Introduction of A New Principle in The Theory of Magnetism, I.

The Classical Derivation of The Meissner Effect in Perfect Conductors

SHUICHI IIDA

Department of Physics, The University of Tokyo, Tokyo, Japan

飯田修一

Synopsis

In the past theory of magnetism, there was an insufficient understanding in the processing of the magnetic energy of the externally applied field. By introducing a correct way of processing this energy, it was clarified that the Meissner effect is a classical property of the perfect conductors. It was proved rigorously that the perfectly diamagnetic state has the minimum local magnetic energies. Miss Van Leeuwen's theorem is wrong because it does not take account the cooperative nature of the magnetic energy of the system. By assuming $-(ev\cdot A)/c$, as the necessary additional to the statistical energy of the system, the London equation $V_{drift} = (e/mc)A$ has been derived directly. The same equation comes out kinematically as well by analyzing the transient collective motion of the conduction electrons in a perfectly conducting cylinder. It was pointed out that the surface state electrons which are runnig in the surface boundary region paramagnetically must have extremely high magnetic energies and can not be present so many thermodynamically and kinematically as well. This is the principal mechanism for having the perfect diamagnetism in perfect conductors. In order to elucidate the electro-thermodynamics of the persistent current system, a two doubly connected perfect conductor system is analyzed which clarifies the meaning of the Zeeman energy, the tricky structure of the magnetic energy, and another delicate electro-thermodynamical situation of the creation of the Meissner effect.

§1. Introduction

Although de Gennes has shown classically[1] that the Meissner state seems to have the lowest total energy, it has been believed that the Meissner effect is not the property of the classical perfect conductors. In recent several years, we have made an extensive study for reorganizing the classical electromagnetism in terms of the Maxwell-Lorentz electromagnetism[2]*, and have found that the superconductor in magnetism, just corres-

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脚注　物理学教室理論関係者と十数回にわたるコメントの交換があります。御必要の場合には関係者の了解を得て読むことができます。

*It will be noted that the complete understanding of this paper may need the understanding of this book, especially, of the contents of Chap. 12.

ponds to the perfect conductor in electricity and it looks very unrealistic that the Meissner effect could be outside of the classical physics. One essential idea originates from the existing belief that the field **H** will not penetrate into the inside of the perfect conductor because of the generation of induction currents at the surface, which will persist at least nearly eternally, because there is no electrical resistance. In this way, presnece of persistent currents is believed to be a virtue of perfect conductors. Because there is no resistence, the Maxwell equations allow this physical situation. Now, since we know that the current in metallic conductors are "the drift currents", theoretically the presence of this persistent current must require the presence of a very fundamental general principle in the interaction between the kinematics of the conduction electrons and the magnetic field. Encouraged by this expectation, we have made a certain study of the existing statistical theories on magnetism and have found that there was serious lack of understanding on the right processing of the magnetic energy of the system. The mistake comes out most distinctly in the statistical treatment of a superconductor. We believe now that quantum theory is necessary in order to get the perfect conduction, but, once perfect conduction could be realized, the Meissner effect comes out simultaneously.
This is the first paper which describes the essential point of the new way of understanding of the phenomena. We shall make the description in terms of classical physics.
The essential point, i.e., the right way of processing the magnetic energy of the system, is irrelevant on the way of the description, i.e., classical or quantum. The second paper which is related to the new thermodynamics on the diamagnet and the superconductor will be published soon.

It is to be mentioned that, in the cases of usual weak magnetisms such as Larmor diamagnetism, although the new theory introduces definitely a new way of understanding, the necessary modification is practically very small.

§2. Magnetic Energy of a Persistent Current System

Now let us start our discussion from the understanding of the magnetic energy of an idealized perfect conductors system with artificially arbitralily assumed persistent current distribution. (Fig. 1)

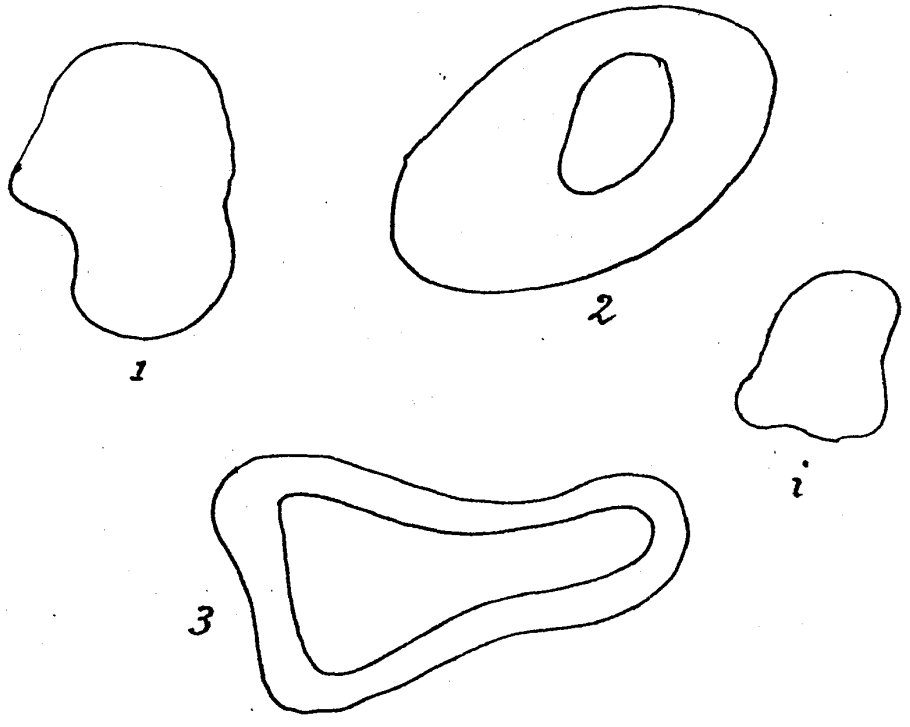


Fig. 1. An arbitrary perfect conductors system.

From the elementary electromagnetism, when there are 1, 2, 3, . . . , i, . . . , N idealized perfect conductors with stationary persistent current, **j(r)**, the self magnetic energy of the system is

$$U_m = \iiint \frac{\mathbf{H}^2}{2}\,dV = \iiint\iiint \frac{\mathbf{j}(\mathbf{r}_1)\cdot\mathbf{j}(\mathbf{r}_2)}{8\pi c^2 r_{12}}\,dV_1 dV_2 \tag{1}$$

$$= \iiint \frac{\mathbf{j}\cdot\mathbf{A}}{2c}\,dV \tag{2}$$

$$= \sum_i \sum_\lambda \oint_{c\lambda_i} \frac{\triangle I\lambda_i \mathbf{A}\cdot d\mathbf{l}}{2c} \tag{3}$$

$$= \sum_i \sum_\lambda \frac{\triangle I\lambda_i \Phi\lambda}{2c} = \sum_i \sum_\lambda \sum_j \sum_\mu \frac{1}{2} L\lambda_i\mu_j \triangle I\lambda_i \triangle I\mu_j \tag{4}$$

in which

$$\mathbf{A}(\mathbf{r}_1) = \iiint \frac{\mathbf{j}(\mathbf{r}_2)}{4\pi c r_{12}}\,dV_2 = \sum_j \sum_\mu \oint_{c\mu_j} \frac{\triangle I\mu_j}{4\pi c r_{12}}\,d\mathbf{l}_2 \tag{5}$$

$$\Phi_{\lambda_i} = \iint_{S_{\lambda_i}} \mathbf{B}\cdot d\mathbf{s} = \oint_{C_{\lambda_i}} \mathbf{A}\cdot d\mathbf{l} \tag{6}$$

$$= \sum_j \sum_{\mu} \oint_{C_{\lambda_i}} \oint_{C_{\mu_j}} \frac{d\mathbf{l}_1\cdot d\mathbf{l}_2}{4\pi c r_{12}} \Delta I_{\mu_j} \tag{7}$$

$$= \sum_j \sum_{\mu} c L_{\lambda_i \mu_j} \Delta I_{\mu_j} \tag{8}$$

$$L_{\lambda_i \mu_j} = \oint_{C_{\lambda_i}} \oint_{C_{\mu_j}} \frac{d\mathbf{l}_1\cdot d\mathbf{l}_2}{4\pi c^2 r_{12}} \tag{9}$$

Here, we employ MKS rationalized Gauss unit system for convenience, (We call this system MKS Physical or MKSP system)[2] and **j(r)** is the current density, **A(r)** the vector potential, c_{λ_i} one of the very fine differential closed current path λ on the conductor *i* with the total current ΔI_{λ_i}, d**l** the line element of the path, Φ_{λ_i} the total magnetic flux of c_{λ_i}, and $L_{\lambda_i \mu_j}$ the mutual inductance of the two paths, c_{λ_i} and c_{μ_j}. We have subdevided all the artificially assumed persistent currents into a number of differential closed paths c_{λ_i}. It is noticed that the possibility of having the overlaps or the crosses between these paths is not denied. The Maxwell equations allow for nature to have the presence of the strict superposition principle and, physically, this corresponds to a superposition of the drift velocity of the conduction electrons.

Obviously, in the present case threre is no gauge ambiguity in the vector potential **A**.

Now, let us look for the minimum magnetic energy state of the system under the idealized assumption that any current distribution which is consistent with the Maxwell equation is possible. Since we have idealized that the conductors are perfect conductors, the possibility of the presence of mathematical surface current is not denied. The variation of U_m with respect to the variation in ΔI_{λ_i}, $\delta(\Delta I_{\lambda_i})$, is

$$\delta U_m = \sum_i \sum_{\lambda} \sum_j \sum_{\mu} \frac{1}{2} L_{\lambda_i \mu_j} \cdot [\delta(\Delta I_{\mu_j}) \Delta I_{\lambda_i} + \Delta I_{\mu_j} \cdot \delta(\Delta I_{\lambda_i})]$$

$$= \sum_i \sum_{\lambda} \sum_j \sum_{\mu} L_{\lambda_i \mu_j} \Delta I_{\mu_j} \delta(\Delta I_{\lambda_i})$$

$$= \sum_i \sum_{\lambda} \Phi_{\lambda_i} \delta(\Delta I_{\lambda_i}) \tag{10}$$

Here we have assumed for convenience that the geometry of each path C_{λ_i} is invariant. Eq. (10) means that when

$$\Phi_{\lambda_i} \neq 0 \tag{11}$$

then mathematically we can always reduce the magnetic energy U_m by changing the current intensity ΔI_{λ_i} in such a way as

$$\Phi_{\lambda_i} \cdot \delta(\Delta I_{\lambda_i}) < 0 \tag{12}$$

Therefore, in the minimum energy state, all the Φ_{λ_i} should be zero. Namely

$$\Phi_{\lambda_i} \equiv 0 \tag{13}$$

When we require Eq. (13) strictly, then from Eq. (1)

$$U_m = 0, \quad \mathbf{H} = 0 \tag{14}$$

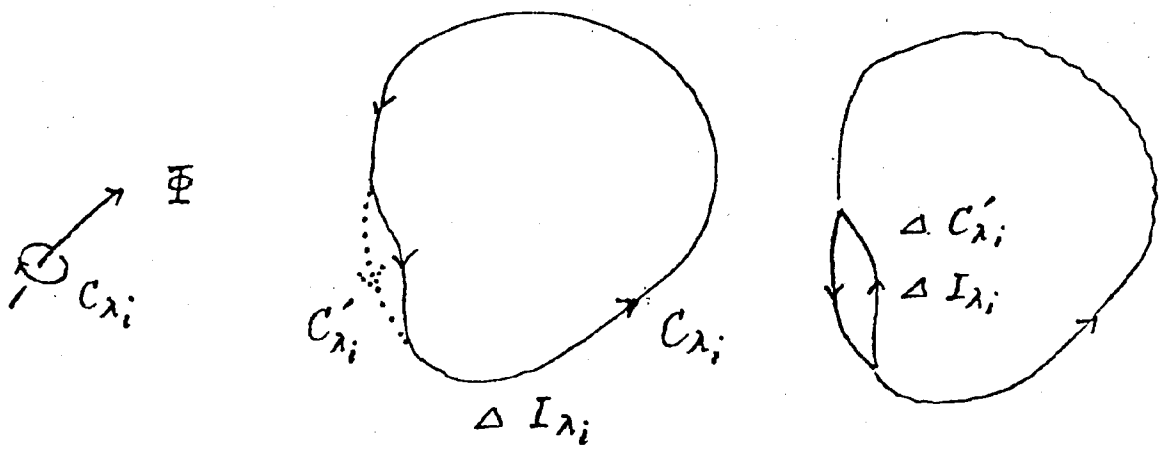
and there is no persistent current remaining.

