiv:0803.0796 [hep-ph]]; A. Adulpravitchai, A. Blum and C. Hagedorn, JHEP **0903**, 046 (2009) [arXiv:0812.3799 [hep-ph]].

[18] G. C. Branco, J. M. Gerard and W. Grimus, Phys. Lett. B **136**, 383 (1984); C. Luhn, S. Nasri and P. Ramond, J. Math. Phys. **48**, 073501 (2007) [arXiv:hep-th/0701188]; I. de Medeiros Varzielas, S. F. King and G. G. Ross, Phys. Lett. B **648**, 201 (2007) [arXiv:hep-ph/0607045]; E. Ma, Mod. Phys. Lett. A **21**, 1917 (2006) [arXiv:hep-ph/0607056]; Phys. Lett. B **660**, 505 (2008) [arXiv:0709.0507 [hep-ph]].

[19] H. Ishimori, T. Kobayashi, H. Okada, Y. Shimizu and M. Tanimoto, arXiv:0811.4683 [hep-ph]; J. A. Escobar and C. Luhn, J. Math. Phys. **50**, 013524 (2009) [arXiv:0809.0639 [hep-th]].

[20] H. Abe, T. Kobayashi and H. Ohki, JHEP **0809**, 043 (2008) [arXiv:0806.4748 [hep-th]].

[21] H. Abe, K. S. Choi, T. Kobayashi and H. Ohki, Nucl. Phys. B **814**, 265 (2009) [arXiv:0812.3534 [hep-th]].

[22] R. Blumenhagen, M. Cvetic, F. Marchesano and G. Shiu, JHEP **0503**, 050 (2005) [arXiv:hep-th/0502095].

[23] N. Haba, A. Watanabe and K. Yoshioka, Phys. Rev. Lett. **97**, 041601 (2006) [arXiv:hep-ph/0603116].

[24] T. Kobayashi, Y. Omura and K. Yoshioka, Phys. Rev. D **78**, 115006 (2008) [arXiv:0809.3064 [hep-ph]].

[25] G. Seidl, arXiv:0811.3775 [hep-ph].

[26] T. Araki, T. Kobayashi, J. Kubo, S. Ramos-Sanchez, M. Ratz and P. K. S. Vaudrevange, Nucl. Phys. B **805**, 124 (2008) [arXiv:0805.0207 [hep-th]].

[27] T. Araki, K. S. Choi, T. Kobayashi, J. Kubo and H. Ohki, Phys. Rev. D **76**, 066006 (2007) [arXiv:0705.3075 [hep-ph]].