Cp*Li as a Base: Application to Palladium-Catalyzed Cross-Coupling Reaction

of Aryl-X or Alkenyl-X (X = I, Br, OTf, ONf) with Terminal Acetylenes

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Abstract: The reaction of aryl-X or alkenyl-X (X = I, Br, OTf, ONf) with terminal acetylenes in the

presence of a catalytic amount of Pd(OAc)₂ provided the alkynylated products in good yields by

using Cp*Li (Cp* = 1,2,3,4,5-pentamethylcyclopentadienyl) as a base.

Keywords: pentamethylcyclopentadiene, cross-coupling reaction, lithium acetylides, alkynylation

Introduction

Pentamethylcyclopentadienide (Me₅C₅⁻, Cp*⁻) has been used as a ligand of transition

metal complexes for forty years.¹ Although Cp*- has unique properties, such as six-electron

donation and steric bulkiness by the five methyl groups on the five-membered ring, it has not been

used for other purposes. Then we have been exploring applications of Cp*- as a reagent in organic

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synthesis.² Now, we have been interested in Cp^{*-} as a base. We expected that its strong basicity and steric bulkiness would exhibit interesting reactivities. We have found that the reaction of aryl-X or alkenyl-X (X = I, Br, OTf, ONf) with terminal acetylenes in the presence of a catalytic amount of palladium provided the alkynylated products in good yields by using Cp^*Li as a base.

Results and Discussions

Treatment of a suspension of Cp*Li in THF with iodobenzene (1a), 1-octyne (2a), and a catalytic amount of Pd(OAc)₂ at reflux for 1 h gave 1-octynylbenzene (3aa) in 82% yield (Table 1, entry 7). Other lithium bases, such as *n*-BuLi, lithium diisopropylamide (LDA), lithium hexamethyldisilazide (LHMDS), and lithium cyclopentadienide (CpLi) did not assist the coupling reaction efficiently (entries 1–4). However, lithium indenide resulted in formation of 3aa in 24% yield (entry 5). Interestingly, the reaction with lithium tetramethylcyclopentadienide, which lacks one methyl group compared with Cp*Li, gave 3aa in lower yield than that with Cp*Li yet in higher yield than that with CpLi (entry 6).

Although many kinds of alkynylmetals have been used for palladium-catalyzed cross-coupling reaction of aryl-X or alkenyl-X, there are no reports on the cross-coupling reaction with lithium acetylides.³ The coupling reaction with lithium acetylides is difficult because the reaction of a palladium catalyst with an excess amount of lithium acetylides provides lithium palladate **4**, which does not exhibit any catalytic activity (eq 1).⁴ We thought that gradual formation of lithium acetylides by means of Cp*Li would avoid the formation of **4**.

Table 1. Effect of lithium bases on palladium-catalyzed cross-coupling reaction of iodobenzene (1a) with 1-octyne (2a)

ent	ry lithium base	NMR yield /%	
1	n-BuLi	3	
2	lithium diisopropylamide (LDA)	3	
3	lithium hexamethyldisilazide (LHMDS)	3	
4	lithium cyclopentadienide (CpLi)	3	
5	lithium indenide ^a	24	
6	lithium tetramethylcyclopentadienide	55	
7	Cp*Li	82 ^b	

^a Derived from indene and *n*-BuLi. ^b Isolated yield.

To confirm the gradual generation of lithium acetylides, we carried out the following experiments. Treatment of lithium acetylide 5 with Cp*H for 1 h at reflux followed by an addition of benzaldehyde at 0 °C gave a mixture of octynylated alcohol 6 and pentamethylcyclopentadienylated alcohol 7 (eq 2). On the other hand, 7 was obtained selectively by using the reaction mixture of 5 and Cp*H which was treated for 11 h at reflux. These facts indicate that the pKa value of Cp*H is smaller than that of 2a and that a high energy barrier exists

between 2a + Cp*Li and 5 + Cp*H. A similar reaction with cyclopentadiene (CpH), instead of Cp*H, only for 1 h followed by an addition of benzaldehyde did not give 6 but gave Cp adduct-derived compounds such as 6-phenylfulvene. This result means that the energy barrier between 2a + CpLi and 5 + CpH is lower than that between 2a + Cp*Li and 5 + Cp*H. Not only the pKa values of lithium bases but also the high energy barrier is important to promote the cross-coupling reaction. The reaction of a mixture of 2a and 2a a

