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Reachability Computation for Power System Transient Stability

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Abstract— We perform reachability computation for an analysis of continuous and hybrid models for transient stability of the Single Machine-Infinite Bus (SMIB) system. Analysis algorithms developed in this paper are based on the iterative computation of reachable sets for continuous and hybrid systems. They accurately estimate stability regions of the models and thereby can evaluate transient stability of the SMIB system with switching operations.

1. Introduction

Current trends in the introduction of switching capabilities in power systems for fulfilling various operational objectives have made the analysis and control of power systems become a difficult task (see [1, 2]). Especially, transient stability of power systems is one of the major concerns as any improper transient introduced by a switching operation may trigger a sequence of irreversible actions resulting in catastrophic failures and widespread blackouts [3, 4]. Hybrid systems [5, 6, 7] have been proposed for the modeling of systems composed of discrete and continuous dynamical systems. Several researchers have proposed the use of hybrid systems for the analysis and control of power systems with switching phenomena [8, 9, 10, 11, 12, 13]. In [12] the authors proposed to use the model of hybrid automaton and its reachability for transient stability analysis of power systems with switching operations. The analysis was performed by extensive numerical simulations of backward reachable sets in the hybrid model.

In this paper, we propose to extend the computational approach developed in [14] for the analysis of continuous and hybrid models for transient stability of the Single Machine-Infinite Bus (SMIB) system performed in [12]. In [14] the authors proposed a computational approach for the algorithmic estimation of stability regions of nonlinear models with attractive equilibrium points via the use of the Level Set Methods (LSM) for computing reachable sets. For transient stability analysis of a power system, it is necessary to estimate stability regions of a nonlinear model representing electromechanical dynamics of the system [15], which we term the transient stability region. In [16] the authors computed transient stability regions of the continuous-time classical models using the LSM. On the other hand, in this paper we show that the LSM for hybrid automaton makes it possible to estimate transient stability regions of the SMIB system *with* switching operations. This implies that the technique developed for hybrid and embedded systems offers an effective computation platform for power system analysis.

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2. Computational Approach for Transient Stability Analysis

This section introduces mathematical models for transient stability analysis of the SMIB system , which are developed in [12], and the concept of algorithmic analysis considered in this paper.

2.1. Mathematical Model

In the following, we introduce the mathematical definition of hybrid automaton.

Definition 1 (Hybrid Automaton [6])
A hybrid automaton is a collection $H = (Q, X, V, Y, Init, f, D, E, G)$ *where* $Q = \{q_1, \ldots, q_N\}$ *is a set of discrete states;* $X \subseteq \Re^n$ *is a continuous state space;* $V = \{\sigma_1, \ldots, \sigma_N\}$ *is a set of input symbols;* $Y = X$ *is a continuous output space;* $Init \subseteq Q \times X$ *is a set of initial conditions;* $f : Q \times X \to \Re^n$ *assigns to every discrete state a Lipschitz continuous vector field on* X*;* $D : Q \to 2^X \times V$ *assigns each* $q \in Q$ *an invariant set;* $E \subseteq Q \times Q$ *is a collection of discrete transitions;* $G : E \to V$ *assigns each* $e = (q, q') \in E$ *a guard.* $\square$

The hybrid automaton model of the SMIB system with clearing and re-closing operations is shown in Fig. 1. In the model, f describes the so-called classical model with constant voltage behind reactance [15]. $Q = \{q_1, q_2, q_3\}$ consists the three modes with fault-on (q_1), 1 line operation (q_2), and 2 lines operation (q_3). $V = \{\sigma_1, \sigma_2\}$ is the finite set in which σ_1 and σ_2 correspond to the inputs symbols for clearing and re-closing operation, respectively. Notice that the discrete transition of the model is triggered by timed discrete input, $\sigma = \sigma_1$ for $t = t_\mathrm{c}$ and $\sigma = \sigma_2$ for $t = t_\mathrm{r}$ where t_c and t_r correspond to the time instants for clearing and re-closing operation, respectively. In addition, we have $X = S^1 \times \Re$, $x = (\delta, \omega)^\mathrm{T} \in X$, $Init = \{q_1\} \times X$, $D(q) = X$, $\forall q \in Q$, $P_\mathrm{M} = 0.2$, $b = 0.7$, and $k = 0.05$. The parameter settings are based on [12].