Now, as an additional physical condition to the variation procedure of Eq. (10), we impose that only the variations which is obtainable by the piling up of small micro local processes are allowable. Micro local process means that the total variation is confined in a very small microscopic volume and each process must be in the direction to decrease the total magnetic energy. Here it should be noted that, in our consideration, the path λ_i is not necessarily the actual path with a current ΔI_{λ_i}, but we can take any path c_{ν_i} in the conductor i, under the assumption that initially

$$\Delta I_{\nu_i} = 0 \tag{15}$$

This condition has been imposed from a thermo-dynamical point of view. We know that usually thermal changes occur locally so as to reduce the total free energy of the system. Since the magnetic energy is definitely an internal energy of the system and the reduction of this energy will not associate any appreciable decrease of the entropy of the system.

Then, we can conclude immediately that, in the minimum magnetic energy state, there is no flux in the interior of the perfect conductors. Because, when there is a flux, we can always construct a micro loop C_{λ_i} and we can put $\delta(\Delta I_{\lambda_i})$ so as to reduce the total magnetic energy (Fig. 2(a)). In this consideration, we have neglected the electromagnetic reaction of this procedure, $\delta(\Delta I_{\lambda_i})$, for the rest of the system. On this point, we



(a) (b) (c)

Fig. 2. Examples of microprocesses. (a), micro current loop C_{λ_i} generated in the flux Φ. (b), micro change of the path from C_{λ_i} to C'_{λ_i}. (c), the change in (b) is identical to the creation of micro loop $\Delta C'_{\lambda}$ with the same current ΔI_{λ_i}.

should point out that it takes a finite time for the other part of the system ot receive the information, and also, this dicussion itself is the central part of the problem which is being studied in the later part of this paper.

Then we can conclude that, in the "minimum" magnetic energy state, all the persistent current must be on the surface and there is no flux inside of our idealized conductors. The meaning of "minimum" is with respect to the micro processes. (Fig. 2(a), (b), (c)). In our sense, it is easy to verify that the trapped flux state of the multiply connected superconductor also has the minimum magnetic energy.

It will be noticed that, from the Maxwell equations, on any path C_{λ_i}, the equation

$$\oint_{C_{\lambda_i}} \mathbf{E} \cdot d\mathbf{l} = -\frac{1}{c} \frac{\partial \Phi_{\lambda_i}}{\partial t} \tag{16}$$

is present. Therefore, macroscipically, when we assume

$$\mathbf{E} = 0 \tag{17}$$

inside, as a virtue of the perfect conductor, then the flux Φ_{λ_i} can not change macroscopically. There is, however, an interface between the macroscopic continium theory

and the microscopic Maxwell-Lorentz type theory in the micro region on this point and we believe that our assumption of the presence of the micro process which violates Eq. (17) is essentially correct. The meaning of the minimum magnetic energy will be further clarified in §'s 6 and 7.

de Gennes might be the first person who have made minimum energy calculation[1] by taking account the magnetic energy of Eq. (1) and the kinetic energy of the electrons of

$$\iiint \frac{n}{2} \mathrm{m} v^2 \, dv \tag{18}$$

Here, v and n are the drift velicity and the density of the electrons. By employing the variation procedure, he derived the London equation

$$\triangle \mathbf{H} = \frac{n \mathrm{e}^2}{\mathrm{m} c^2} \mathbf{H} \tag{19}$$

directly, which, is known to includes the Meissner effect. Let us make the order estimation of the total energy of Eqs. (1) for a superconductor ring of the radius 30 cm which keeps the magnetic field, which is about 100 Oe at the center. Then we get

$$U_{\mathrm{m}} \sim 10 \, \mathrm{Joule} \tag{20}$$

but

$$\iiint \frac{n}{2} \mathrm{m} v^2 dv \sim 10^{-33} \mathrm{Joule} \ ! \tag{21}$$

for the penetration depth of 300 Å. Although these two values are very different, since the magnetic field energy in the penetrated layer is identical to Eq. (21), his calculation will be correct physically. This procedure, however, when neglected the term of Eq. (18), can not give a meaningful answer, and the procedure neglects a few other complexities, which will be analyzed in §'s 6 and 7.

§3. Kinematical Motion of Electrons in A Perfectly Conducting Cylinder

We have the famous theorem of Miss Van Leeuwen[3), 4)] and also a famous figure by Van Vleck for the cyclotron motion of electrons in a perfectly conducting Cylinder[5), 6)] (Fig. 3 (a)). This figure was believed to show clearly that the diamagnetic effect of the cyclotron motion of most of the free electrons in the cylinder is just cancelled out by

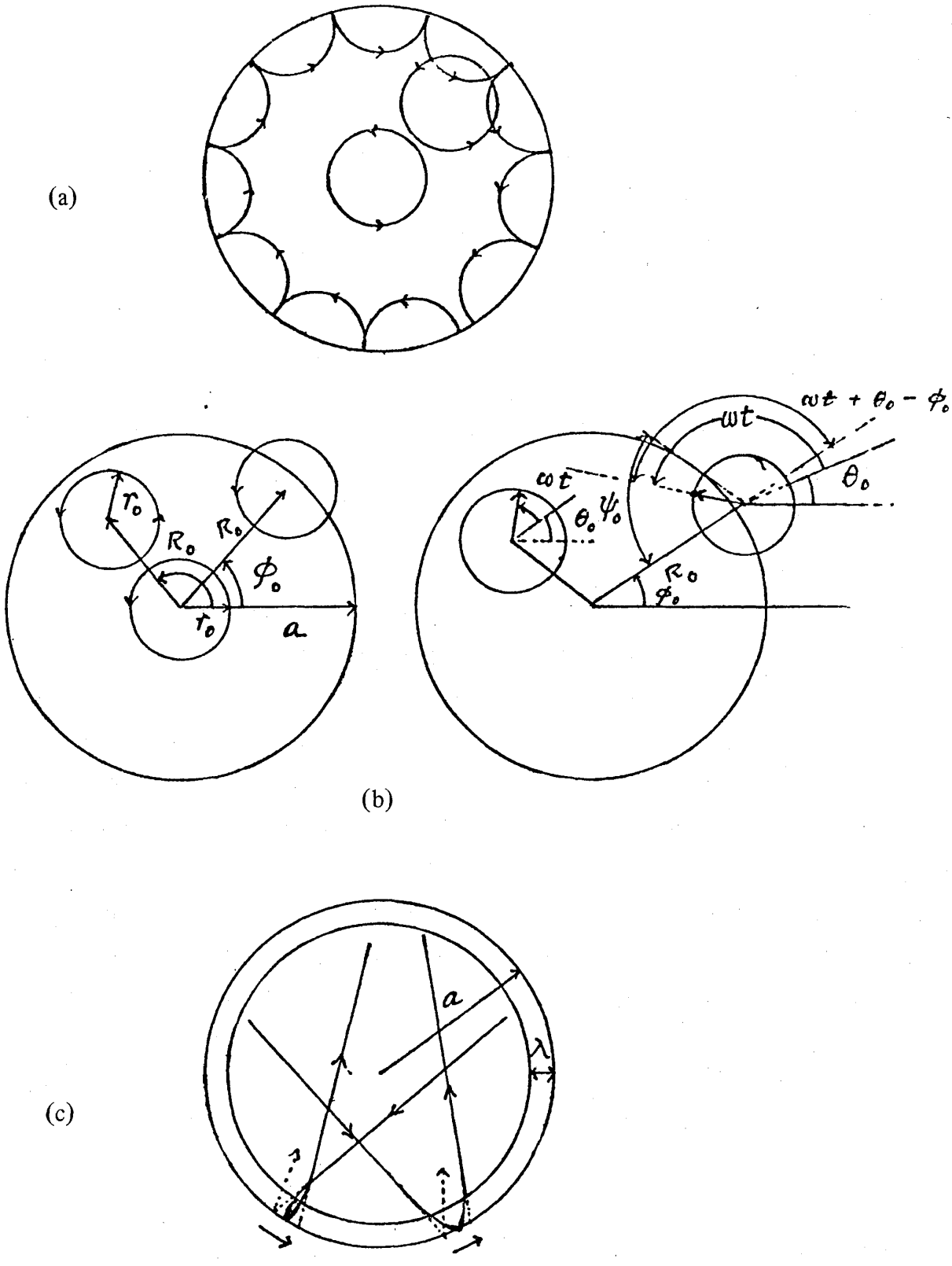


Fig. 3. Two typical cases of kinematical motion of electrons of perfect conductors under the magnetic field. (a), with a constant field, (b), with parameters, and (c), with a penetratration depth of λ.

the special movement of the electrons located near the surface boundary of the cylinder. Let us trace the kinematical motion of electrons in a perfectly conducting cylinder. In order to make the situation clearer, let us assume an infinitely long perfectly conducting cylinder 1 in the magnetic field supplied by another larger infinitely long idealized coaxial cylindrical coil 2. Let us have an attention first to the motion of one electron in cylinder 1. The Hamiltonian of the electron in this case is

$$\mathcal{H} = mc^2 \sqrt{1 + \frac{[\mathbf{p} + \frac{e}{c}\mathbf{A}(\mathbf{r}, t)]^2}{m^2 c^2}} - e\varphi(\mathbf{r}, t) \qquad e > 0 \tag{22}$$

$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{e}{c}\mathbf{A}(\mathbf{r}, t) \tag{23}$$

$$\mathbf{A}(\mathbf{r}, t) = \mathbf{A}_{ext}(\mathbf{r}, t) + \mathbf{A}_{in}(\mathbf{r}, t) \tag{24}$$

Eqs. (22). (23) and (24) are quite general and there is no further requirement besides the perfect cylindrical symmetry. Further restrictions will be added later one by one according to the necessity. Here $\varphi(\mathbf{r}, t)$ represents the electric potential in the conductor, which must have a very sharp decrease near the surface boundary. $\mathbf{A}(\mathbf{r}, t)$ is the vector potential, being composed of the vector potential $\mathbf{A}_{ext}$, which has its source in the outer coil 1, and $\mathbf{A}_{in}$, which has its source in the collective drift motion of the conduction electrons in the internal cylinder itself. $\varphi(\mathbf{r}, t)$ is also dependent on the collective density distribution of the conduction electrons. In the start of the present calculation, however, we disregard these cooperative correlation and $\mathbf{A}(\mathbf{r}, t)$ and $\varphi(\mathbf{r}, t)$ are regarded as externally given functions. The Lorentz force $\mathbf{f}$ is

$$\mathbf{f} = -\frac{e\mathbf{v} + \mathbf{H}}{c} - e\mathbf{E} \tag{25}$$

where

$$\mathbf{E}(\mathbf{r}, t) = -\nabla\varphi(\mathbf{r}, t) - \frac{1}{c}\frac{\partial \mathbf{A}(\mathbf{r}, t)}{\partial t} \tag{26}$$

$$\mathbf{H}(\mathbf{r}, t) = \nabla \times \mathbf{A}(\mathbf{r}, t) \tag{27}$$

Since we assume a perfect cylindrical symmetry, the problem is essentially two dimensional as is shown in Fig. 3. Let us use the cylindrical coordianate (ρ, ϕ, z). Then there is a relativistically invariant general principle[7] that the total angular momentum of many electron system is a constant of motion when the axial symmetry can be maintained, even if the electromagnetic fields are time dependent. Therefore,

$$p_\phi = (m\rho\omega - \frac{e}{c}A_\phi) \times \rho = \text{constant} \tag{28}$$