Table 2 summarizes the results obtained by the Cp*Li-mediated reaction of aryl iodides 1 with terminal acetylenes 2. Aromatic iodide 1 bearing an electron-donating group (Table 2, entry 7) or an electron-withdrawing group (entry 8) afforded 3 in high yields. Surprisingly, ester (entry 10) or cyano moiety (entry 11) on the aromatic ring did not undergo nucleophilic addition reaction of

Cp*Li or lithium acetylides during thermal conditions. Reaction of *meta*- or *ortho*-substituted 1 also proceeded smoothly (entries 12, 13). 1-Naphthyl iodide could be used in this reaction (entry 14). The reaction with phenylacetylene (2c) or trimethylsilylacetylene (2e) afforded 3 in good yields by using 15 mol% of triphenylphosphine (entries 3, 5).

Table 2. Reactions of various aryl iodides 1 with terminal acetylenes 2

Cp*Li (1.5 equiv) R ² Pd(OAc) ₂ (7.5 mol%)						
	Cp*L H_C_C_R^2 Pd(C_TI	DAc) ₂ (7.5 mol%) HF, reflux, 1 h	C			
R ¹	1 2 (1.5 equiv)		R ¹ 3			
entry	\mathbb{R}^1	\mathbb{R}^2	isolated yield /%			
1	H (1a)	n-C ₆ H ₁₃ (2a)	82 (3aa)			
2	1a	c-C ₆ H ₁₁ (2b)	85 (3ab)			
3	1a	Ph (2c)	93 (3ac) ^a			
4	1a	TES (2d)	69 (3ad) ^b			
5	1a	TMS (2e)	72 (3ae) ^{a,b}			
6	<i>p</i> -Me (1b)	2a	81 (3ba)			
7	<i>p</i> -OMe (1c)	2a	82 (3ca)			
8	<i>p</i> -CF ₃ (1d)	2a	70 (3da)			
9	<i>p</i> -Cl (1e)	2a	71 (3ea)			
10	<i>p</i> -COOEt (1f)	2a	83 (3fa)			
11	<i>p</i> -CN (1g)	2a	74 (3ga)			
12	<i>m</i> -Me (1h)	2a	83 (3ha)			
13	o-Me (1i)	2a	83 (3ia) ^c			
14	1-Naphthyl iodide (1	j) 2a	67 (3ja)			

^a 15 mol% of PPh₃ was added. ^b The reaction was carried out at 50 °C for 2 h. ^c The reaction was carried out for 1.5 h.

Not only aryl iodides but also aryl bromides, triflate, or nonaflate could be also used as substrates in the Cp*-mediated reaction (Table 3). In these cases, triphenylphosphine was essential as a ligand.

Table 3. Reactions of various aryl bromides, triflates or nonaflates with 1-octyne (2a)

R X	Р	p*Li (1.5 equiv) d(OAc) ₂ (7.5 mol%) Ph ₃ (15 mol%) THF, reflux, 1 h	R	.C ^E .C n-C ₆ H ₁₃
entry	X	R	isolated	Lyield /%_
1	Br	Н	79	(3aa)
2	Br	<i>p</i> -OMe	74 ((3ca) ^c
3	Br	p-CF ₃	79	(3da)
4	$\mathrm{OTf}^{\mathrm{a}}$	Н	58	(3aa)
5	ONf^b	Н	88 ((3aa) ^c

 $^{^{}a}$ Tf = trifluoromethanesulfonyl b Nf = 1,1,2,2,3,3,4,4,4-nonafluoro-1-butanesulfonyl c The reaction was carried out for 1.5 h.

By utilizing the advantages of Cp*Li as a base, we find two applications. The reaction of 2-iodobenzoic acid with **2** provided isocoumarin **8** and (*Z*)-3-alkylidenephthalide **9** in good yields (eq 4).⁵ We also found that Cp*Li can be used as a bulky base to prepare lithium enolates. With this reactivity, we accomplished one-pot synthesis of enynes **12** from ketone **10** *via* nonaflate **11** in good yields (eq 5).