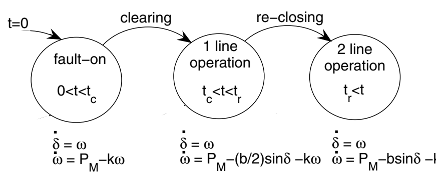


Figure 1: Switching sequence governing the SMIB system with clearing and re-closing operations.

In order to perform stability analysis, an unsafe set $G \subseteq X$ is defined for capturing the subset in the state space in which the power system cannot be safely operated. As stated in [12], G can be defined as

$G \triangleq \{x \in X \,|\, \Omega_1(x) \leq 0\}$ where $\Omega_1(x) = \omega_c^2 - \omega^2$ with $\omega_c = \pi$, and the usable part UP can be characterized as $UP = \{x \in \partial G \,|\, \mp 2\pi(P_M - b \sin\delta \pm \pi k) < 0\}$. Define an implicit functions $\Omega_2(x) = \pm 2\pi(P_M - b\sin\delta \pm \pi k)$. Hence, $UP = \partial G \setminus B$ where $\partial G = \{x \in X \,|\, \Omega_1(x) = 0\}$ and $B = \{x \in X \,|\, \Omega_2(x) \leq 0\}$ can be constructed via the use of logical combinations of the associated implicit functions.

In the next subsection, we will propose a computational approach for the algorithmic estimation of transient stability regions of the the hybrid model and the continuous model by considering only the 2 lines operation mode. The proposed approach is easily extended to the analysis of hybrid automaton model: see Sec. 3.2.

2.2. Algorithmic Analysis

Consider an ordinary differential equation in the form:

$$\dot{x}(t) = f(x(t)), \; x(0) \in P, \; t \geq 0 \tag{1}$$

where the set $P = \{x \in X | J(x, 0) \leq 0\}$ with $J : X \times \Re \rightarrow \Re$. In the Level Set Methods (LSM) [17, 18], the interface of P, ∂P, is defined as the zero level set of the higher dimensional function $J(x,t)$ at time t, *i.e.* $J(x,t) = 0$, $\forall t$. Furthermore, the interface evolution of P can be depicted by the Level Set Equation (LSE):

$$\dot{J}(x,t) + \nabla J(x,t) \cdot f(x) = 0. \tag{2}$$

Hence, the forward reachable set of (1) starting from P with time Δt can be represented as: $Post_{\Delta t}(P) = \{x \,|\, \exists \tau \in [0, \Delta t], J(x,\tau) \leq 0\}$. Since the system (1) is time-invariant, the backward reachable set of (1) starting from P with time Δt can be represented as: $Pre_{\Delta t}(P) = \{x \,|\, \exists \tau \in [0, \Delta t], J'(x,\tau) \leq 0\}$ where $J'(x,t)$ is the solution of (2) with $f(x) := -f(x)$. Hence, the forward and backward reachable sets can be computed by solving the corresponding LSE.

As shown in [17, 18], the LSE can be solved numerically by using the Method of Lines (MOL) normally with the state space partitioned by a Cartesian grid, and the MOL assumes that the spatial discretization can be separated from temporal discretization in a semi-discrete manner that allows the temporal discretization of the LSE to be treated independently as an ordinary differential equation. There exist combinations of spatial discretization scheme and temporal spatial discretization scheme to ensure the convergence of numerical solutions.

