

New Frame in Physics,
New Thermodynamic Principles, and
Classical Derivation of the Meissner-Ochsenfeld Effect[†]

by

Shuichi Iida

Department of Physics, University of Tokyo
Bunkyo-ku, Tokyo 113, Japan

(1983年4月26日受理)

まえがき

新統一体系物理学の概要を示す英文報告を掲載させて戴く。新統一体系物理学の成立は10年に亘る努力の成果であって、理解には相当の努力を必要とするものと考え。一行で書いて居ることも、実際計算すると半日以上掛るという内容も入っているが、或る程度の頁数に収めないと逆に全貌が判らなくなる恐れがあるので御了承戴きたい。その詳細は多くの未公表論文の中に説明されているが、それらが、永年に亘って公表出来ないという事情が現在存在している。その内容は先月号「新体系物理学の発展と、近藤氏へ、Ⅶ」の中で説明される。なお、「近藤氏へ、Ⅶ」は近藤氏の「飯田氏へ、Ⅶ、Ⅷ、及び追記」の全部に対する回答であるが、「飯田氏へ、Ⅷ」及び「追記」の内容は筆者には理解不可能である。もちろん本英文報告は本格的論文であって、以上の諸報告より、はるかに重要である。

新体系物理学は物理学における新しい鉱床の露頭の発見であって、そこから有効な鉱石を掘り出すのは世界の人達の仕事である。本論文で説明するマイスナー効果理解の他、Stern - Gerlachの実験と観測の理論に関して、見事な応用が最近成立したことをお知らせできる。新体系物理学に関して、疑問、コメント、新しい知識などが、生れた際は、御遠慮なく御一報戴ければ幸いである。

IIDA, Shuichi; Dept. of phys., Univ. of Tokyo

[†] This is an exceptional paper which will bring the new concepts in physics, abandoning a few of the old common understandings. Therefore, there have been serious publication difficulties for which the author has not been responsible. Since a few tens of referees and friends, both domestic and international, had already presented their various critiques and opinions on several parts of this paper and they are well taken into consideration in this paper, the author would like to request for this printing to be regarded as an official publication of the new frame, which is to be referred officially in future.

なお本研究に関係して多くの時間を割かれた内、外の友人に厚く御礼申し上げる。又物性研究誌、とくに長岡洋介氏に厚く感謝させて戴く。レフェリーをしないで、必ず論文を掲載してゆく物性研究誌の編集方針は実に貴重なものであって、こうした常識を崩す研究は、物性研究誌の編集方針無くしては、その育成が不可能であると断言することを憚らないものである。

Abstract

This is a brief review on the new frame of physics. Based on the persistent current model of the electron, a new rigorously selfconsistent theory of classical electromagnetism, in which the magnetic field energy and the energy transfer by induction have been rigorously taken into account, is proposed. Using this new frame in electromagnetism, rigorous unification of the theories of the Maxwell-Lorentz electromagnetism and the quantum physics is developed, concluding that the classical Lagrangian equations, being regarded as operator equations, are accurately valid in the frame of the quantum theory. Equivalence and inequivalence of the Hamiltonian and Lagrangian formalisms in both classical and quantum theories are analyzed. The principle of action through medium requests the advantage of the Lagrangian formalism over the Hamiltonian, predicting its importance for macroscopically inhomogeneous systems; the Meissner effect presents a typical example. By using the Coulomb gas as an example, the proofs have been given that the Meissner effect is a classical property of the system which can sustain persistent currents. The principle of classification and the transient energy principle are proposed.

§1. Introduction

In 1911, Kamerlingh Onnes found the superconductivity and, in 1933, Meissner and Ochsenfeld found that the superconductors exhibit almost perfect diamagnetism with diamagnetic current at the surface. The phenomenon was described phenomenologically by London and London (1935) in terms of the so-called London equations.

In 1957, Bardeen, Cooper and Schrieffer have shown quantum mechanically, that why these metals can sustain persistent currents, together with a quantum mechanical explanation of the Meissner-Ochsenfeld effect.

Now, in 1974, we proposed a persistent current model of the electron (Iida 1974), which has introduced a continuous effort to reorganize the classical theory of electromagnetism (Iida 1975a). There had been a serious inconsistency for the momentum-energy density relation of the electrons in the existing classical theory of electromagnetism (e. g., Feynman et al. 1964), which

has related also to the current theory of quantum physics, where the divergence difficulties are present and the role of both the magnetic field energy and the transfer of electromagnetic energy by induction was implicit. Naturally these efforts requested to clarify the interface between classical and quantum physics, which lead finally to the proposal of a new frame in physics for the materials (Iida 1981c, 1982a, 1982b, 1982c, 1982d) together with a conclusion that the Meissner-Ochsenfeld effect is a classical property of a material which can sustain persistent currents, retaining many studies and debates (Iida 1975a, 1975b, 1977, 1978, 1979, 1980, 1981a, 1981b, 1981c, 1982a; Kondo 1978, 1979, 1981, 1982).