Cp*Li (2.5 equiv)
Pd(OAc)₂
(7.5 mol%)
THF, reflux, 1 h
R²
(7.5 mol%)
R²

$$R^2$$

 R^2
 R^2

$$\begin{array}{c} \text{1) Cp*Li (2.5 equiv)} \\ \text{2) NfF* (1.2 equiv)} \\ \text{THF, 0 °C} \\ \end{array} \begin{array}{c} \text{ONf} \\ \end{array} \begin{array}{c} \text{Pd(OAc)}_2 \text{ (7.5 mol\%)} \\ \text{PPh}_3 \text{ (15 mol\%)} \\ \text{reflux, 1 h} \\ \end{array} \\ \text{12} \\ \text{12a: R = } n\text{-}C_6H_{13}, 64\% \\ \text{12b: R = Ph, 70\%} \\ \text{12c: R = TMS, 69\% (50 °C, 2 h)} \end{array}$$

Experimental

Instrumentation and Materials

¹H NMR (500 MHz) and ¹³C NMR (125.7 MHz) spectra were taken on a Varian UNITY INOVA 500 spectrometer unless otherwise noted. Chemical shifts (δ) are in parts per million relative to chloroform at 7.26 ppm for ¹H and at 77.0 ppm for ¹³C. IR spectra were determined on a SHIMADZU FTIR-8200PC spectrometer. TLC analyses were performed on commercial glass plates bearing 0.25-mm layer of Merck silica gel 60F₂₅₄. Silica gel (Wakogel 200 mesh) was used for column chromatography. Gel permeation chromatography (GPC) was performed by LC-908 (Japan Analytical Industry Ltd., two in-line JAIGEL-2H, toluene, 7.8 mL/min, UV and RI detectors). Elemental analyses were carried out at the Elemental Analysis Center of Kyoto University.

Unless otherwise noted, materials obtained from commercial suppliers were used without further purification. 1,2,3,4,5-Pentamethyl-1,3-cyclopentadiene was purchased from Kanto Chemical Co. THF (dehydrated, stabilizer free) was purchased from Kanto Chemical Co., stored under nitrogen, and used without distillation. All reactions were carried out under argon atmosphere.

General Procedure for Palladium-Catalyzed Cross-Coupling Reaction of Aryl Iodides 1 with Terminal Acetylenes 2 Using Cp*Li: A solution of *n*-BuLi in hexane (1.60 M, 0.47 mL, 0.75 mmol) was added to a solution of 1,2,3,4,5-pentamethyl-1,3-cyclopentadiene (0.13 mL, 0.80 mmol) in THF (5 mL) at 0 °C. The mixture was stirred for 10 min at the same temperature. Pd(OAc)₂ (8.4 mg, 0.038 mmol), 2a (82.7 mg, 0.75 mmol) in THF (0.5 mL), and 1a (102 mg, 0.50 mmol) in THF (0.5 mL) were added to the reaction mixture. The resulting mixture was stirred for 1 h at reflux. The reaction mixture was quenched with saturated aqueous NH₄Cl, and the mixture was extracted with hexane. The combined organic parts were washed with brine, dried over anhydrous Na₂SO₄, and concentrated *in vacuo* to give a crude oil. The oil was purified by chromatography on silica gel (Wakogel C-200, hexane) to afford 3aa (75.5 mg, 0.41 mmol, 82%).

General Procedure for Palladium-Catalyzed Cross-Coupling Reaction of Aryl Bromide, Aryl Triflate, Aryl Nonaflate with 1-Octyne (2a) Using Cp*Li: A solution of *n*-BuLi in hexane (1.60 M, 0.47 mL, 0.75 mmol) was added to a solution of 1,2,3,4,5-pentamethyl-1,3-cyclopentadiene (0.13 mL, 0.80 mmol) in THF (5 mL) at 0 °C. The white suspension was stirred for 10 min at the same temperature. Pd(OAc)₂ (8.4 mg, 0.038 mmol), PPh₃ (19.7 mg, 0.075 mmol), 1-octyne (82.7

mg, 0.75 mmol) in THF (0.5 mL), and bromobenzene (78.5 mg, 0.50 mmol) in THF (0.5 mL) were added to the reaction mixture, and the mixture was stirred for 1 h at reflux. The reaction was terminated with saturated aqueous NH₄Cl, and the mixture was extracted with hexane. The combined organic layers were washed with brine, dried over anhydrous Na₂SO₄, and concentrated *in vacuo* to give a crude oil. Chromatographic purification on silica gel (Wakogel C-200, hexane) afforded 3aa (73.4 mg, 0.39 mmol, 79%).