Table 1: The Proposed Algorithm

1. Set $S_0 = P$. Set $t = 0$.
2. Repeat $k = 0, 1, 2, \cdots$

   Compute $S_{k+1} = S_k \cup Pre_{\Delta t}(S_k)$

   Until $S_{k+1} \subseteq S_k$ or $k\Delta t > T$.
3. Set $\hat{S} = S_{k+1}$

In this paper, we propose to use the backward reachability algorithm reported in [14] for the estimation of stability regions for the continuous model (1). The algorithm is shown in Tab. 1. In the algorithm, starting from the initial set S_0, the backward reachable set is computed iteratively for an integer multiple of Δt. There are two termination conditions, either the computation has reached a fixed point solution, or the total amount of time exceeds the specified limit T. The algorithm terminates due to the two following reasons. Firstly, the algorithm terminates at a fixed point solution, *i.e.* $S_{k+1} \subseteq S_k$ for some k, which indicates that there is no new state can be reached. However, the termination can also happen due to the finite representation of reachable sets on a Cartesian grid. Secondly, if the number of iterations exceeds the predefined limit, the algorithm will terminate due to the second condition. Given an initial set P, the proposed algorithm attempts to synthesize a subset of X such that if a state starts from the subset, it can eventually reach P (at most) in bounded time T. As defined in [6], the property $\Diamond P$ is *True* if $\exists t \geq 0 \; x(t) \in P$, but *False* otherwise. Therefore, the proposed algorithm is also regarded as the $\Diamond$ algorithm by considering $\Diamond$ for *eventually* ($\exists$). In the computation for enabling the transient stability analysis, the fifth order upwind HJ-(W)ENO, Hamilton-Jacobi (Weighted) Essentially Non-Oscillatory, approximation scheme and the third order TVD (Total Variation Diminishing) Runge-Kutta are used to ensure numerical convergence. Furthermore, the executable code in each computation is generated by ReachLab [19] platform automatically and deployed in a quad-core machine for implementation.

3. Transient Stability Analysis of the SMIB System

In this section, we use the $\Diamond$ algorithm for the estimation of transient stability regions of the continuous and hybrid models for the SMIB system.

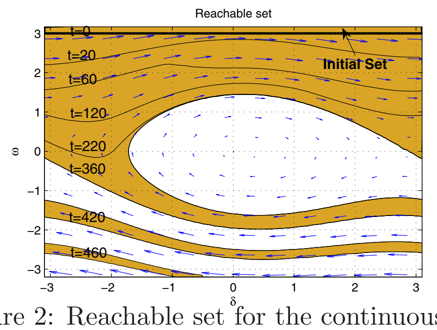


Figure 2: Reachable set for the continuous case.

3.1. Continuous Case

Consider the continuous dynamics in the the mode of 2 lines operation (q_3). We are interested to perform the estimation of transient stability regions by using the $\Diamond$ algorithm. Define the set of interest P as $[-\pi, \pi] \times [\pi - 0.1, \pi + 0.1]$. By applying the $\Diamond$ algorithm, the backward reachable set for the continuous model is computed iteratively as shown in Fig. 2. The analysis set X is $[-\pi, \pi] \times [-\pi, \pi]$, the grid size Δx is 0.02×0.02, the time step Δt is 0.01, and the computation time is about 3 hours. As the time step increases, the backward reachable set expands into the state space and approaches to the stability region of the asymptotically stable equilibrium point $x_e = (\sin^{-1}(P_M/b), 0)$. For finite time, the complementary set to the reachable set contains the stability region and hence a necessary condition of transient stability of the SMIB system. The complementary set gives an over-approximation of the stability region in the continuous case. The present result in Fig. 2 coincides with the previous one in [12] based on the direct numerical integration of the continuous model.

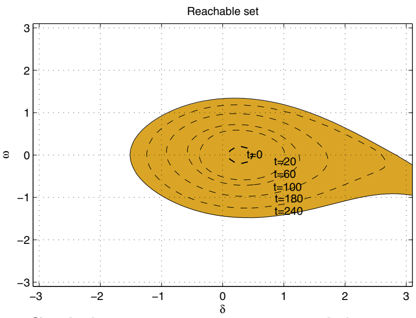


Figure 3: Stability region estimated by applying the $\diamond$ algorithm for the continuous case.

