TWO-DIMENSIONAL ANDERSON LOCALIZATION UNDER STRONG MAGNETIC FIELDS

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§1. Introduction

When we consider the problem of the localization in random potentials, one of the most convenient methods is to investigate the low frequency behaviour of the dynamical diffusion coefficient. If it vanishes for $\omega \to 0$, then the system is considered to be insulating and the corresponding states are localized. On the other hand, if it is finite at $\omega = 0$, the system is conducting and the states are extended.

Vollhardt and Wölfle (VW) proposed a microscopic formulation to calculate the dynamical diffusion coefficient self-consistently. In two dimensions, they have obtained the following form of the dynamical diffusion coefficient in the low frequency limit,

$$D(\omega,E) = -i\omega A_1(E) + \omega^2 A_2(E) + \cdots \tag{1}$$

where A_1 and A_2 are positive functions of the Fermi energy E. That is, $D(0,E) = 0$ irrespectively of E. This conclusion is consistent with that obtained by Abrahams, Anderson, Licciardello and Ramakrishnan by the renormaization group theoretical treatment. The coefficient A_1 is found to be the square of the localization length and an exponentially increasing function for large values of E.

The VW formulation has been extended by Yoshioka, Fukuyama and the present author to the case with a weak magnetic field. They have also obtained the dynamical diffusion coefficient in the form of eq. (1). The effect of the magnetic field is to increase A_1 and A_2, which means that the magnetic field weakens the localization. This is consistent with the theory of the negative magnetoresistance by Hikami, Larkin and Nagaoka. The fact that $D(\omega=0) = 0$ even in the presence of the magnetic field, however, implies that the weak magnetic field does not completely destroy the two-dimensional localization. Then a naive question arises, how about the two-dimensional Anderson localization under strong magnetic fields?. In this note we discuss about it by using a similar method as VW'.

§2. Self-Consistent Calculation of Dynamical Diffusion Coefficient

Precise investigation of the VW formulation gives us the notion that the most essential point is the renomalization of the diffusion process. This renormalization corresponds to take account of the diffusive motion of electrons between successive scatterings by an impurity, and the diffusive motion of electrons is characterized by the dynamical diffusion coefficient which should be determined self-consistently. The same idea can be applied in the presence of the strong magnetic fields.

In the following we take the following model of two-dimensional electrons scattered by short-range impurities under a perpendicular magnetic field,

$$H = \int dr\, \psi^{\dagger}(r) [\, (p + \frac{e}{c}A)^2/2m + u \,\Sigma\, \delta(r - R_i)\,]\psi(r) \tag{2}$$

where $\psi^{\dagger}$ and ψ are electron field operators, A the vector potential, u the strength of the impurity potentials distributed randomly at $\{R_i\}$. In order to describe the diffusion process, we consider the density relaxation function $\phi(q,\omega,E)$ at T = 0 with E the Fermi energy. ϕ is expressed in terms of the retarded and advanced single particle Green functions as

$$\phi(q,\omega,E) = \Sigma\, \rho_{\alpha\alpha'}(q)\, \varphi_{\alpha\alpha'}(q,\omega,E)(i/2\pi) \tag{3}$$

$$\varphi_{\alpha\alpha'}(q,\omega,E) = \Sigma\, \langle G^R(\alpha,\beta,E+\omega) G^A(\beta',\alpha',E)\rangle \rho_{\beta'\beta}(-q) \tag{4}$$

$$\rho_{\alpha\alpha'}(q) = \langle\alpha|\, \exp(iqr)\, |\alpha'\rangle \quad ; \ |\alpha\rangle \quad |N,X\rangle \ \text{the Landau state} \tag{5}$$

The bracket in eq. (4) expresses the impurity average. By decomposing the average of the product of Green functions into products of the averaged Green functions as usual, it is straight forward to obtain a Bethe-Salpeter type of equation for $_{\alpha\alpha'}$. Summing up this equation with respect to α and α' and using the Ward identity which relates the ireversible vertex corrections and the self-energies of the single-particle Green functions, we obtain the following equation,

$$\omega\phi(q,\omega,E) + \phi_j(q,\omega,E) = -N(E) \qquad [\ N(E) \text{ the density of states at } E\ ] \tag{6}$$

with

$$\phi_j(q,\omega,E) = \Sigma\, (\varepsilon_{\alpha} - \varepsilon_{\alpha'})\rho_{\alpha\alpha'}(q)\, \varphi_{\alpha\alpha'}(q,\omega,E)(i/2\pi) \tag{7}$$

