6. ラフニング転移

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ラフニング相転移は今から30年以上も前に結晶成長の問題に関連してその可能性が指摘された。即ち結晶速度がある温度 T_R を境にして急激に増大することが結晶表面の構造の変化に結びつけられた。この様な物性物理でも地味な研究が普通の意味での対称性の破れを伴なわない新しい型の相転移として脚光をあびて来たのは5年程前であるが，これがクォークの閉じ込めに応用されるようになった事はこの問題に多少かゝわりあった者にとって大きな驚きであった。研究会では初めにこの問題の初等的レビューを行った。たゞし新しい話として Ref. 1）と2）に引用した論文を紹介した。私の話の後半では，ラフニング転移と直接結びつく問題ではないが相境界面の運動を記述する一つの模型について話した。内容についてはこの文の後にある英文のサマリーを参照されたい。ここでは表面がある形をとる確率分布の時間変化を追求するのでそのような表面のとりうる状態の数を数える事が問題になる。素粒子の方でも周囲を固定した時の面の数が問題にされているのは大変興味深い（川合氏の話）。

1) J. E. Avron et al. Phys. Rev. Lett **45** (1980) No. 10.
2) J. M. Luck, J. Physique–Lettrer **42** (1981) L–275.

Satistical Dynamics of Interface

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I. Introduction

An interface between two coexisting phases can be regarded as a simplest example of topological excitations, other familiar examples being vortex lines in fluids, dislocations and disclinations in crystals etc.[1] Such topological excitations appear as singular solutions of the equations of motion or the equations for equilibrium states of nonlinear continuum model systems in the limit of strong nonlinearity. Such topological excitations are very important in the fully developed hydrodynamic turbulence as well as in the late stages of phase transition kinetics where nonlinearity of the problem plays a crucial role. Here a simple stochastic model called the kinetic drumhead model is presented[2,5] which describes random motions of an interface together with its applications.

II. Derivation of the model

We start from the well-known time-dependent Ginzburg-Landau model[3] which is expressed in the form of a nonlinear Langevin equation for the non-conserved local order parameter $S(x, t)$ as

$$\frac{\partial}{\partial t}S(\boldsymbol{x},\ t) = -L\ \{\ \nabla^2-\tau)\ S+\frac{g}{6}S^3)\ +\ \zeta(\boldsymbol{x},\ t) \tag{1}$$

where ζ is an appropriate random force representing a thermal noise. Singular solutions are obtained in the limit of strong nonlinearity $g\rightarrow\infty$, which in turn requires $\tau\rightarrow\infty$ in order to keep the equilibrium value of the order parameter finite[4]; $S_{\text{eq}}=\pm\ [\ 6\tau/g]^{1/2}$. The resulting stochastic equation for the position of the interface $z = f(\boldsymbol{r}, t)$ with $\boldsymbol{x} = (z, \boldsymbol{r})$ can be expressed again as a nonlinear Langevin equation as follows:

$$\frac{\partial}{\partial t}f(\boldsymbol{r},\ t) = \sqrt{1+(\partial_r f)^2}\ \ [\ LK+\zeta_{\text{f}}\ ] \tag{2}$$

where $K\equiv\partial_r\cdot[\partial_r f/1+(\partial_r f)^2)$ is the mean curvature of the interface and ζ_{f} is the random force. Detailed discussions about the derivation and the properties such as the Euclidean invariance of (2) can be found in Refs 2 and 5.

III. Applications

A. Critical dynamics

The model (2) allows an expansion in $(d-1)$ of the dynamic critical exponent $z(d)$ in the vicinity of $d = 1$ where d is the spatial dimensionality of the hulk system. Such an expansion was recently obtained by Bausch et al. See Ref. 5, for further details.

B. Kinetics of phase transitions

Here we are interested in the whole process of approach to equilibrium when the system is suddenly quenched from a high temperature disordered equilibrium state to an unstable low temperature state accompanyed by enomous supra-thermal fluctuation.[3,6] One of the fundamental problems here is the understanding of the newly discovered scaling low where the normalized variance of the Fourier components of the local under parameter fluctuations exhibit a scaling behavior $k(t)^{-d}F(g/k(t))$ where $k(t)^{-1}$ is the varying length scale characterizing the spatial pattern of fluctuations after a certain transient period.[6] Earlier theoretical attempts at this problem based or decoupling ideas[8] or singular perturbation theory[3,9] met only partial success in coping with strong nonlinearity. The approach which starts from (2) hence complements the earlier theories. We report here only some preliminary results obtained so far. First, the scaling behavior can be understood rather easily. We first rewrite (2) in the form of a Fokker-Planck type equation for the probability distribution functional for the position of interface $P(\{f\}, t)$. We then seek a scaling type solution $P(\{f\}, t) = \hat{P}(\{\phi\})$ where $\phi(y) = k(t) f(r, t)$ with $y = k(t)r$. We then find that it $\dot{k}(t)/k(t)^3$ is a negative constant (which we take to be $-L$ by an appropriate choice of unit of time), the Fokker-Planck equation allows a scaling solution where the term coming from the random noise term of (2) drops out. Hence $k(t) = (2Lt)^{-1/2}$ for sufficiently long times. The scale-invariant functional now satisfies the condition