General Procedure for Palladium-Catalyzed Annulation of 2-Iodobenzoic Acid with Terminal Acetylenes 2 Using Cp*Li: A solution of *n*-BuLi in hexane (1.60 M, 0.78 mL, 1.25 mmol) was added to a solution of 1,2,3,4,5-pentamethyl-1,3-cyclopentadiene (0.20 mL, 1.3 mmol) in THF (5 mL) at 0 °C. The mixture was stirred for 10 min at 0 °C. Pd(OAc)₂ (8.4 mg, 0.038 mmol), 2a (82.7 mg, 0.75 mmol) in THF (0.5 mL), and 2-iodobenzoic acid (124 mg, 0.50 mmol) in THF (0.5 mL) were added to the reaction mixture, and the mixture was stirred for 1 h at reflux. After being quenched with water, the mixture was extracted with ethyl acetate. The combined organic parts were washed with brine, dried over anhydrous Na₂SO₄, and concentrated *in vacuo* to give a crude oil. Purification on silica gel (Wakogel C-200, hexane – ethyl acetate = 20:1) afforded a mixture of 8a and 9a (8a:9a = 68:32, 80.8 mg, 0.35 mmol, 70%).

General Procedure for Palladium-Catalyzed One-Pot Synthesis of Enynes 12 from 10 via 11: A solution of *n*-BuLi in hexane (1.60 M, 0.78 mL, 1.25 mmol) was added to a solution of 1,2,3,4,5-pentamethyl-1,3-cyclopentadiene (0.20 mL, 1.3 mmol) in THF (5 mL) at 0 °C. The mixture was stirred for 20 min at the same temperature. Cyclohexanone (49.1 mg, 0.5 mmol) in THF (0.5 mL) was added to the resulting white suspension, and the mixture was stirred for 30 min

at 0 °C. 1,1,2,2,3,3,4,4,4-Nonafluoro-1-butanesulfonyl fluoride (181 mg, 0.60 mmol) in THF (0.5 mL) was added to the reaction mixture, and the mixture was stirred for 30 min at 0 °C. 1-Octyne (2a, 82.7 mg, 0.75 mmol) in THF (0.5 mL), PPh₃ (19.7 mg, 0.075 mmol), and Pd(OAc)₂ (8.4 mg, 0.038 mmol) were added to the reaction mixture, and the mixture was stirred for 1 h at reflux. Water was added to quench the reaction, and the mixture was extracted with hexane. The combined organic parts were washed with brine, dried over anhydrous Na₂SO₄, and concentrated *in vacuo* to give a crude oil. The oil was purified by chromatography on silica gel (Wakogel C-200, hexane) to afford 12a (60.3 mg, 0.32 mmol, 64%).

Preparation of Phenyl 1,1,2,2,3,3,4,4,4-nonafluoro-1-butanesulfonate: A solution of *n*-BuLi in hexane (1.60 M, 3.44 mL, 5.5 mmol) was added to a solution of phenol (471 mg, 5.0 mmol) in THF (17 mL) at 0 °C. The mixture was stirred for 10 min at the same temperature. 1,1,2,2,3,3,4,4,4-nonafluoro-1-butanesulfonyl fluoride (1.81 g, 6.0 mmol) in THF (1 mL) was added to the reaction mixture, and the mixture was stirred for 30 min at room temperature. The reaction was quenched with water, and the mixture was extracted with ethyl acetate. The combined organic parts were washed with brine, dried over anhydrous Na₂SO₄, and concentrated *in vacuo* to give a crude oil. The oil was purified by chromatography on silica gel (Wakogel C-200, hexane – ethyl acetate = 40:1) to afford phenyl 1,1,2,2,3,3,4,4,4-nonafluoro-1-butanesulfonate (1.52 g, 4.0 mmol, 81%).