Here

$$\omega = \dot{\phi} \tag{29}$$

and

$$\mathbf{A}(\rho, t) = A_\phi(\rho, t)\rho\nabla\phi \tag{30}$$

in which $\rho\Delta\phi$ is a circumferential unit vector. The instantaneous contribution to the magnetic moment expected from this electron[8] is

$$\mu = \frac{1}{2c} r \times (-e\mathbf{v}) = -\frac{e}{2c}\rho^2\omega\nabla z = -\frac{e}{2mc} m\rho^2\omega\nabla z \tag{31}$$

which is proportional to the mechanical part of the angular momentum of Eq. (28). Here ∇z is a unit vector along z. Then, there is an important physical conclusion that if the radial location of the electron ρ does not change so much and when the vector potential A_ϕ changes, the electron must changes its mechanical angular momentum by the amount of

$$\Delta(eA_\phi\rho) / c \tag{32}$$

This relation represents the principal kinematical mechanism by which the conduction electron responds diamagnetically to the applied magnetic field. We know, however, that there is the Ehrenfest theorem, which denies the change of the magnetic moment, provided that the magnetic field is uniform and there is no static electric field. Now in general,

$$A = A_\phi \rho \nabla\phi = f(\rho, t)\rho \cdot \rho\nabla\phi \tag{33}$$

$$\mathrm{H} = H(\rho, t)\nabla z = [f'(\rho, t)\rho + 2f(\rho, t)]\nabla z \tag{34}$$

Further, when we neglect the presence of $\mathbf{A}_{in}$,

$$\mathbf{A} = \mathrm{A}_\phi \rho \nabla\phi = \frac{1}{2}\mathrm{H}_0(t)\rho \cdot \rho\nabla\phi \tag{35}$$

$$\mathrm{H} = \mathrm{H}_0(t)\nabla z.$$

Then Eq. (28) becomes

$$p_\phi = (m\omega - \frac{eH_0}{2c})\rho^2 = \text{constant} \tag{36}$$

This is a very important equation and it must be noted that Eq. (63) is effective all the time, even so $\varphi(\rho, t)$ is present.

The Ehrenfest case is a free electron, or, $\varphi = 0$, in a slowly time dependent uniform magnetic field.

Let us assume the same in our cylindrical space which has a steep boundary which reflects the electron elastically. As is shown in Fig. 3. (b), when we stop temporarily the change of the magnetic field, the orbit of the electron must be expressed as

$$\left.\begin{aligned} x &= R_0\cos\phi_0 + r_0\cos(\omega_0 t + \theta_0) \\ y &= R_0\sin\phi_0 + r_0\sin(\omega_0 t + \theta_0) \end{aligned}\right\} \tag{37}$$

$$\rho^2 = R^2 + r_0^2 + 2R_0 r_0\cos(\omega_0 t + \theta_0 - \phi_0) \tag{38}$$

where R_0 and ϕ_0 are the polar coordinates of the center of the circular orbit and, r_0 and θ_0 are the radius and the phase constant of the orbit. Of course here

$$\omega_0 = \frac{eH_0}{mc} \tag{39}$$

is the cyclotron frequency and

$$v_0 = r_0\omega_0 \tag{40}$$

is the velocity. Since the angular velocity in the cylinder

$$\omega = \dot{\phi} = \frac{d}{dt}\left(\tan^{-1}\frac{y}{x}\right)$$

$$= \frac{\omega_0 r_0 \left[R_0\cos\left(\omega_0 t + \theta_0 - \phi_0\right) + r_0\right]}{\rho^2} . \tag{41}$$

the relation of Eq. (36) in this time segment is

$$p_\phi = p_{\phi,\,\mathrm{mech}} + p_{\phi,\,\mathrm{rad}}$$

$$= m\omega_0 r_0 \left[R_0\cos\left(\omega_0 t + \theta_0 - \phi_0\right) + r_0\right]$$

$$- \frac{eH_0}{2c}\left[R_0^2 + r_0^2 + 2R_0 r_0\cos\left(\omega_0 t + \theta_0 - \phi_0\right)\right]$$

$$= -\frac{eH_0}{2c}\left(R_0^2 - r_0^2\right) = \text{invariant} \tag{42}$$

Now, all the electron orbits must be classified into I), the interior orbits and II), the boundary orbits. Let us analyze them separately

I), interior electron orbit.

Denoting the radius of the cylinder as a,

$$R_0 + r_0 < a . \tag{43}$$

the orbit is a full circle. When $R_0 \neq 0$ the relation (42) becomes

$$p_\phi = \frac{eH_0}{2c} r_0^2 \text{ constant} \tag{44}$$

so that

$$r_0 = \sqrt{\frac{2cp_\phi}{e}} \cdot \frac{1}{H_0} \qquad \left(\frac{\Delta r_0}{r_0} = \frac{1}{2}\frac{\Delta H_0}{H_0}\right) \tag{45}$$

$$v_0 = r_0\omega_0 = \frac{1}{m}\sqrt{\frac{2ep_\phi}{c}}\sqrt{H_0} \qquad \left(\frac{\triangle v_0}{v_0} = \frac{\triangle H_0}{2H_0}\right). \tag{46}$$

When $R_0 = 0$, the change of the velocity v_0 during the transient time per one circle should depend on

$$\oint f_\theta dt = -\oint eE_\theta dt = e\oint \mathbf{E}_\theta \cdot \frac{dl}{v}$$

$$= + \frac{e}{v} \cdot \frac{1}{c}\frac{\partial \Phi}{\partial t} \tag{47}$$

and be identical to the foregoing case. Here, f_θ and E_θ are the components of the electromagnetic force and electric field which are parallel to a transient time pseudo-circular orbit. Therefore

$$r_0 \propto \frac{1}{\sqrt{H_0}}, \quad \frac{eH_0}{2c} r_0^2 = \text{const}. \tag{48}$$

so that from Eq. (42)

$$\frac{eH_0}{2c} R_0^2 = \text{const}. \tag{49}$$

$$R_0 \propto \frac{1}{\sqrt{H_0}} \qquad \left(\frac{\Delta R_0}{R_0} = \frac{1}{2}\frac{\Delta H_0}{H_0}\right). \tag{50}$$

This is identical to the Ehrenfest case. But, as we see that there is a definite pinching effect as expressed in Eqs. (50), (45) and (48). Therefore, as a collective motion, this simple description can not be true as discussed soon.

II) boundary electron orbit.

when

$$R_0 - r_0 < a < R_0 + r_0 \tag{51}$$

the electron orbit has collisions to the surface boundary. In this case, althouth the electron collides to the surface boundary, Eqs. (37) are still effective. r_0, ω_0 and R_0 are

still the constants, and, only ϕ_0 and θ_0 change after each collision. As shown in Fig. 3(b), let us denote the arc angle of the orbit as $2\psi_0$. In this case, during the transient time, the f_θ is

$$f_\theta = -eE_\theta = (-e)\left(-\frac{1}{e}\frac{\partial A}{\partial t}\cdot\frac{v}{v}\right) = \frac{e\dot{H}}{2c}\left[R_0\cos(\omega_0 t+\theta_0-\phi_0)+r_0\right] \quad (52)$$

There is an important physical situation for these boundary electrons that the component of f_θ which decelaratates the velocity dominates effectively. Namely, by integrating Eq. (52) with time, we get

$$\frac{\triangle v_0}{v_0} = \frac{\triangle H_0}{2H_0}\left[1-\frac{R_0}{r_0}\frac{\sin\psi_0}{\psi_0}\right] \quad (53)$$

as the change of the velocity between the two successive collision during the transient time. Here $2\psi_0$ is the arc angle of the orbit as shown in Fig. 3(b). From Eq. (40)

$$\frac{\triangle r_0}{r_0} = -\frac{\triangle H_0}{2H_0}\left[1+\frac{R_0}{r_0}\frac{\sin\psi_0}{\psi_0}\right] \quad (54)$$

and, from Eq. (42)

$$\frac{\triangle R_0}{R_0} = -\frac{\triangle H_0}{2H_0}\left[1+\frac{r_0}{R_0}\frac{\sin\psi_0}{\psi_0}\right] \quad (55)$$

By comparing Eqs. (46) and (53), we can see that, in contrast to the interior electron, when

$$\frac{R_0}{r_0}\frac{\sin\psi_0}{\psi_0} > 1. \quad (56)$$

the boundary electron receives the decelaration in its velocity, and it can be enormous, when

$$\frac{R_0}{r_0}\frac{\sin\psi_0}{\psi_0} \gg 1. \tag{57}$$

From the Fermi energy, if we assume

$$v_0 = 10^{6\sim 4}\mathrm{m/sec} \tag{58}$$

then

$$r_0 = \frac{v_0}{\omega_0} = \frac{v_0 mc}{eH_0} \sim \frac{6}{h}\times 10^{-(3\sim 5)}\mathrm{cm} = \frac{6}{h}\times 10^{(5\sim 3)}\mathrm{\AA} \tag{59}$$

Here, h is the magnetic field intensity as expressed in kilo Oersteds. This means that the condition (57) is satisfied usually. It is to be noticed that in this case, different from the case of the interior electron, the pinching effect is not obvious. As we see soon, there is a definite antipinching effect also.

Now, by utilizing these results of the idealized single electron orbit, let us analyze the expected magnetism for the collective motion of the electrons in the present idealized situation.

It will be obvious that when $H_0 = 0$, all the electrons are the boundary electrons. When we apply a very small field $H_0 \neq 0$, then all the electrons will still be the boundary electrons but, the trajectory of each electron is now represented by two parameters R_0 and r_0. When we increased H_0 further, we must expect several very interesting phenomena. Let us trace these one by one. First, we shall trace the result under the assumption that all the electrons are completely independent. Then, for the interior electrons, from Eqs. (48) and (31), the effective diamagnetic magnetic moment

$$\mu = -\frac{e^2 r_0^2}{2mc^2} H_0 \tag{60}$$

is an invariant. For the boundary electron,

$$\mu = -\frac{e^2 r_0^2}{2mc^2} H_0^2 + \frac{e^2 r_0 R_0}{2mc^2} H_0 \frac{\sin\psi_0}{\psi_0} \tag{61}$$

can be obtained from Eqs. (31) and (42). When

$$R_0 \gg r_0 \tag{62}$$

the second term of Eq. (61) dominates which is paramagnetic definitely. With increasing H_0, however, this paramagnetic moment decreases rapidly, as shown in Eqs. (54) and (55). With the increase of H_0, ψ_0 increases as

$$\frac{d\psi_0}{dH_0} = \frac{1}{2H_0 \sin\psi_0}\left[\frac{a^2}{R_0 r_0} + \frac{\{a^2(R_0^2+r_0^2)-(R_0^2-r_0^2)^2\}\sin\psi_0}{2R_0^2 r_0^2 \psi_0}\right] > 0 \tag{63}$$

and when

$$\psi_0 = \pi \tag{64}$$

then the paramagnetic orbital motion with the paramagnetic contribution to the magnetic moment ends, and, the boundary electron converts to the interior electron. The diamagnetic moment as expressed by the first term of Eq. (61) becomes a constant, with keeping the value at this instant.

This description is not true actually because there are definitely collective correlation of these electron movements. The first one is the electric Coulomb correlation against the pinching effect. For the interior electrons, there is a definite pinching effect which is identical to the case of plasma. In a metal, we know that no appreciable pinching can be possible, because strong Coulomb correlation must adjust the distribution of the electrons uniform. In the first approximation, this Coulomb correlation must be represented by the transient creation of the radial electric field

$$\mathbf{E}(\rho, t) = E(\rho, t)\nabla\rho.$$

Then, we can still assume the validity of Eqs. (37), (39), (40) and (42). But, not of Eqs. (48), (49) and (50). As the result, let us assume that the center of mass, or R_0, does not change. Then from Eq. (42)

$$r_0 = \sqrt{R_0^2 + \frac{2cp_\phi}{eH_\phi}} \tag{65}$$

and, from Eq. (60), we get

$$\mu = -\frac{e^2}{2mc^2}\left(R_0^2 H_0 + \frac{2cp_\phi}{e}\right) \tag{66}$$

Namely, we get a diamagnetic term which are propoetional to H_0 and a constant term. Since initially the average of p_ϕ for all the electrons will be zero, the first term only must be effective. In this way we get a very reasonable constant diamagnetic susceptibility in this case, but, this is against the Ehrenfest theorem.