In the continuous case, we can apply the $\diamond$ algorithm as proposed in [14] by choosing a sufficiently small set of initial conditions in the neighborhood of the asymptotically stable equilibrium point x_{e}. The analysis set X is $[-\pi, \pi] \times [-\pi, \pi]$, the initial set is a circle whose center is $(0.289752,\ 0)$, the radius is 0.2, the grid size Δx is 0.05×0.05, the time step Δt is 0.01, and the computation time is about 2 hours. Fig. 3 shows the result of the $\diamond$ algorithm for the estimation of stability region of power systems. The $\diamond$ algorithm expands the reachable set from the initial set. This algorithm terminated until the reachable set stops growing. The dashed boundaries inside the *yellow* region are the reachable sets for each time period. The reachable sets also approach to the true stability region as time increases. The set for each time is contained in the true region and hence gives a sufficient condition of the transient stability. It becomes conservative for short computation time.

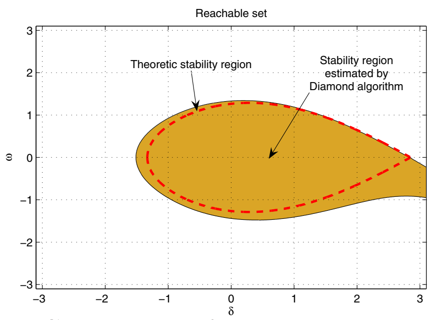


Figure 4: Comparison of stability regions estimated with the $\diamond$ algorithm and the closest u.e.p. method. Theoretical stability region denotes the sufficient condition based on the closet u.e.p. method.

Before closing the continuous case, we compare the two different results obtained by the $\diamond$ algorithm and the closet u.e.p. method. Fig. 4 shows the comparison of stability regions estimated with the $\diamond$ algorithm and the theoretical sufficient condition based on the closet u.e.p method. The inner dashed boundary colored in *red* corresponds to the sufficient condition based on the closest u.e.p. method. The region colored in *yellow* is the stability region estimated with the $\diamond$ algorithm. In Fig. 4 we see that the stability region estimated with the $\diamond$ algorithm, on one hand, is close to the true region characterized by the stable manifold of the saddle equilibrium point $(\pi - x_{\mathrm{e}}, 0)$ and becomes less conservative than the sufficient condition. But, on the other hand, it includes the states that can eventually reach the stable manifold. Due to the finite representation, the estimation error is observed in the comparison of the region. This is mainly due to the size of the grid used in computation. However, according to the Lax-Richtmyer equivalence theorem [18], the combination of the fifth order HJ WENO scheme and third order TVD Runge-Kutta used in this paper can be shown to be *convergent*, *i.e.* the correct solution can be obtained as $\Delta x \to 0$ and $\Delta t \to 0$. Hence, the estimation error can be reduced by using finer finite representations in both space and time domains.

3.2. Hybrid Case

Although the $\diamond$ algorithm is developed for continuous model, for the hybrid automaton for the SIMB system *with* switching operations, we can still use the $\diamond$ algorithm for the transient stability analysis by slight modification. Since each discrete transition is triggered by time, the basic structure of the algorithm remains. However, only the dynamics needs to be changed once a discrete transition occurs.

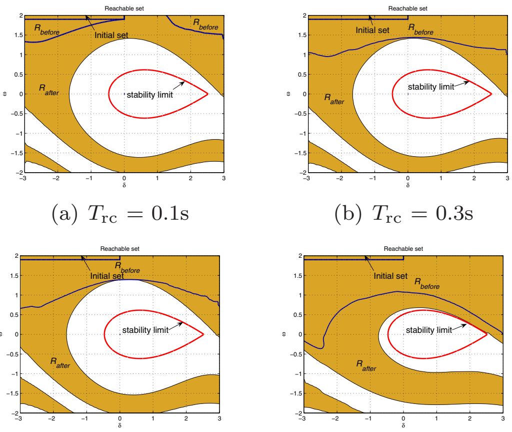
In this paper we use the modified $\diamond$ algorithm to estimate the transient stability region in the hybrid case. Based on the $\diamond$ algorithm as shown in Tab. 1, the modifications are made in Step 2. by 1) inserting if-then-else statements as shown in below in between the *Repeat* and *Compute* statements for modeling the discrete transitions:

if $k\Delta t = 0$, then $q = q_1$
else if $k\Delta t = t_{\mathrm{c}}$, then $\sigma = \sigma_1$ and $q = q_2$
else if $k\Delta t = t_{\mathrm{r}}$, then $\sigma = \sigma_2$ and $q = q_3$
end if

and 2) redefining $f(x) := f(q, x)$ as the vector field for the computation of $Pre_{\Delta t}(S_k)$ in the *Compute* statement.