Characterization Data

The compounds 3aa, 3ac and 3ae are commercially available. The products 3ab, 3ad, 3ba, 3ba, 3ac and 3ae are commercially available.

3ca, ⁸ 3da, ⁸ 3ea, ⁹ 3ga, ⁸ 3ha, ⁸ 3ia, ⁸ 3ja, ¹⁰ 12a, ¹¹ 12b, ¹² and 12c¹² can be found in the literature. ethyl 4–(1–octynyl)benzoate (3fa): IR (neat) 2932, 2859, 1720, 1608, 1466, 1367, 1306, 1272, 1175, 1106, 1097, 1021, 857, 770, 697 cm⁻¹; ¹H NMR (CDCl₃) δ 0.91 (t, J = 7.0 Hz, 3H), 1.28–1.64 (m, 8H), 1.39 (t, J = 7.0 Hz, 3H), 2.42 (t, J = 7.0 Hz, 2H), 4.36 (t, J = 7.0 Hz, 2H), 7.41–7.45 (m, 2H), 7.93–7.97 (m, 2H); ¹³C NMR (CDCl₃) δ 14.04, 14.30, 19.50, 22.53, 28.54, 28.60, 31.33, 61.01, 80.12, 93.90, 128.81, 129.13, 129.33 (×2), 131.39 (×2), 166.20. Found: C, 78.92; H, 8.78%. Calcd for C₁₇H₂₂O₂: C, 79.03; H, 8.58%.

phenyl 1,1,2,2,3,3,4,4,4-nonafluoro-1-butanesulfonate: IR (neat) 1488, 1427, 1354, 1230, 1204, 1145, 1133, 1036, 891, 772, 686 cm⁻¹; 1 H NMR (CDCl₃) δ 7.28–7.31 (m, 2H), 7.37–7.42 (m, 1H), 7.44–7.48 (m, 2H); 13 C NMR (CDCl₃) δ 107.44–118.48 (m, ×4), 121.35 (×2), 128.35, 130.25 (×2), 149.89. Found: C, 31.87; H, 1.50%. Calcd for $C_{10}H_{5}F_{9}O_{3}S$: C, 31.93; H, 1.34%.

3-hexylisocoumarin (8a) and (*Z*)-3-heptylidenephthalide (9a) (68:32): IR (neat) 2929, 2857, 1782, 1733, 1658, 1484, 1161, 1024, 983, 759, 691 cm⁻¹; ¹H NMR (CDCl₃) δ 0.89 (t, *J* = 7.5 Hz, 3H), 1.24–1.74 (m, 8H), 2.47 (dt, *J* = 7.5, 7.5 Hz, 2×0.32H), 2.52 (t, *J* = 7.5 Hz, 2×0.68H), 5.64 (t, *J* = 7.5 Hz, 1×0.32H), 6.25 (s, 1×0.68H), 7.32–8.27 (m, 4H); ¹³C NMR (CDCl₃) δ 14.01, 14.05, 22.49, 22.56, 25.81, 26.85, 28.66, 28.96, 29.22, 31.48, 31.60, 33.51, 102.82, 109.76, 119.59, 120.10, 124.43, 124.98, 125.22, 127.49, 129.27, 129.48, 134.17, 134.67, 137.63, 139.58, 145.58, 158.33, 163.11, 167.21. Found: C, 78.13; H, 7.83%. Calcd for C₁₅H₁₈O₂: C, 78.23; H, 7.88%.

3-cyclohexylisocoumarin (**8b**) and (**Z**)-**3-cyclohexylmethylenephthalide** (**9b**) (**63:37**): IR (nujol) 1776, 1727, 1718, 1649 cm⁻¹; ¹H NMR (CDCl₃) δ 1.28–2.08 (m, 10H), 2.41–2.48 (m, 1×0.63H), 2.79–2.88 (m, 1×0.37H), 5.50 (d, J = 9.5 Hz, 1×0.37H), 6.23 (s, 1×0.63H), 7.34–8.27 (m, 4H);

¹³C NMR (CDCl₃) δ 25.63, 25.86 (×3), 25.92 (×2), 30.56 (×2), 32.85 (×2), 35.28, 41.84, 100.85, 115.04, 119.64, 120.26, 124.37, 125.20, 125.22, 127.45, 129.27, 129.42, 134.15, 134.60, 137.74, 139.78, 144.16, 162.35, 163.12, 167.25. Found: C, 79.13; H, 7.20%. Calcd for C₁₅H₁₆O₂: C, 78.92; H, 7.06%. Mp 78.0–79.0 °C.