For the boundary electrons, the average radial distance ρ can be calculated as

$$\frac{d\bar{\bar{\rho}}}{dH_0} = \frac{1}{2H_0}\left[\frac{a-\bar{\bar{\rho}}}{\psi_0 \sin\psi_0}\left\{\frac{a^2}{R_0 r_0} + \frac{\sin\psi_0}{\psi_0}\,\frac{a^2(R_0^2+r_0^2)-(R_0^2-r_0^2)^2}{2R_0^2 r_0^2}\right\}\right.$$
$$\left. -\bar{\bar{\rho}}\left\{1+\frac{\sin\psi_0}{\psi_0}\left[\frac{R_0^2+r_0^2}{2R_0 r_0} - \frac{(R_0^2-r_0^2)^2}{2R_0 r_0}\,\frac{1}{\bar{\bar{\rho}}}\left(\frac{\bar{\bar{1}}}{\rho}\right)\right]\right\}\right] \tag{67}$$

It is easy to see that the first term is always positive but the second is negative. The former, however, becomes infinite when

$$\psi_0 \to \pi$$

or, just before the boundary electron converts to an interior electron. This means that, for the boundary electrons, there is a definite anti-pinching effect, as compared with the interier electrons. Therefore the pinching effect must be mild than the case of the interior electrons.

Now we regard that these analysis are almost enough to convince that the Coulomb correlation effect does not introduce any appreciable change to the derived characters for the paramagnetic boundary electrons against the increase of the applied field, i. e., the very strong decelaration of the velocity and the conversion to the interior electrons.

By approving these characters of the boundary electrons, now we have to consider the next most important factor, i.e., the correlation in the average magnetic field, or, the exclusion of the applied magnetic field from the inteior. Since this is the central problem of this paper, we think that it is better to leave the discussion on this problem in the later part of this paper.

In conclusion, we can say that, since Eq. (36) is effective in general, and, under the

presence of Coulomb correlation, the increase of the magnetic field definitely decreases the radiation angular momentum of Eq. (36), it must associate the increase in the mechanical angular momentum, which is identical to the increase of the diamagnetic moment.

Now we know that there is a famous theory of Miss. Van Leeuwen[7], which is just against this statement for the equilibrium state. What is wrong in the Miss Van Leeuwen's theorem[9] is the absence of the correct processing of the magnetic energy of the applied magnetic field itself in its Boltzman's factor. One of the physical implications of this energy has been already shown in §2. As will be shown in this and the next papers[18], the correct introduction of the magnetic energy in the system eliminates the considered contradiction completely and the application of the magnetic field brings the system into a new thermally equilibrium perfectly diamagnetic state adiabatically.

It is to be mentioned that the associated induced macroscopic diamagnetic current at the boundary is essentially identical to the Eddy current in usual conductors. We show soon that, in usual conductors, the current damps out quite quickly, but for the perfect conductors, the transient state itself is already approxiamtely in termal equilibrium, which can be maintained eternally.

§4. Thermostatistical Dynamics of the Collective Motion of Electrons in A Perfectly Conducting Cylinder.

Now let us go to the next step where we introduce the vector potential $\mathbf{A}_{in}$ of Eq. (24) clearly, taking into account the magnetic effect of the collective cooperative motion of the electrons. Although the rigorous thermodynamical derivation is given in the other paper[10], we have to admit that, when the cooperative motion of the electrons is to be taken account, the magnetic energy of the system has to be introduced in the statistical thermodynamics. Of course we know from the argument of §'s 2 and 3 that there is a definite electromagnetic and kinematic tendency to create $\mathbf{A}_{in}$ the electron system in order to realize the less energy state.

Let us have an attention ot the thermodynamical behavior of a single electron. Now the magnetic energy of a single electron will be deduced from the energy expression of Eqs. (1) and (2) as follows. Let us denote the contribution to the macroscopic $\mathbf{j}$ and $\mathbf{A}$ from the considered one electron as

$$\mathbf{j}_e,\ \mathbf{A}_e \tag{68}$$

then

$$U_m = \iiint \frac{(\mathbf{j}'+\mathbf{j}_e)\cdot(\mathbf{A}'+\mathbf{A}_e)}{2c}$$

$$= \iiint \frac{\mathbf{j}'\cdot\mathbf{A}'}{2c}dV + \iiint \frac{\mathbf{j}_e\cdot\mathbf{A}_e}{2c}dV + \iiint \frac{\mathbf{j}'\cdot\mathbf{A}_e+\mathbf{j}_e\cdot\mathbf{A}'}{2c}dV$$

$$= \iiint \frac{\mathbf{j}'\cdot\mathbf{A}'}{2c}dV + \iiint \frac{\mathbf{j}_e\cdot\mathbf{A}_e}{2c}dV + \iiint \frac{\mathbf{j}_e\cdot\mathbf{A}'}{c}dV \tag{69}$$

where

$$\mathbf{j}'+\mathbf{j}_e=\mathbf{j},\ \mathbf{A}'+\mathbf{A}_e=\mathbf{A},\ \mathbf{j}_e \ll \mathbf{j},\ \mathbf{A}_e \ll \mathbf{A} \tag{70}$$

and

$$\iiint \frac{\mathbf{j}'\cdot\mathbf{A}_e}{2c}dV = \iiint\iiint \frac{\mathbf{j}'(\mathbf{r}_1)\cdot\mathbf{j}_e(\mathbf{r}_2)}{8\pi c r_{12}}dV_1dV_2$$

$$= \iiint \frac{\mathbf{j}_e\cdot\mathbf{A}'}{2c}dV \tag{71}$$

The first term of Eq. (69) is independent of the kinematical motion of the considered electron. The second term is the small self interaction energy and can be neglected. Therefore we get as the effective magnetic interaction energy

$$\iiint \frac{\mathbf{j}_e\cdot\mathbf{A}'}{c}dV = -\frac{e}{c}\mathbf{v}\cdot\mathbf{A} \tag{72}$$

Here, we have approxiamted $\mathbf{A}'$ with $\mathbf{A}$. The procedures adopted in obtaining Eqs. (69), (71), and (72) are just identical to the procedures to get the electrical potential energy of

$$-e\varphi(\mathbf{r}) \tag{73}$$

from the total electric energy

$$U_e = \iiint \frac{\rho(\mathbf{r})\varphi(\mathbf{r})}{2} dV = \iiint \iiint \frac{\rho(\mathbf{r}_1)\cdot\rho(\mathbf{r}_2)}{8\pi r_{12}} dV_1 dV_2 \tag{74}$$

It will be noticed that the expression of the magnetic energy of Eq. (72) appears also in the Lagrangian of the electron in an electromagnetic field as[13)]

$$L = -mc^2\sqrt{1-\frac{v^2}{c^2}} + e\varphi - \frac{e}{c}\mathbf{v}\cdot\mathbf{A}$$

$$\simeq -mc^2 + \frac{1}{2}mv^2 + e\varphi - \frac{e}{c}\mathbf{v}\cdot\mathbf{A} \tag{75}$$

It will be noted that here the vector potential $\mathbf{A}(\mathbf{r})$ is treated as a physical entity just like as the electric potential $\varphi(\mathbf{r})$. Foretunately, in our simplest case of a cylindrical conductor there is no gauge arbitraliness. (see Eq. (33)).

There is an argument that this kind of the energy has been already included in the employed Hamiltonian. The most advanced Hamiltonian might be the one used by Heitler[11)]. We know, however, that the only one place in which externally applied static vector potential, $\mathbf{A}_{ext}$, appears is the place of Eq. (22). Now from the Maxwell-Lorentz equation, we have

$$-c\iint_s \mathbf{e}\times\mathbf{h}\cdot d\mathbf{s} = \iiint_v \left[\frac{\partial}{\partial t}\left(\frac{e^2+h^2}{2}\right) + \mathbf{e}q\mathbf{v}\right] dV \tag{76}$$

Here, S is the surface of arbitrary volume V and, e and h are the microscopic Lorentz electric and magnetic fields and, q and $\mathbf{v}$ are the microscopic charge density and its velocity. The left side equation represents the electromagnetic energy flow entered from the surface S. Although the Poynting vector

$$c\mathbf{e}\times\mathbf{h} \tag{77}$$

cannot always be regarded as the energy flow, the above interpretation is regarded correct[11)], since the true radiation flow $\mathbf{R}$

$$\mathbf{R} = c(\mathbf{e}\times\mathbf{h}) + \mathbf{R}^* \tag{78}$$

in which

$$\nabla\cdot\mathbf{R}^* = 0 \tag{79}$$

Then, we get

$$-c\iint_s \mathbf{e}\times\mathbf{h}\cdot d\mathbf{s}=\iiint_v \frac{\partial}{\partial t}\left(\frac{\mathbf{e}^2+\mathbf{h}^2}{2}\right)dV+\sum_i \frac{dT_i}{dt} \tag{80}$$

Here

$$T_i=\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}=mc^2\sqrt{1+\frac{\left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)^2}{m^2c^2}} \tag{81}$$

are the kinetic energies of the electrons, which is identical to Eq. (22). Eqs. (76). (80) and (81) are the very strict relativistic equations and Eq. (80) is regarded that the radiation energy entered. from the surface is converted into the increase of the electromagnetic energy in space and the increase of the kinetic energy of the particles. This shows clearly that the kinetic energy of Eq. (22) or (81) is independent of the magnetic energy in the space.

Another point which must be worthwhile to mention is the relation between the electromagnetic energies of microscopic and macroscopic fields. The Maxwell-Lorentz electromagnetic fields $\boldsymbol{e}$ and $\boldsymbol{h}$ are

$$\begin{aligned}\mathbf{e}&=\mathbf{E}+\mathbf{e}'\\ \mathbf{h}&=\mathbf{H}+\mathbf{h}'\end{aligned} \tag{82}$$

in which

$$\begin{aligned}\mathbf{E}&=\bar{\bar{\mathbf{e}}},\quad \mathbf{e}'=\mathbf{e}-\bar{\bar{\mathbf{e}}}\\ \mathbf{H}&=\bar{\bar{\mathbf{h}}},\quad \mathbf{h}'=\mathbf{h}-\bar{\bar{\mathbf{h}}}\end{aligned} \tag{83}$$

so that

$$\begin{aligned}\bar{\bar{\mathbf{e}}}'&=0\\ \bar{\bar{\mathbf{h}}}'&=0\end{aligned} \tag{84}$$

Then[2)]

$$\iiint_v \frac{\mathbf{e}^2+\mathbf{h}^2}{2}\,dV = \iiint_v \frac{\mathbf{E}^2+\mathbf{H}^2}{2}\,dV + \iiint_v \frac{\mathbf{e}'^2+\mathbf{h}'^2}{2}\,dV \tag{85}$$

In this paper, we have considered only the first term of the right side equation of Eq. (85), which means that we have assumed that the short range interaction term

$$\iiint_v \frac{\mathbf{e}'^2+\mathbf{h}'^2}{2}\,dV \tag{86}$$

keeps a constant value. This assumption can be justified because we are dealing with only the slight uniform drift modificaiton of the kinematical motion of the electrons. There is a physical reason that, microscopically, this uniform slight modification of the velocity does not affect to this term. The macroscopic magnetic energy term of Eq. (85) can be converted into Eq. (69), through Eq. (1).