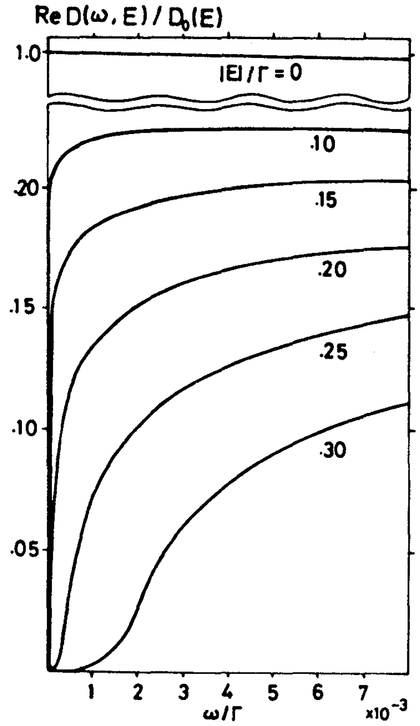
which is nothing but the correlation between the current and the density. It is straightforward to show by using the number conservation that ϕ_j is propotional to $q^2\phi$ in the small q limit. In other words, ϕ is written in the form,

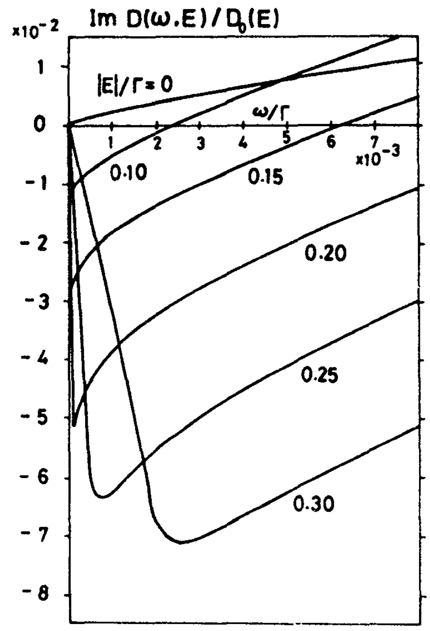
$$\phi(q,\omega,E) = -N(E)/[\omega + iD(\omega,E)q^2] \ . \tag{8}$$

The coefficient D is found to have the meaning of the diffusion coefficient. In fact, by introducing an appropriate approximation for the irreducible vertex correction, we can derive a relation between D and the irreducible vertex corrections. An approximation which takes account of the effective diffusive motion between scatterings leads to the following form of the self-consistent equation for $D(\omega,E)$,

$$D(\omega,E) = \frac{2\pi^2 \ell^4 n_i u^2 N(E)[(1-n_i u^2 P^2 w_1')(1/2\pi\ell^2 + Pw_1) + n_i u^2 P^2 S^2 w_1'(w_0 - w_1)]}{(1-n_i u^2 P^2 w_1')^2 - n_i u^2 PS^2 w_1'(1-2n_i u^2 P^2 w_1')} \tag{9}$$

where we have assumed that the Fermi energy lies within the lowest Landau subband and that the subband width is much smaller than the subband splitting (i.e. the strong field limit); n_i is the impurity concentration, ℓ the magnetic length, $P = G^R(E)G^A(E)$ and $S = G^R(E) + G^A(E) = 2\,\mathrm{Re}[G^R(E)]$ with the impurity averaged Green functions for the lowest Landau subband G^R and G^A, and w_0, w_1 and w_1' are integrals including the diffusion process. Especially w_0 has the logarithmic singularity in the small ω limit. Because of this singularity D must have the form of eq. (1) as far as $S \neq 0$ and we arrive at the conclusion that the states are localized except for the subband center which is defined as the energy at which S vanishes. In the figures below we show the frequency dependences of Re D and Im D obtained from eq. (9), where $D_0(E) = 2\pi\ell^2 n_i u^2 N(E)$ and Γ is the subband width; $E = 0$ corresponds to the subband center.





§3. Conclusion

By calculating self-consistently the dynamical diffusion coefficient of two-dimensional electron system under strong magnetic fields, we showed that all the stats except for the subband center are localized, which is consistent with the numerical computation by Ando.