This paper presents a brief summary of these results and is hoped to initiate a new frame in physics, which will have an effect to the field of quantum electrodynamics in future. Since the most dramatical change from the current understanding has happened in the orbital diamagnetism of the electrons, the emphasis will be placed on the classical derivation of the Meissner-Ochsenfeld effect. In order to clarify a crux of the Meissner effect, we believe that we have found a new thermal law called the transient energy principle, which will also be introduced.

Different from the current understanding, we conclude that quantum theory is necessary in order to get perfect conduction, but once the material can sustain persistent currents, the Meissner effect is an its classical consequence.

Recently Edwards (1981) has reported that the London equation is derivable classically by using a modification of the action principle and mentioned that the Meissner effect is a classical property of perfect conductor. Since his work is mainly on the transient behavior of classical charged particles and did not analyze the effect of the surface boundary of the material, as well as the thermal stability of the effect, we insist that he did not prove that the Meissner effect is a classical property of perfect conductors and probably he knows our work (Iida 1975b, 1977, 1981c). The difference will be clarified also in this paper.

§2. Brief explanation of the new frame in classical electromagnetism

As has been mentioned, there has been a serious inconsistency in the accepted theory of classical electromagnetism. In 1974, we have found the vortex ring classical model of the electron (Iida 1974[†], we refer it as VR hereafter), in which the electron is approximated by a tiny persistent ring current with a continuous charge density, having the size of the order of the Compton wave length, i. e., 10^{-2} \AA . There is a quite common misunderstanding about the classical size of the electron.

[†] A small mistake has been found in the paper. The values of $(6.33 \text{ or } 4.75) \times 10^{-378} \text{ m}$ in p. 1586 should be replaced with $(1.241 \text{ or } 0.967) \times 10^{-386} \text{ m}$.

Different from the case of the α -particle for the Rutherford scattering, after admitting the superposition and the Pauli principle for the two approaching electrons, there has been no classically explainable experimental result which can support the very small size, much smaller than the above, electron, so that this is the only possible minimum classical size which can be supported theoretically as well as experimentally. With the use of this concept for the electron and starting from the Maxwell-Lorentz field equations, the success has followed, in which a consistent unified theory of classical electromagnetism was obtained (Iida 1975a, 1978, 1981a, 1981c, 1982a). Now we regard that the world consists of electrons, nuclei, and electromagnetic fields. As the representative of the charged particles, we take the electron and approximate it by the VR model. The real quantal electron is approximated by three steps. In the first step, the VR is represented by the original tiny ring current and charge densities with its electromagnetic fields, $(\mathbf{e}_i, \mathbf{h}_i)$, for the i -th electron. In the next step, if the spin state is α , the state is represented by an ensemble, having the angular momentum vectors distributing uniformly over the upper hemisphere (Iida 1974). In the third step, we take the second ensemble in which the center of the VR's distributes according to the quantal probability, $\psi^*(r, t) \psi(r, t)$, moving in accordance with the probable orbital electric current of $\psi^*(-e\mathbf{v}) \psi = (-e/2m)[\psi^* \{(\hbar/i) \nabla - (-e/c)A\} \psi + \psi \{(-\hbar/i) \nabla - (-e/c)A\} \psi^*]$. Thence, we get the electric current density four vector, $\{\mathbf{j}_i, i c \rho_i\}$, for the i -th electron, \mathbf{j}_i being composed of spin and orbital currents. Here, we use i for i of the four space (Iida 1975a). We define $(\mathbf{e}_i, \mathbf{h}_i)$ as the resultant total ensemble average for the i -th electron in the Maxwell-Lorentz world. Our final Maxwell-Lorentz electromagnetic fields are $(\mathbf{e}, \mathbf{h}) = (\sum_i \mathbf{e}_i, \sum_i \mathbf{h}_i)$, in which the free electromagnetic wave is defined by $i = 0$.

Then the Maxwell-Lorentz equations for (\mathbf{e}, \mathbf{h}) can be decomposed as

$$\nabla \times \mathbf{e}_i = -\frac{1}{c} \frac{\partial \mathbf{h}_i}{\partial t}, \quad \nabla \cdot \mathbf{h}_i = 0 \quad (1)$$

$$\nabla \times \mathbf{h}_i = \frac{1}{c} \frac{\partial \mathbf{e}_i}{\partial t} + \frac{\rho_i \mathbf{v}_i}{c} + \frac{\mathbf{I}_i}{c}, \quad \nabla \cdot \mathbf{e}_i = \rho_i,$$

in which, ρ_i is the charge density of i -th particle and \mathbf{I}_i is its persistent closed current part and $\rho_0 = \mathbf{I}_0 = 0$. There can be free wave components for $i \neq 0$, depending on the physical situation of the system. We use the MKSP system (MKS rationalized Gauss system) for the units (Iida 1974, 1975a). Defining the fictitious magnetic shell function \mathbf{m}_i by