Now, the thermodynamical Hamiltonian of the electron will be

$$\mathcal{H}_{\text{thermo}} = \frac{1}{2m}\left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)^2 - e\varphi - \frac{e}{c}\mathbf{v}\cdot\mathbf{A} \tag{87}$$

$$= \frac{1}{2m}\mathbf{p}^2 - e\varphi - \frac{e^2}{2mc^2}A^2 \tag{88}$$

$$\left(m\mathbf{v} = \mathbf{p} + \frac{e}{c}\mathbf{A}\right) \tag{89}$$

This expression is identical to a part of the Helmboltz free energy of the system in the rigorous thermodynamical treatment[10]. In this paper, however, this should be regarded as a reasonable physical assumption. Let us have the attention to the electrons which will pass or has passed a single fixed location. Then, in a thermal equilibrium state, if physically consistent, the p of Eq. (88) will have a tendency towards a symmetrical distribution in the phase space, because for the electron, $\mathbf{A}$ is a given quantity and $+p$ and $-p$ give the identical contribution to the total energy of the system. If this could be realized, then,

$$\bar{\mathbf{p}} = 0 = m\bar{\mathbf{v}} - \frac{e}{c}\mathbf{A} \tag{90}$$

$$\bar{\mathbf{v}} = \frac{e}{mc}\mathbf{A} \tag{91}$$

We believe that this is the correct classical thermodynamical derivation of the London

equation and is a fundamental mechanism which makes the Maxwell equations compatible with the persistent current system.

We can show also another very simple argument that Eq. (91) is kinematically consistent. One of the general equations of a single electron is[13)]

$$\frac{dp}{dt} = \nabla\left(-\frac{e}{c}\mathbf{v}\cdot\mathbf{A}\right) + e\nabla\phi = -\frac{e}{c}\nabla\left[\mathbf{v}_\phi A_\phi\right] + e\frac{\partial\phi}{\partial\rho}\nabla\rho \tag{92}$$

$$= -\frac{e\rho\omega}{c}\left[f(\rho, t) + \rho f'(\rho, t)\right]\nabla\rho + e\frac{\partial\phi(\rho, t)}{\partial\rho}\nabla\rho. \tag{93}$$

This means that the differential change of the general momentum p is not present along the circumferential direction. Exact integration of Eq. (93) should yield again Eq. (28). Namely, if the electron comes from the region where $A_\phi = 0$, then, where $A_\phi \neq 0$,

$$\omega\rho = v_\phi = v_\phi^* + \frac{e}{mc}A_\phi \tag{94}$$

$$\Delta v_\phi = v_\phi - v_\phi^* = \frac{e}{mc}A_\phi \tag{95}$$

in which v_ϕ^* is the v_ϕ which should be present when A_ϕ was still abscent there. Eq. (95) is identical to Eq. (91).

We believe that these are the dynamical mechanism, which makes the magnetic shielding by the drift currents in a perfect conductor possible.

We know that in general cases when multiply connected perfect conductors are involved, Eq. (91) is not always correct. In these cases, selfconsistency cannot be obtained by Eq. (91) and this is related to find out the proper technical gauge for the given mathematical problem. This problem, which has been known and analyzed well, will not be discussed further in this paper. In the last of this section, let us criticize the ficticious figure of Fig. 3. (a). In this figure, the magnetic field $\mathbf{H}_0$ is assumed to be present initially at all over the cylinder. Now, it becomes clear that, this artificial situation never happens in nature, because the introduction of the magnetic field associates the circular electric field which creates a mechanical angular momentum together with the associated diamagnetic moment. As a mathermatical ficiton, if we assume the presence of this situation, then the paramagnetic electrons at the boundary which repeat collisions to the boundary should have extremely higher magnetic energies of Eq. (72). Elementary

calculation can give easily, that, in a magnetic field of 10^3 Oe, the extra magnetic energy of one of these electrons can be as high as

$$10^{1\sim 2}\,\text{ev} \tag{96}$$

for the specimen with a centimeter size and a few electron volts Fermi energy. Therefore, as a result of the thermal fluctuation, their number diminishes with a certain time constant. The annihilation of the surface electrons must associates the creation of the diamagnetic drift surface current one by one and the penetrated magnetic flux is pushed out step by step towards the final perfectly diamagnetic state. In this case, the change of the total mechanical angular momentum of the conduction electrons should be balanced by the corresponding change in the angular momentum of the lattice. This means that the condition of the perfect elastic collision should not be maintained strictly.

Here, we should mention one evidence for the present theory. It is well known that high temperature plasma responds to the magnetic field diamagnetically. The observed diamagnetism is considerable, ranging up to several ten percent of the perfect diamagnetism. In the plasma, the temperature is so high that the system must be classical and the thermal equilibrium will be attained.

It is also pointed out that this treatment brings and essential doubt to the usual treatment [14), 15), 16), 17)] of the Landau diamagnetism, or the free electron diamagnetism. Because the calculations usually do not take account the magnetic energy of Eq. (72).

§5. An Example of the Collective Motion of Electrons with the Meissner Effect

In this section, we shall show one of the typical example of the classical collective motion of the electrons with the Meissner effect in a perfectly conducting material.

In Fig. 3. (c), we show typical example in which the magnetic field just penetrates the skin depth λ. All the electrons inside will make straight line motions, but as a result of magnetic Lorentz force of Eq. (26), they exhibit curved locuses near the boundary as shown in the figure. Obviously from Eq. (95) the locus near the boundary always shift to the counter-clock wise direction compared with without the magnetic field, indicating the possible drift velocity of the Meissner effect.

In order to make the physical situation clearer and avoid the unnecessary mathematical confusion, initially we assume that all the electrons are located inside of the

cylinder

$$\rho \leq a - n\lambda \qquad (n \sim 10)$$

uniformly with the thermally equilibrium uniform distribution of the velocities to all directions. This situation means that all the electrons are the boundary electrons as defined in §3, but, at the same time, there is no true surface state paramagnetic electrons initially. The so-called surface electrons, which will be running in the field penetrated thin surface region with very high speeds must have very high extra magnetic energies of Eq. (96), and are excluded in this idealized model.

It will be also pointed out that, from the stochastic theory, we know that entirely uniform density distribution of the electrons will be realized instantly after the start. In this case, however, since we assume perfectly free motion of the electrons, all the electrons always comes back to the situation similar to the original, once after each collision to the boundary. This makes the calculation very simple.

Let us calculate the effect by using a plane boundary model as shown in Fig. 4. The vector potential of Eq. (33) is now have the form*

$$\mathbf{A} = A_y(x)\nabla y$$

and $A_y(x)$ decreases steeply to zero when x becomes $n\lambda$. Then

$$\mathbf{H} = \nabla x \mathbf{A} = \frac{dA_y(x)}{dx}\nabla z .$$

---

*When the field is uniform, the vector potential is

$$\mathbf{A}(\mathrm{r}, \mathrm{t}) = (A_\mathrm{x}, A_\mathrm{y}, 0) = \frac{\mathrm{H(t)}}{2}(-\mathrm{y}, \mathrm{x}, 0),$$

and even if

$$A_\mathrm{x} \ll A_\mathrm{y}$$

A_x can not be neglected. In the present problem, however, since $\frac{dA_\mathrm{y}}{dx}$ is so large, we can safely neglect A_x.

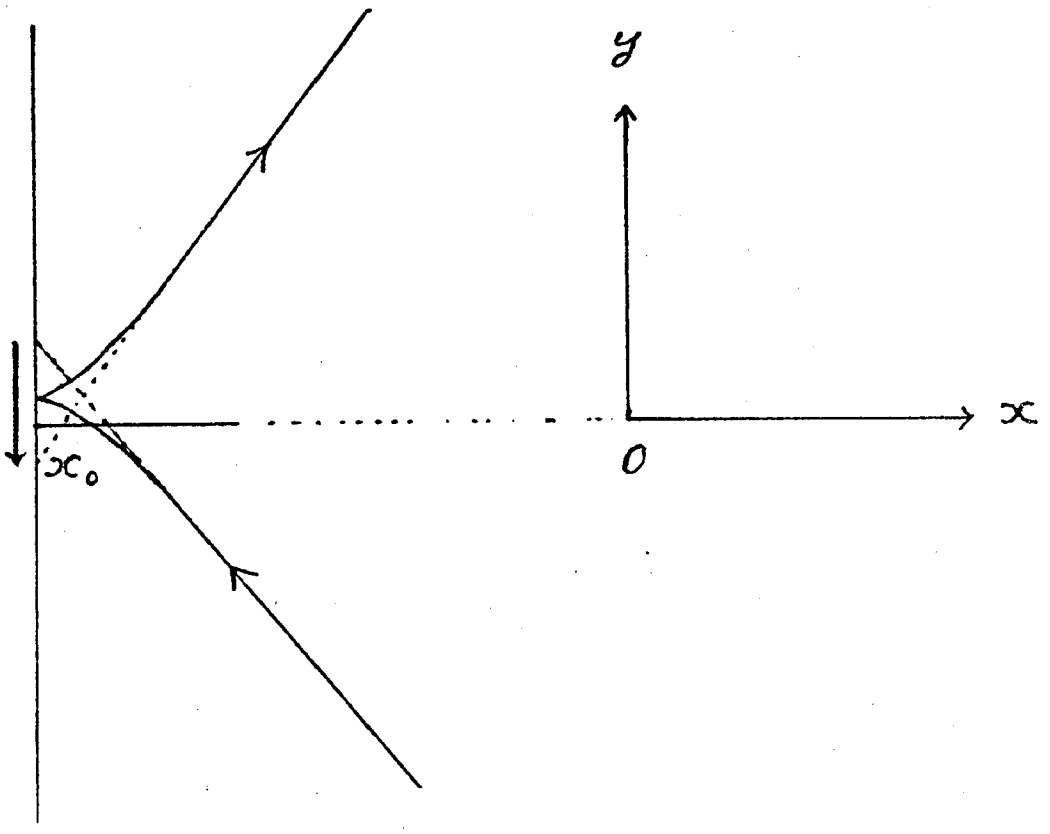


Fig. 4. A plane boundary model of the electron kinematics.

Now, from Eq. (95) the drift velocity, $v_y \Delta y$

$$v_y \nabla y = v_{drift} = \frac{e}{mc} A_y(x) \nabla y \tag{97}$$

is independent of the initial conditions. Then the resultant drift current

$$j = -e \cdot \mathbf{v}_{drift} \cdot n = \frac{ne^2}{mc} A_y(x) \nabla y. \tag{98}$$

Here n is the number of the conduction electron per unit volume. The self-consistency equation is

$$c\nabla \times H = -c \frac{d^2 A_y}{dx^2} \nabla y = -\frac{ne^2}{mc} A_y(x) \nabla y \tag{99}$$

$$\frac{d^2 A_y}{dx^2} = \frac{ne^2}{mc^2} A_y \tag{100}$$

Therefore

$$A_y = A_y^0 \exp\left[-\frac{(x-x_0)}{\lambda}\right] = -H_0\lambda \exp\left[-\frac{(x-x_0)}{\lambda}\right] \tag{101}$$

$$\lambda = \sqrt{\frac{mc^2}{ne^2}} \tag{102}$$

$$H = H_0 \exp\left[-\frac{(x-x_0)}{\lambda}\right] \nabla z. \tag{103}$$

Thus we have obtained the London equations directly.

§6. Magnetic Energy and Zeeman Energy of A two Persistent Current System

In §2, we have derived that the Meissner state has the minimum magnetic energy under the assumption that each localized $\delta(\Delta I_{\mu_k})$ can be assumed to be mutually not correlated. Now, we shall analyze the problem further in detail and rigorously. We shall see that there are very delicate and complexed structures.

Let us assume that there are two doubly connected idealized perfect conductors 1 and 2, as shown in Fig. 5. From the analysis of §3, the minimum magnetic energy state has the Meissner effect, i. e., all the current are on the surface and there is no flux inside. We assume that this state is realized. Now in state I, we assume

$$\Phi_1^I \neq 0, \quad \Phi_2^I = 0 \tag{104}$$

Then the total current I_1^I and I_2^I are composed of

$$\Delta I_{\mu_1^I} = I_1^I \Delta i_{\mu_1^I}, \quad \Delta I_{\mu_2^I} = I_2^I \Delta i_{\mu_2^I} \tag{105}$$

$$\sum_{\mu} \Delta i_{\mu_1^I} = 1, \quad \sum_{\mu} \Delta i_{\mu_2^I} = 1 \tag{106}$$

In state II,

$$\Phi_2^{II} \neq 0, \quad \Phi_1^{II} = 0 \tag{107}$$

$$\Delta I_{\lambda_2^{II}} = I_2^{II} \Delta i_{\lambda_2^{II}}, \quad \Delta I_{\lambda_1^{II}} = I_1^{II} \Delta i_{\lambda_1^{II}} \tag{108}$$

$$\sum_{\lambda} \Delta i_{\lambda_2^{II}} = 1, \quad \sum_{\lambda} \Delta i_{\lambda_1^{II}} = 1 \tag{109}$$

Here we have used the same notations with those of §2 and $\Delta i_{\mu_k^I}$ is a normalized differentical current in the path μ_k in state I.