The numerical result by applying the modified $\diamond$ algorithm to the hybrid automaton model with one switching σ_2 is shown in Fig. 5. The analysis set X is $[-\pi, \pi] \times [-2, 2]$, the initial set is $[-\pi, 0] \times [2 - 0.2, 2]$, the grid size Δx is 0.05×0.05, the time step Δt is 0.01, and the computation time is about 3 hours. In each figure, the non-colored (*white*) region corresponds to the stability region estimated by applying the modified $\diamond$ algorithm to the hybrid automaton model. The *red* closed loop, denoted by "stability limit," stands for the sufficient condition based on the closest u.e.p. method for the 1 line operation mode and still becomes a sufficient condition in the hybrid case (see [12]).

Notice that R_{before} is the subset of X from which any state reaches to ∂G before the re-closing; and R_{after} is the one from which any state reaches to ∂G after the re-closing. If a smaller gird size is chosen in this figure, the estimation error could be reduced. The numerical result coincides with the previous one in [12] based on the direct numerical integration of the hybrid automaton model. Thus the LSM is also effective for the estimation of transient stability regions of the SMIB system with switching operations. Figure 5 describes how the reachable set changes as the re-closing period $T_{\mathrm{rc}} = t_{\mathrm{r}} - t_{\mathrm{c}}$ increases where T_{rc} is the parameter that varies the effect of re-closing operation σ_2 to the transient stability. As T_{rc} increases, the non-colored region approaches the sufficient condition of stability region for the continuous case, which is based on the closet u.e.p. method. This is exactly true because the limit of infinite T_{rc} ($T_{\mathrm{rc}} \to \infty$) yields the continuous case without the re-closing operation, which is modeled by the switching (σ_2) from the 1 line operation (q_2) to the 2 lines one (q_3). In Fig. 5 we see that the estimation of transient stability regions based on the LSM is, needless to say, less conservative than that in the continuous case based on the closest u.e.p. method. This clarifies that by explicitly considering the switching operation, namely, hybrid nature of system operation, we can accurately evaluate transient stability of the SMIB system. Thus the LSM for hybrid systems provides an effective computation platform for power system analysis.



(a) $T_{\mathrm{rc}} = 0.1$s (b) $T_{\mathrm{rc}} = 0.3$s

(c) $T_{\mathrm{rc}} = 0.5$s (d) $T_{\mathrm{rc}} = 0.8$s

Figure 5: Reachable sets in the hybrid case.

4. Conclusion

In this paper we proposed reachability computation for the analysis of continuous and hybrid models for transient stability of the SMIB system. Analysis algorithms were developed for the estimation of transient stability regions of the models via the iterative computation of reachable sets. LSM was then used for enabling reachable set computation for the continuous and hybrid models with nonlinear ordinary differential equations. The estimations of transient stability regions based on the proposed algorithms were compared with their theoretical sufficient conditions based on closet u.e.p. method. Thereby we show that the proposed algorithms are effective for the estimation of transient stability regions in both the continuous and hybrid models and provide a new computational approach to the direct analysis of transient stability in power systems *with* switching operations. The accuracy of approximation of transient stability regions can be improved if finer finite representations in both space time domains are used in the numerical methods. Future directions are to extend the developed algorithm to analysis of multi-machine power systems, to resolve the computational complexity in the case of high-dimensional nonlinear differential equations, and to use the algorithm for controller synthesis for enhancement of transient stability under (uncertain) disturbances.

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