$$\int dya \frac{\partial}{\partial\phi}\left[\frac{-\phi + y\cdot\nabla\phi}{a} + \hat{K}\right]\hat{P}(\{\phi\}) = 0 \tag{3}$$

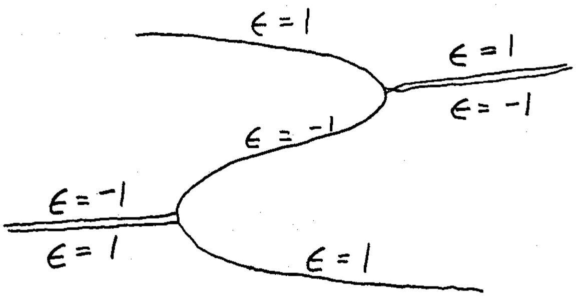
where $\hat{K} \equiv \nabla\cdot[\nabla\phi/a]$ is the scaled mean curvature with $\nabla = \partial/\partial y$ and $a = \sqrt{1+(\nabla\phi)^2}$. The first and second terms of (3) represent spurious and real displacements of interface $\phi(y)$, respectively, where the former is caused by the scale change $k(t)$ and the latter by the curvature. Thus the condition (3) shows that these two displacements compensate in maintaining a steady state distribution $\hat{P}\{\phi\}$ in an expanding universe. Here it is interesting to note an analogy between the present problem (with the direction of time reversed) and the three-dimensional strong Navier-Stokes turbulence. Both are governed by deterministic nonlinear equations of motion which are expected to show chaotic behaviors. Singular solutions of there equations are important in the limit of high Reynolds number or strong nonlinearity. Such singular solutions are not yet fully understood in the case of Navier-Stokes turbulence (vortex lines, vortex sheets, etc.)[10] whereas they are well-known in the form of interface in our case. "Topological excitations" (singular solutions) in both problems are expected to show stretching and folding behaviors that characterize a turbulent motion.[11] Hence the study of phase transition kinetics should be useful to elucidate some aspects of the Navier-stokes turbulence. We have also found a constant of the motion $\int dya$ (the total area of interface) under some assumption where the integral is over the entire invariant space but is not over a space of y which shrinks with the changing length scale. However this appears to be not quite sufficient to obtain an explicit form for the invariant distribution $\hat{P}$ in analogy with the equilibrium distribution which is a function of constants of the motion such as the Hamiltonian. This is due to the fact that the time variations of ϕ is not measure-preserving. It may be neccessary to reconsider the stationarity condition in the expanding universe more carefully by taking note of the actual volume change of the system and the concomitant change in the function space of f and/or ϕ which are required to ensure the stationarity. Here the problem has some formal similarity to obtaining the fixed point Hamiltonian in critical phenomena. The problem is now under active investigation and the results will be reported elsewhere.

Finally we exhibit formula that connects the experimentally accessible equal-time pair correlation of the Fourier component of the order parameter fluctuation $G(q)$ to $f(r)$:

$$G(q) = (2|S_{\mathrm{eq}}|)^2 q^{-2} [1 + (q_\perp/q_z)^2) V^{-1} \iint d\boldsymbol{r}_1 \, d\boldsymbol{r}_2 \, e^{-iq\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)} \times \langle \sum_\alpha \sum_\beta \varepsilon_\alpha \varepsilon_\beta e^{-iq_z[f_\alpha(\boldsymbol{r}_1)-f_\beta(\boldsymbol{r}_2)]} \rangle \tag{4}$$

where $q_\perp$ and q_z are the components of the vector q parallel and perpendicular to the $\boldsymbol{r}$-plane, respectively. Here we have generalized the model so that there can be more than one interfaces which are designated by α and β. ϵ_α is the polarity of the interface α which is +1 or −1 according to the sign of S above the interface.

Note that possible multivaluedness of the function $f(r)$ can be handled by introducting interfaces with different polarities as is shown in the figure where of course interfaces of opposite polarities cancel whenever they touch each other.



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