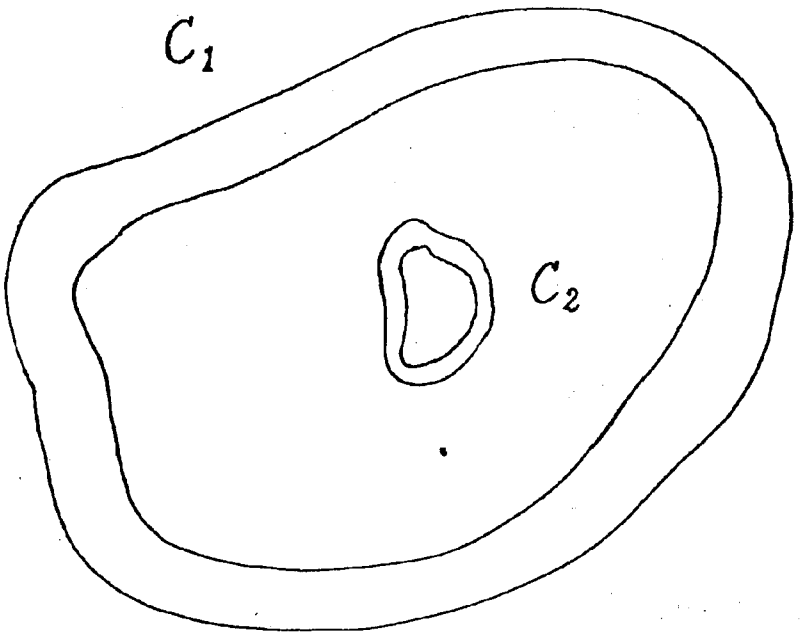


Fig. 5. Two doubly connected perfect conductors.

Then by superposing state I and II, we get

$$\Phi_1 = \Phi_1^I = \Phi_{\lambda_1^{II}}^I = \sum_{\mu} c L_{\lambda_1^{II}\mu_1^I} \Delta i_{\mu_1^I} I_1^I + \sum_{\mu} c L_{\lambda_1^{II}\mu_2^I} \Delta i_{\mu_2^I} I_2^I \tag{110}$$

in which λ_1^{II} means one of the path C_{λ_1} of 1 conductor and, of course, Φ_1 is independent of the choice of the path, so far as it surrounds the hole, and

$$L_{\lambda_j^{\xi}\mu_k^{\eta}} = \oint_{C_{\lambda_j^{\xi}}} \oint_{C_{\mu_k^{\eta}}} \frac{dl_\alpha \cdot dl_\beta}{4\pi c^2 r_{\alpha\beta}} \tag{111}$$

Then Φ_1 can be transformed further

$$\Phi_1 = c\left(\sum_{\lambda}\sum_{\mu} L_{\lambda_1^{II}\mu_1^I} \Delta i_{\lambda_1^{II}} \Delta i_{\mu_1^I}\right) I_1^I + c\left(\sum_{\lambda}\sum_{\mu} L_{\lambda_1^{II}\mu_2^I} \Delta i_{\lambda_1^{II}} \Delta i_{\mu_2^I}\right) I_2^I \tag{112}$$

and similarly we get

$$\Phi_2 = c\left(\sum_{\mu}\sum_{\lambda} L_{\mu_2^{\mathrm{I}}\lambda_2^{\mathrm{II}}} \Delta i_{\mu_2^{\mathrm{I}}} \Delta i_{\lambda_2^{\mathrm{II}}}\right) I_2^{\mathrm{II}} + c\left(\sum_{\mu}\sum_{\lambda} L_{\mu_2^{\mathrm{I}}\lambda_1^{\mathrm{II}}} \Delta i_{\mu_2^{\mathrm{I}}} \Delta i_{\lambda_1^{\mathrm{II}}}\right) I_1^{\mathrm{II}} \tag{113}$$

$$0 = c\left(\sum_{\lambda}\sum_{\mu} L_{\lambda_2^{\mathrm{II}}\mu_1^{\mathrm{I}}} \Delta i_{\lambda_2^{\mathrm{II}}} \Delta i_{\mu_1^{\mathrm{I}}}\right) I_1^{\mathrm{I}} + c\left(\sum_{\lambda}\sum_{\mu} L_{\lambda_2^{\mathrm{II}}\mu_2^{\mathrm{I}}} \Delta i_{\lambda_2^{\mathrm{II}}} \Delta i_{\mu_2^{\mathrm{I}}}\right) I_2^{\mathrm{I}} \tag{114}$$

$$0 = c\left(\sum_{\mu}\sum_{\lambda} L_{\lambda_1^{\mathrm{I}}\mu_2^{\mathrm{II}}} \Delta i_{\mu_1^{\mathrm{I}}} \Delta i_{\lambda_2^{\mathrm{II}}}\right) I_2^{\mathrm{II}} + c\left(\sum_{\mu}\sum_{\lambda} L_{\mu_1^{\mathrm{I}}\lambda_1^{\mathrm{II}}} \Delta i_{\mu_1^{\mathrm{I}}} \Delta i_{\lambda_1^{\mathrm{II}}}\right) I_1^{\mathrm{II}} \tag{115}$$

Now let us denote

$$\begin{aligned}
&\sum_{\lambda}\sum_{\mu} L_{\lambda_1^{\mathrm{II}}\mu_1^{\mathrm{I}}} \Delta i_{\lambda_1^{\mathrm{II}}} \Delta i_{\mu_1^{\mathrm{I}}} = \bar{L}_{11} \\
&\sum_{\lambda}\sum_{\mu} L_{\lambda_1^{\mathrm{II}}\mu_2^{\mathrm{I}}} \Delta i_{\lambda_1^{\mathrm{II}}} \Delta i_{\mu_2^{\mathrm{I}}} = \bar{L}_{12} \\
&\sum_{\mu}\sum_{\lambda} L_{\mu_2^{\mathrm{I}}\lambda_2^{\mathrm{II}}} \Delta i_{\mu_2^{\mathrm{I}}} \Delta i_{\lambda_2^{\mathrm{II}}} = \bar{L}_{22} \\
&\sum_{\lambda}\sum_{\mu} L_{\lambda_2^{\mathrm{II}}\mu_1^{\mathrm{I}}} \Delta i_{\lambda_2^{\mathrm{II}}} \Delta I_{\mu_1^{\mathrm{I}}} = \bar{L}_{21}
\end{aligned} \tag{116}$$

then $\bar{L}_{11}$, $\bar{L}_{12}$, $\bar{L}_{22}$, and $\bar{L}_{21}$ are the quantities which are dependent only on the mutual locations of the conductors 1 and 2 and do not depend on the values of Φ_1 and Φ_2. From Eqs. (102)–(106), and

$$I_1 = I_1^{\mathrm{I}} + I_1^{\mathrm{II}}, \quad I_2 = I_2^{\mathrm{I}} + I_2^{\mathrm{II}}. \tag{117}$$

we get

$$\begin{aligned}
\Phi_1 &= cI_1 \frac{\bar{L}_{11} + \dfrac{\bar{L}_{12}\bar{L}_{21}}{\bar{L}_{22}}}{\bar{\Delta}} + cI_2 \frac{\bar{L}_{12} + \bar{L}_{21}}{\bar{\Delta}} \\
\Phi_2 &= cI_2 \frac{\bar{L}_{22} + \dfrac{\bar{L}_{12}\bar{L}_{21}}{\bar{L}_{11}}}{\bar{\Delta}} + cI_1 \frac{\bar{L}_{12} + \bar{L}_{21}}{\bar{\Delta}}
\end{aligned} \tag{118}$$

$$\bar{\Delta} = 1 - \frac{\bar{L}_{21}^2}{\bar{L}_{11}\bar{L}_{22}} \tag{119}$$

Therefore

$$\begin{aligned} \Phi_1 &= c(L_{11}I_1 + L_{12}I_2) \\ \Phi_2 &= c(L_{21}I_1 + L_{22}I_2) \\ L_{12} &= L_{21} \end{aligned} \tag{120}$$

are obtained rigorously. From Eq. (120)

$$\begin{aligned} I_1 &= \frac{1}{c}(k_{11}\Phi_1 + k_{12}\Phi_2) \\ I_2 &= \frac{1}{c}(k_{21}\Phi_1 + k_{22}\Phi_2) \end{aligned}$$

where

$$(k) = (L)^{-1} = \frac{1}{\Delta_L}\begin{pmatrix} L_{22} & -L_{12} \\ -L_{21} & L_{11} \end{pmatrix} \tag{121}$$

$$\Delta_L = L_{11}L_{22} - L_{12}^2 \tag{122}$$

Finally we have

$$U_m = \frac{1}{2c}(\Phi_1 I_1 + \Phi_2 I_2) \tag{123}$$

$$= \frac{1}{2}(L_{11}I_1^2 + 2L_{12}I_1I_2 + L_{22}I_2^2) \tag{124}$$

$$= \frac{1}{2c^2\Delta_L}(L_{22}\Phi_1^2 - 2L_{12}\Phi_1\Phi_2 + L_{11}\Phi_2^2) \tag{125}$$

Now, although Eqs. (123), (124), and (125) have been derived for two perfect conductors, we shall regard these equation as the fundamental magnetic energy equations of two current systems. These currents could be persistent currents, but not necessarily be

$$\delta\Phi_1 = \delta\Phi_2 = 0. \tag{126}$$

Which is the case for type I super conductor. The electron itself can be regarded as a tiny persistent current[18] with the radius of

$$r_0 = g\frac{h}{mc} \tag{127}$$

The currents could be the ordinary conduction currents for which the current source are necessary. But, so far as the magnetic energy concerned, Eqs. (123), (124), and (125) must be a good representation

Now let us apply variation analysis. The variations are for the total currents and for the mutual configuration.

$$\delta U_m = L_{11}I_1\delta I_1 + L_{12}(I_1\delta I_2 + I_2\delta I_1) + L_{22}I_2\delta I_2 + \frac{I_1^2}{2}\delta L_{11} + I_1I_2\delta L_{12} + \frac{I_2^2}{2}\delta L_{22} \tag{128}$$

$$= I_1\frac{\delta\Phi_1}{c} + I_2\frac{\delta\Phi_2}{c} - I_1I_2\delta L_{12} - \frac{I_1^2}{2}\delta L_{11} - \frac{I_2^2}{2}\delta L_{22} \tag{129}$$

We shall derive the Zeeman energy expression of a magnetic moment in a magnetic field. For this purpose, we assume that conductor 1 is very large and conductor 2 is very small, located at the center of the large hole of conductor 1. This means that conductor 2 represents a permanent magnetic moment and conductor 1 represents the source of the magnetic field applied to this magnetic moment. Then we can safely neglect

$$\frac{I_1^2}{2}\delta L_{11} \tag{130}$$

and

$$\frac{I_2^2}{2}\delta L_{22} \tag{131}$$

The reason is as follows. From Eq. (126), Eq. (130) indicates the change in the total magnetic energy when the orientation of conductor 2 have been changed, under the condition that total current $I_2 = 0$. Elementary calculation can give easily that this is identical to the change in the demagnetization energy of a magnetized body with the magnetization $\mathbf{M} = \mathbf{H}_{21}$ and with the identical shape and location to conductor 2. Here $\mathbf{H}_{21}$ is the magnetic field induced by the current I_1 at the location of conductor 2. This kind of energy should be neglected in the problem of the Zeeman energy, i. e., in the case when conductor

2 has a permanent magnetic moment of considerable magnitude. Considerable means

$$H_{21}V_2 \ll \mu_2 \tag{132}$$

here V_2 is the volume of conductor 2 and μ_2 it its magnetic moment. Eq. (131) indicates the change in the total magnetic energy when the position and location of conductor 1 have been changed under the ocndition that total current $I_1 = 0$. Since conductor 1 is far apart from conductor 2, physically this energy must be also neglegible as compared the Zeeman energy in which $I_1 \neq 0$.

