Quantised Hall effect and localisation in two dimensions

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The quantised Hall effect (von Klitzing et al 1980) is remarkable in that the quantum plateau of the Hall conductivity, which provides e^2/h precisely, comes from localisation due to randomness, which usually smears out a sharp transition in a system. Laughlin (1981) was first to introduce a gauge-transformation approach to this phenomena to be followed by Halperin(1982) and Imry(1982). However, whilst this approach certainly gives a correct result in the disorder-free case, the situation for the disordered system with localisation has not been clarified in terms of the gauge transformation. We elucidate this point to show that the quantised Hall plateau (Aoki and Ando 1981) indeed results from the gauge considerations. This is closely related to the nature of localisation of the states in this system, which is quite different from ordinary systems since the Landau quantisation in two dimensions makes the electronic structure peculiar. We point out that we can use the dependence of states on a gauge transformation as a measure of localisation in systems in magnetic fields.

The geometry of the system is shown in figure 1. Varying the magnetic flux Φ, which penetrates the opening of the ribbon, causes a gauge transformation in this Aharonov-Bohm type setting. The discrete set of values for the centre of cyclotron motion, X, is changed into $X\ (0)+(\Phi/\phi_0)\Delta X$ with $\Delta X=2\pi\ell^2/L$ and $\phi_0=hc/e$. The total energy of the system, E_T, and the total Hall current, I, are related (Byers and Yang 1961) by $I=c\partial E_T/\partial\Phi$. Figure 2 shows a numerical result for the eigenstates as a function of Φ in the absence of an electric field. The total number of states for the ground Landau subband is taken to be $N_L=256$. It is seen that the Φ-dependence of the state reflects the degree of localisation, which is the weakest at the band centre. The result for the Hall current for a finite electric field (figure 3) clearly shows a plateau behaviour arising from the localisation (Aoki 1982).

Now we can use the sensitivity of an eigenstate against a gauge transformation to study the localisation (Aoki 1983b). The sensitivity is defined as a ratio of the energy shift ($\Delta\epsilon$) due to the transformation to the mean level spacing in a similar manner as Licciardello and Thouless (1975). Then we can study the sample-size dependence of the Thouless number. The result (figure 4) shows that the states are exponentially localised as $\exp(-\alpha L)$. We plot the localisation length defined by $1/\alpha$ against energy in figure 5, which shows a steep rise of the localisation length towards the band centre, suggestive of its divergence at the band centre as discussed by Ono(1982) and Ando(1983).

If the localisation length diverges at the band centre, it is expected that the state at this particular energy shows some critical behaviour. It is an interesting question, especially for the case of short-range scatterers for which the percolation picture is invalid. We propose that the state should have a self-similar, filamentary structure with a scale invariance, which is reminiscent

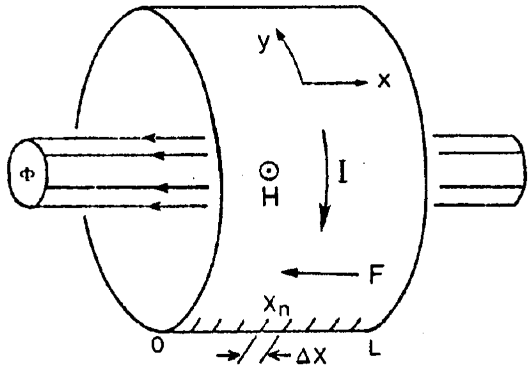

Figure 1. The Hall current I in the loop geometry with a magnetic field H, an electric field F and a magnetic flux Φ through the cylinder. A discrete set of values is allowed for X.

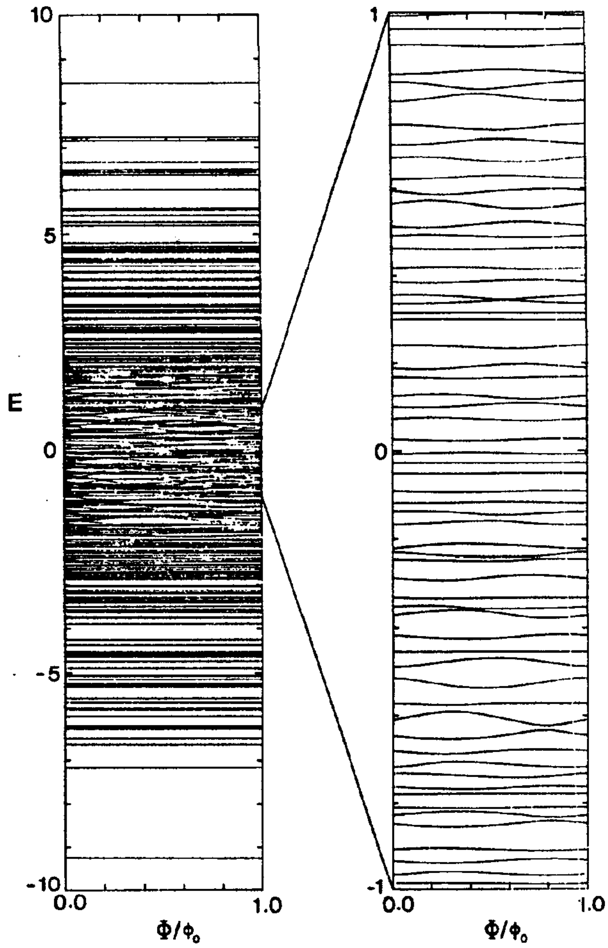

Figure 2(right). All the energy levels are plotted against the flux Φ over one period of flux quantum. The wiggle of the lines is exaggerated by a factor of ten: no two lines do infact cross as is shown in the enlarged diagram in a real scale.