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References and notes

- 1) Initial examples of the synthesis of Cp*-metal complexes: (a) King, R. B.; Bisnette, M. B. J. Organomet. Chem. 1967, 8, 287. (b) Röhl, H.; Lange, E.; Gössl, T.; Roth, G. Angew. Chem. 1962, 74, 155.
- 2) (a) Yagi, K.; Yorimitsu, H.; Oshima, K. *Tetrahedron Lett.* **2005**, *46*, 4831. (b) Uemura, M.; Yorimitsu, H.; Oshima, K. *Tetrahedron Lett.* **2006**, *47*, 163. (c) Uemura, M.; Yagi, K.; Iwasaki, M.; Nomura, K.; Yorimitsu H.; Oshima, K. *Tetrahedron* **2006**, *62*, 3523. (d) Uemura, M.; Yorimitsu, H.; Oshima, K. *Chem. Lett.* **2006**, *35*, 408. (e) Iwasaki, M.; Morita, E.; Uemura, M.; Yorimitsu, H.; Oshima, K. *Synlett* **2007**, 167.
- 3) (a) Negishi, E.; Xu, C. *In Handbook of Organopalladium Chemistry for Organic Synthesis*; Negishi, E. Ed.; Wiley-Interscience: New York, 2002; Vol. 1, pp 531. (b) Negishi, E.; Anastasia, L.

- Chem. Rev. 2003, 103, 1979.
- 4) Negishi, E.; Akiyoshi, K.; Takahashi, T. Chem. Commun. 1987, 477.
- 5) The stereochemistry of **9** was determined by comparison with their closely related analogue. Bellina, F.; Ciucci, D.; Vergamini, P.; Rossi, R. *Tetrahedron* **2000**, *56*, 2533.
- 6) Gong, J.; Fuchs, P. L. J. Am. Chem. Soc. 1996, 118, 4486.
- 7) Selina, A. A.; Karlov, S. S.; Gauchenova, E. V.; Churakov, A. V.; Kuz'mina, L. G.; Howard, J. A. K.; Lorberth, J.; Zaitseva, G. S. *Heteroatom Chem.* **2004**, *15*, 43.
- 8) Torres, G. H.; Choppin, S.; Colobert, F. Eur. J. Org. Chem. 2006, 1450.
- 9) Cai, M.-Z.; Song C.-S.; Huang, X. Synth. Commun. 1997, 27, 1935.
- 10) Alonso, D. A.; Nájera C.; Pacheco, M. C. Adv. Synth. Catal. 2003, 345, 1146.
- 11) Barluenga, J.; Yus, M.; Concellon, J. M.; Bernad, P.; Alvarez, F. J. Chem. Res. (S) 1985, 128.
- 12) Stille, J. K.; Simpson, J. H. J. Am. Chem. Soc. 1987, 109, 2138.

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$$\label{eq:constraint} \begin{split} Cp*Li \ \ as \ \ a \ \ Base: \ \ Application \ \ to \ \ Palladium-Catalyzed \ \ Cross-Coupling \ \ Reaction \ \ of \ \ Aryl-X \ or \ Alkenyl-X \ (X=I,Br,OTf,ONf) \ with \ Terminal \ Acetylenes \end{split}$$

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$$\begin{array}{c} X \\ \downarrow \\ R^{1} \\ X=I, \, Br, \, OTf, \, ONf \end{array} + \begin{array}{c} C \\ H \\ C \\ \end{array} \xrightarrow{C} \begin{array}{c} C \\ C \\ THF, \, \Delta \end{array} + \begin{array}{c} C \\ R^{1} \\ R^{1} \end{array}$$