Now let us analyze the meaning of the first two terms of Eq. (129). From the Maxwell equation

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{133}$$

Integrating Eq. (133) over the area $\mathbf{S}_{\mu_j}$ which is bounded by the path C_{μ_j}, we get

$$\iint_{S_{\mu_j}} \nabla \times \mathbf{E} \cdot d\mathbf{s} = -\frac{1}{c}\frac{\partial}{\partial t}\iint_{S_{\mu_j}} \mathbf{B} \cdot d\mathbf{s}$$

$$\oint_{C_{\mu_j}} \mathbf{E} \cdot d\mathbf{l} = -\frac{1}{c}\frac{\partial \Phi_{\mu_j}}{\partial t} \tag{134}$$

Multiplying the current ΔI_{μ_j}

$$\oint_{C_{\mu_j}} \mathbf{E} \cdot \Delta I_{\mu_j}\, d\mathbf{l} = -\frac{1}{c}\Delta I_{\mu_j}\frac{\partial \Phi_{\mu_j}}{\partial t} \tag{135}$$

When Φ_{μ_j} can be assumed to be independent of specific path μ_j, then

$$-\frac{1}{c} I_j \delta\Phi_j = \iiint_{V_j} \mathbf{E} \cdot \mathbf{j}\delta t dV_j = \delta A_j \tag{136}$$

δA_j is the work given to the current I_j through the induced electric field **E**. When the current is being supplied by a source, then the work of the energy is given to the source. When the current is a persistent current and the current has another mechanism

of keeping the energy, such as the kinetic energy of the conduction electrons, then

$$\delta A_j = \delta G_j \tag{137}$$

i. e., transforms into the increase in the potential-like energy of the persistent current system. When the conductor is an ideal perfect conductor, it will be not possible to change $\delta \Phi_j$, so that

$$\delta A_j = 0 \tag{138}$$

In general, for the persistent current system, we have from Eq. (129)

$$\delta (U_m + G_1 + G_2) = -I_1 I_2 \delta L_{12} = \delta^* (- I_1 I_2 L_{12}) \tag{139}$$

where δ^* indicates the variation with keeping the values of I_1 and I_2 constant. When conductor 1 represents an electron spin[18] then

$$-I_1 I_2 L_{12} = - \frac{\bar{L}_{21} I_1 I_2 + \bar{L}_{12} I_1 I_2}{1 - \dfrac{\bar{L}_{21}^2}{\bar{L}_{11}\bar{L}_{22}}} \tag{140}$$

$$= -\oint_{\lambda_1}\oint_{\mu_2} \frac{d\mathbf{l}_\alpha \cdot d\mathbf{l}_\beta}{4\pi c^2 r_{\alpha\beta}} I_1 I_2 = -\oint_{\mu_2} \frac{\mathbf{A}(\mathbf{r}_\alpha)}{c} \cdot I_2 d\boldsymbol{l}_\alpha \tag{141}$$

$$= -\iint_{S_2} \nabla \times \mathbf{A}(\mathbf{r}_\alpha) \cdot \frac{I_2}{c} d\mathbf{s}_\alpha = -\mathbf{H}_{21} \cdot \frac{I_2 \mathbf{S}_2}{c} = -(\mathbf{H}_{21} \cdot \mu_2) \tag{142}$$

Here, we have made a few reasonable approxiamtions such as

$$\frac{\bar{L}_{21}^2}{\bar{L}_{11}\bar{L}_{22}} \ll 1. \tag{143}$$

This means that the right-side expression of Eq. (139) just represents the variation of the Zeeman energy. This is the rigorous derivation of the Zeeman energy expression in persistent current electrodynamics. It behaves just like as an effective Hamiltonian the variation of which represents the variation of the total energy of the system.

Here we should mention about the relation of our result to the normally accepted analysis of the atom. The one electron Hamiltonian of an atom with a possible orbital magnetic moment μ is

$$\mathcal{H} = \frac{(\mathbf{p}+\frac{e}{c}\mathbf{A}(\mathbf{r}))^2}{2m} - e\phi(\mathbf{r}) \tag{144}$$

We know that this Hamiltonian is accurate to be able to derive correct kinematical equations of the electron. The total energy of the system, however, must be

$$U = \mathcal{H} + U_m + G_1 \tag{145}$$

Here,

$$U_m = \iiint \frac{\mathbf{h}^2}{2}\, dV \tag{146}$$

is the magnetic energy of the system and G_1 is the potential-like energy of the source of the applied magnetic field. Then

$$\mathcal{H} = G_2 \tag{147}$$

in Eq. (139), so that

$$\delta U = \delta\mathcal{H} + \delta U_m + \delta G_1 = \delta[-(\mathbf{H}_{21}\cdot\boldsymbol{\mu}_2)] \tag{148}$$

Namely, there is a reason to assume the Zeeman expression as the effective Hamiltonian. Further when an atom with many electrons is in a diamagnetic state,

$$\mathbf{p} \gg \frac{e}{c}(\mathbf{A}_{ext} + \mathbf{A}_{int}) \tag{149}$$

and

$$\frac{e}{c}\mathbf{A}_{ext}(\mathbf{r}) \gg \frac{e}{c}\mathbf{A}_{int}(\mathbf{r}) \tag{150}$$

Here, $\mathbf{A}_{int}$ is the average vector potential induced by the Larmor diamagnetism of the electrons. Then it is possible to neglect $\mathbf{A}_{int}(\mathbf{r})$ in the Hamiltonian, which can be written as[19]

$$\mathcal{H} = \sum_j \frac{\boldsymbol{p}_j^2}{2m} - \sum_j e_j \phi_j - \int^{H_{ext}} \boldsymbol{\mu} \cdot d\mathbf{H}_{ext} \tag{151}$$

$$\boldsymbol{\mu} = -\frac{e}{2c}\sum_j \mathbf{r}_j \times \mathbf{v}_j = -\frac{e}{2mc}\mathbf{J} - \frac{e^2}{2m^2c^2}\mathbf{H}_{ext} \cdot \Gamma \tag{152}$$

$$\mathbf{J} = \sum_j \mathbf{r}_j \times \mathbf{p}_j \tag{153}$$

$$\Gamma = m\sum_j (r_j^2 \mathbf{I} - \mathbf{r}_j \mathbf{r}_j) \tag{154}$$

Here μ is the magnetic moment of the system and J is the angular momentum. It is an important physical situation that, in this approzimation, $\mathbf{r}_j$ and $\mathbf{p}_j$ can be kept constant while $\mathbf{H}_{exp}$ is being increased. Eq. (151), however, is just a mathematical identity and it does not mean that the same expression is applicable to the general case in which $\mathbf{A}_{int}$ cannot be neglected. Thus we see that, although mathematically, Zeeman energy type expression can appear frequently, the implication of the expression is different from case to case. In the present macroscopic case, Eqs. (149) and (150) can not be assumed, so that there is no simple realtion like as Eq. (151).

§7. Thermo-Electromagnetic Dynamics of A Two Persistent Current System

Now we have from Eqs. (123), (124), and (125),

$$U_m = \frac{1}{2}(L_{11}I_1^2 + 2L_{12}I_1I_2 + L_{22}I_2^2) \qquad I_1,\ I_2 \tag{155}$$

$$= \frac{1}{2c^2\Delta L}(L_{22}\Phi_1^2 - 2L_{12}\Phi_1\Phi_2 + L_{11}\Phi_2^2) \qquad \Phi_1,\ \Phi_2 \tag{156}$$

$$= \frac{1}{2}\left(L_{11} - \frac{L_{12}^2}{L_{22}}\right)I_1^2 + \frac{1}{2L_{22}}\frac{\Phi_2^2}{c^2} \qquad I_1,\ \Phi_2 \tag{157}$$

$$= \frac{1}{2L_{11}}\frac{\Phi_1^2}{c^2} + \frac{1}{2}\left(L_{22} - \frac{L_{12}^2}{L_{11}}\right)I_2^2 \qquad \Phi_1,\ I_2 \tag{158}$$

We found that the minimum magnetic energy state is quite different from case to case. At first we fix the locations of the two conductors C_1 and C_2 in their most symmetrical locations as shown in Fig. 6, and ocnsider only the variation of I_1, I_2, and Φ_1, Φ_2. From the nature of L_{jk}, we know that this does not violate our theoretical analysis. There are five typical cases.

	Fixed Quantities	Min. U_m Conditions	Type
1.	I_1,	$\Phi_2 = 0$	Meissner
2.	I_2, $\lvert I_2 \rvert$	$I_1 \uparrow\downarrow I_2$	Meissner-like (Zeeman)
3.	I_1, $\lvert \Phi_2 \rvert$	$\pm \Phi_2$	no dependence (Zeeman)
4.	Φ_1,	$I_2 = 0$ ($\Phi_2 = cL_{21}I_1$)	Normal Conductor
5.	Φ_1, $\lvert \Phi_2 \rvert$	$\Phi_1 \uparrow\downarrow \Phi_2$	Zeeman

(159)

Here, Meissner of Zeeman means that the corresponding lowest U_m state has parallel or antiparallel magnetic moments of the two conductors. The meaning of the bracket in cases 2 and 3 will be explained later. We can analyze our problem of the generation

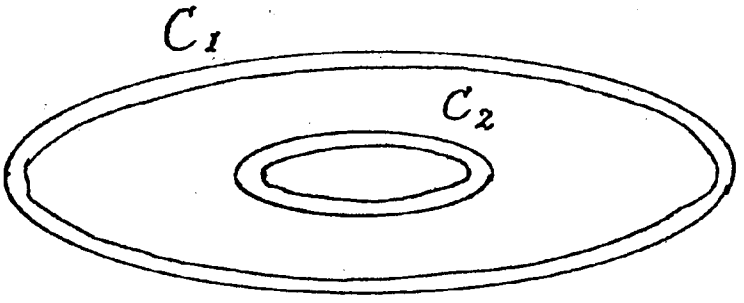


Fig. 6. Most typical locations of two conductors C_1 and C_2.

of the Meissner state by using this two conductor system. The typical situation is shown in Fig. 7, where C_2 is a spherical shell with a small hole at the two poles. Under the application of a uniform field from C_1, if the conductor C_2 prefers to have no flux, Φ_2, then this indicates the preference to the Meissner state. Now we can see in expression (159) that the balance is quite delicate and the difference in the conditions yields completely different results. But this is not unusual because in the electrical case of a two conductor system with the electric charges of Q_1 and Q_2 and the potentials

of V_1 and V_2, we have just the same situation in which equations

$$U_e = \frac{1}{2}(P_{11}Q_1^2 + 2P_{12}Q_1Q_2 + P_{22}Q_2^2) \tag{160}$$

$$= \frac{1}{2}(C_{11}V_1^2 + 2C_{12}V_1V_2 + C_{22}V_2^2)$$

$$= \frac{1}{2\Delta_p}(P_{22}V_1^2 - 2P_{12}V_1V_2 + P_{11}V_2^2) \tag{161}$$

$$(P_{ij} > 0)$$

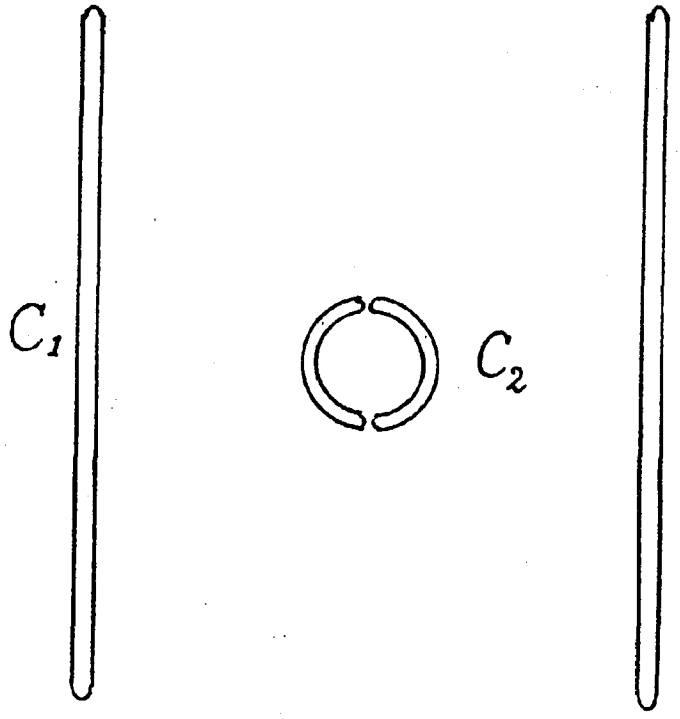


Fig. 7. One typical configuration of two conductors C_1 and C_2 which can check the Meissner effect.

are present. In our case, however, different from the case of the electricity, the minimum energy state does not immediately correspond to the final equilibrium state to be realized. For instance, in case 2, we know that the Lorentz force acts to conductor 2 to rotate towards the minimum Zeeman energy state. What is actually realized is not uniquely determined by the minimum magnetic energy requirement alone. This is another point we hope to add to the analysis of de Gennes[1]. The actual process can be analyzed by

the two very different physical actions. They are 1), the action of Lorentz electromagnetic force and 2), the thermal free energy minimum processes. In each situation, the two actions seem to play the role cooperatively. Now, most important cases are cases 1 and 2. Fixed I_1 means that the external field $\mathbf{H}^{ext}$ is constant. Since coil 2 can not know immediately the induced change of coil 1, this is the physical situation for coil 2. In case 1, there is no requirement on coil 2 and the minimum magentic energy corresponds to the Meissner state of $\Phi_2 = 0$. Case 3 is the case where coil 2 has a permanent magnetic moment. In this case when we require Φ_2 = constant, then from Eq. (157) there is no change of the energy.