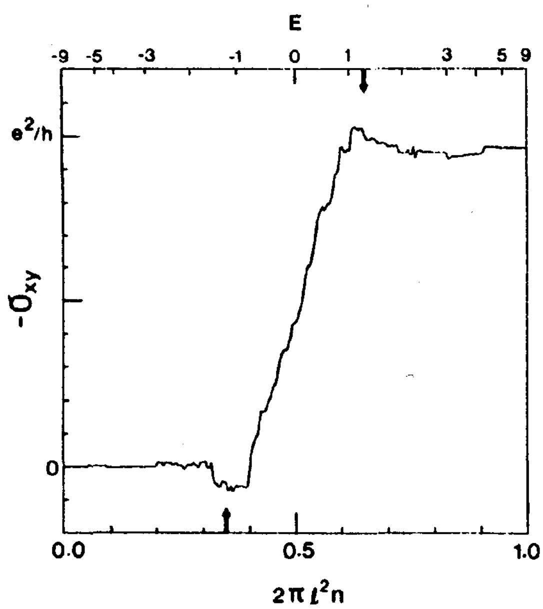

Figure 3. The Hall conductivity agains the Landau-band filling. The arrows indicate the transition to extended behaviour for the finite sample size considered here.

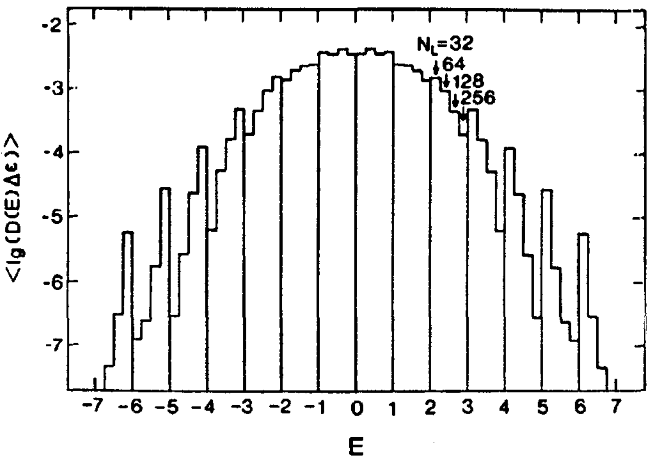

Figure 4. Logarithmic plot of the Thouless number. Within each energy bin, the system size is increased from N_L=32 to 256.

of a fractal (Aoki 1983a). A numerical example of the eigenstate with energy close to the band centre (figure 6) suggests that the long-range network structure may influence the transport properties of the system.

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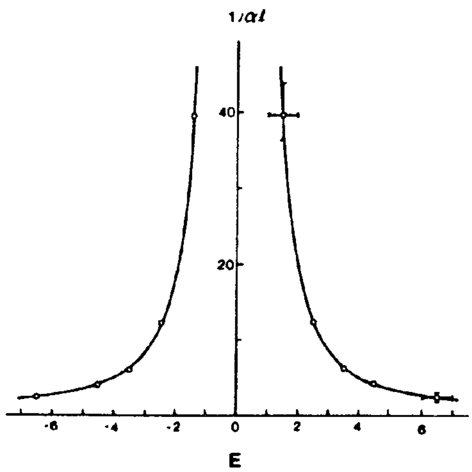

Figure 5. The quantity, $1/\alpha$, with $\langle D(E)\Delta\varepsilon\rangle \sim \exp(-\alpha L)$ in figure 4 is plotted versus energy.

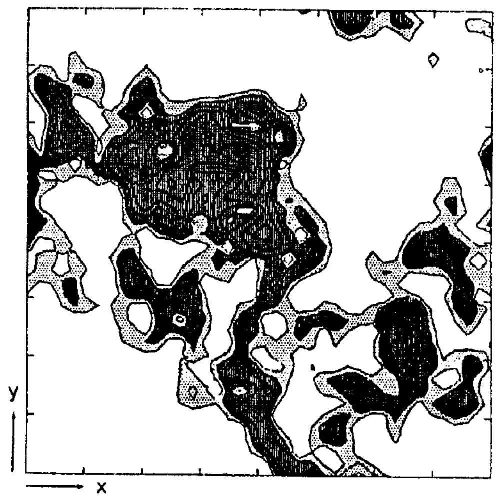

Figure 6. An eigenstate with energy $\varepsilon = E_0 - 0.007$ is shown. The absolute value of the normalised wavefunction in the area of $40\ell \times 40\ell$ is shown with contour heights $0.1/2^{n/2}$ $(n=0,\ldots,5)$.