As we mentioned already, the consideration of the magnetic energy only cannot give the final answer to the problem. We shall analyze the problem further in detail.

A. Zeeman's situation

This situation happens when Φ_2 is conditioned to be finite. This means that there is a permanent magnetic moment and the Lorentz force acts so as to reduce the Zeeman energy

$$\delta^*(-\mu_2 \cdot \mathbf{H}_{21}) < 0 \tag{162}$$

Cases 2, 3 and 5 correspond to this situation. When there is another system which can balance the change of the angular momentum and the change of the energy of Eq. (162) liberated, the Lorentz force can really generate the kinematical motion of conductor C_2. This is the situation of the macroscopic conductors. In this case, the lattice becomes the necessary additional system and Φ_1 and Φ_2 become parallel in the final lowest energy configuration. Electromagnetic radiation energy in this case is just too small. When, however, there is no such additional sytem for the adjustment, then the action of the Lorentz force is just to introduce a Larmor precession. This is the case of the free electron spin and also of the orbital motion of the electrons in a free atom or ion. Larmor precession associates the Larmor diamagnetism. Therefore we should know that the direct action of the Lorentz force is also diamagnetic. Of course, in the case of electron spin, when it is in the higher Zeeman energy state, there is a possibility to emitt the Zeeman energy in a form of electromagnetic radiation and goes to the lower Zeeman energy state spontaneously.

It must be noted in these cases that the final stable configurations does not

correspond to the minimum U_m state.

The final state has even the highest U_m under the given condition. This is the physical situation which is present in the world of electromagnetism. The additional energy is, of course, supplied from the sources which are working to keep I_1 and / or I_2 constant.

B. Meissner's situation

This is the situation when there is no condition for the value of Φ_2 or I_2, and it corresponds to cases 1 and 4. For conductor 2 concerned, there is no difference between cases 1 and 4, because conductor 2 can only see the vector potential and the magnetic field present now and near around and, not that in future or at far distant. In case 1, as shown in Eq. (157), the magentic energy becomes minimum with zero Φ_2. In case 4, however, the minimum magnetic energy corresponds to zero I_2 and not zero Φ_2. Physically, however, when conductor 2 is simply connected, the final state must be identical. We shall analyze the problem hereafter.

Although we know that it is completely ficticious, for the simplicity, let us start our consideration for the case of Fig. 3. (a). Then initially, $\Phi_2 \neq 0$, and all the conduction electrons are making the kinematical motions of Fig. 3 (a). In the next instant, however, there must be a definite thermal micro processes towards the state of Fig. 3 (c), because as is shown by Eq. (72), it affords to each electron the less local magnetic energy. Different from the case of the electric potential of Eq. (73), the action is not supplied simply by the Lorentz force alone, but it must be assisted by thermal fluctuation. Probably, as we see already in §2, most important change of states will be for the electrons in the surface states, and in this case, the interaction with the lattice becomes important. We believe that this must happen, because it creates definitely the lower local magnetic energy state. As we have already seen in §'s 3 and 4, if the magnetic field is applied from outside, this change of states happens dynamically and adiabatically, by means of the action of the induced transient electric field **E**. Now in case 1, the situation is simple because, in this way, the system attains the minimum U_m state and the excess magnetic energy will be returned to the source which is working to maintain the I_1 constant. In case 4, however, the initial U_m^i and Φ_1^j are

$$U_m^i = \frac{1}{2} L_{11} (I_1^i)^2, \quad \Phi_1 = cL_{11} I_1^i \tag{163}$$

and the final U_m^f

$$U_m^f = \frac{1}{2} L_{11} (I_1^i)^2 \frac{1}{1 - \frac{L_{12}^2}{L_{11} L_{22}}} > U_m^i \tag{164}$$

$$I_1^f = \frac{1}{1 - \frac{L_{12}^2}{L_{11} L_{22}}} I_1^i \tag{165}$$

Therefore the total magnetic energy has increased during the transient processes. As we know already, the action of the magnetic energy is very tricky and this situation will be not against the thermodynamics, because we are dealing with an essentially non-equilibrium thermal processes. As shown in Fig. 8, the initial building up of the Meissner state in conductor C_2 will be made with the consumption of the magnetic energy of the surface state electrons and this information with an electromagnetic energy travels to C_1 with the speed of light. Then since C_1 is a perfect conductor, by Eq. (138), C_1 can not pass this electromagnetic energy to outside and an increase in I_1 will be induced. This second information then travels back to C_2, but C_2 also can not absorb this electromagnetic energy and this energy is now spent to build up U_m.

Now let us criticize the previous treatment of Landau's diamagnetism[14), 15), 16), 17)] again. It was said usually that, because of Miss Van Leeuwen's theorem, there is no diamagnetism classically in the Landau's case, but there is a Landau's diamagnetism quantum mechanically. Here, we propose that these statements are essentially wrong, and we can say that there are always orbital diamagnetism in the Landau's case classically and quantum mechanically as well, but the amount should be calculated very carefully taking into account the coherent length of the orbital motion, the role of electron spins, the electron-electron mutual correlations, Zeeman energy, and / or the magentic energy of the form of Eq. (72). Since, in these cases, there are strong interactions with the lattice, we don't believe that there are surface state paramagnetic electrons, when, however, they are assumed to be present, we should take into account the positive magnetic energy of Eq. (72). Therefore, we must be very careful to judge whether the previous calculations are correct or not.

It is also noted that, in these calculations, we must know the rigorous statistical

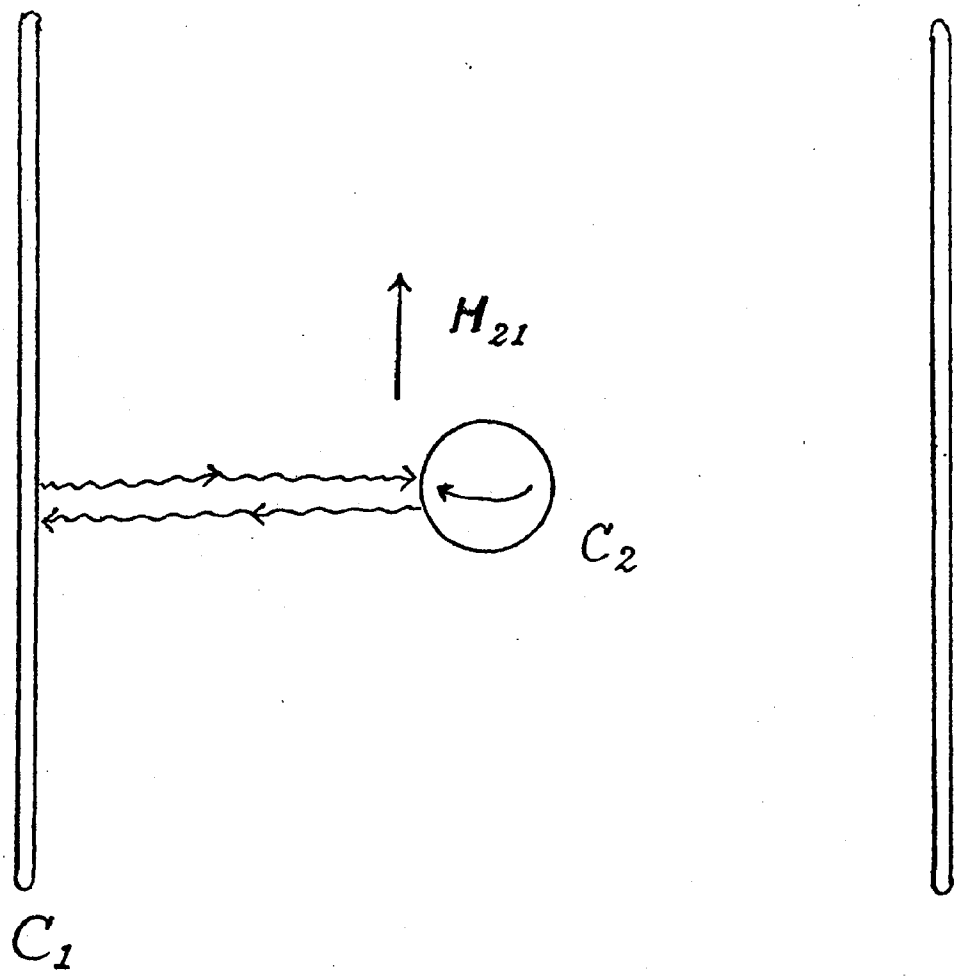


Fig. 8. Thermoelectromagnetic dynamic building up of the Meissner state in a perfect conductor C_2 located in the magnetic field of a perfect conductor C_1.

thermodynamics of the system, the introduction of this problem will be given in the next paper.[10]

It is to be mentioned that the thermodynamics of the system with the elements having permanent magentic moments, and the system without permanent magentic moments must be different.[20),21)] This fact has been known already, but, there had been no detailed analysis on this problem. In the next paper, we shall provide the answer to this important problem. It is to be mentioned that, as the result, fortunately the old treatment is still approximately correct, when the resultant magnetization is not so large (When neglecting the order of the susceptibility κ against one.).

Here, we predict that the magnetism of small particle superconductors will be most

interesting. Since, in small particles, the positive magnetic energies of paramagnetic surface electrons becomes comparable with the kinetic energies and we can expect the mixing effects of these energies for the magnetism of the system.

§8. Beautiful Symmetry of the Classical Electromagnetism

Now we have proved the existence of the Meissner effect in our perfect conductors in classical electromagnetism. In this way, we have obtained a beautiful symmetry between electricity and magnetism. The role of the conductors in electricity is now parallel to the superconductors in magnetism. The surface charge in electricity just corresponds to the surface currents in magnetism. The correspondences are

$$\begin{aligned} &\mathbf{E} = 0,\ \varphi,\ \nabla,\ \nabla\cdot,\ \nabla\times,\ \rho,\ \sigma \text{ etc.} \\ &\mathbf{B} = 0,\ \mathbf{A},\ \nabla\times,\ \nabla\cdot,\ \mathbf{j},\ \mathbf{k} \text{ etc.} \end{aligned} \tag{166}$$

Here, σ and $\mathbf{k}$ indicate surface charge density and surface current density. We propose that, in future, the presence of the superconductor with the Meissner effect should be introduced in the education of classical electromagnetism, because, in this way we are able to understand the intrinsic electromagnetic structure of the world very quickly and accurately.

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