$$\nabla \times \mathbf{m}_i = \frac{\mathbf{I}_i}{c}, \quad (2)$$

to replace \mathbf{I}_i/c in Eq. (1), summing over i , and averaging macroscopically, we get the Maxwell

equations. Namely,

$$\begin{aligned}\bar{\bar{e}} &= E, \quad \bar{\bar{h}} = B, \quad \bar{\bar{m}} = \sum_i \bar{\bar{m}}_i = M, \\ \frac{\bar{\bar{\rho v}}}{c} &= \frac{(\bar{\bar{\rho v}})_{\text{free}}}{c} + \frac{(\bar{\bar{\rho v}})_{\text{bound}}}{c} = \frac{j}{c} + \frac{1}{c} \frac{\partial P}{\partial t}, \\ \frac{\bar{\bar{I}}}{c} &= \boldsymbol{\nabla} \times \bar{\bar{M}}, \quad \bar{\bar{\rho}} = \bar{\bar{\rho}}_{\text{free}} + \bar{\bar{\rho}}_{\text{bound}} = \rho^F - \boldsymbol{\nabla} \cdot \bar{\bar{P}}.\end{aligned}\tag{3}$$

There are additional equations for the ripple electromagnetic fields.

From Eqs. (1) and (2), and integrating over an arbitrary volume V with the surface S , we get

$$\begin{aligned}& -\iint_S c \sum_{j \neq i} \mathbf{e}_j \times (\mathbf{h}_i - \mathbf{m}_i) \cdot d\mathbf{S} \\ &= \iiint_V \sum_{j \neq i} \left[(\mathbf{e}_j \cdot \frac{\partial \mathbf{e}_i}{\partial t} + \mathbf{h}_i \cdot \frac{\partial \mathbf{h}_j}{\partial t}) + \mathbf{e}_j \cdot \rho_i \mathbf{v}_i + (-\mathbf{m}_i) \cdot \frac{\partial \mathbf{h}_j}{\partial t} \right] dV,\end{aligned}\tag{4}$$

and

$$-\iint_S c \mathbf{e}_i \times \mathbf{h}_i \cdot d\mathbf{S} = \iiint_V \left[\frac{\partial}{\partial t} \frac{(\mathbf{e}_i^2 + \mathbf{h}_i^2)}{2} + \mathbf{e}_i \cdot (\rho_i \mathbf{v}_i + \mathbf{I}_i) \right] dV.\tag{5}$$

In Eq. (4), i can be either fixed or summed. Using Eq. (4), we have an identity of

$$\iiint_V \sum_{j \neq i} (-\mathbf{m}_i) \cdot \frac{\partial \mathbf{h}_j}{\partial t} dV = \sum_{j \neq i} \left[\iiint_V \mathbf{e}_j \cdot \mathbf{I}_i dV + \iint_S c \mathbf{e}_j \times \mathbf{m}_i \cdot d\mathbf{S} \right].\tag{6}$$

In Eq. (6), by considering the case where S does not cut \mathbf{I}_i 's or \mathbf{m}_i 's, it is easy to see that the equation represents the energy transfer by induction to the persistent currents, \mathbf{I}_i , or, the magnetic moment, \mathbf{m}_i . Therefore, the right side terms of Eq. (4) represent the time change of the electromagnetic interaction energies, the rate of work done to $\rho_i \mathbf{v}_i$, and the rate of work done to \mathbf{I}_i . Accordingly, the left side term should show the electromagnetic energy flow rate through the surface S .

In Eq. (5), we regard in the new frame that, besides the free radiation component, the electromagnetic self-energy of particle i , $\iiint_V [(\mathbf{e}_i^2 + \mathbf{h}_i^2)/2] dV$, can not be separated from the relativistic self energy of the particle. This view is different from the usual starting view of the field theory (e. g., Nishijima 1969), and, therefore, the renormalization procedure becomes not necessary. Different from the physics of elementary particles, we claim that this is a useful approximation for the physics of materials. Eq. (5) includes the time change of this part of the self energy of particle i . Initially, we disregard Eq. (5), except the free radiation for which $i = 0$, and consider only Eq. (4), because, in a usual material, the ratio of the weight of Eqs. (5) to (4) is $N : N^2$, where N is the number of particle i in a small volume ΔV . Then, we get from Eqs. (4) and (5)

$$\begin{aligned}
& - \iint_S c \mathbf{E} \times (\mathbf{B} - \mathbf{M}) \cdot d\mathbf{S} \\
& = \iiint_V \left[\mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} + \mathbf{B} \cdot \frac{\partial \mathbf{B}}{\partial t} + \mathbf{E} \cdot \mathbf{j} + \mathbf{E} \cdot \frac{\partial \mathbf{P}}{\partial t} - \mathbf{M} \cdot \frac{\partial \mathbf{B}}{\partial t} \right] dV, \quad (7)
\end{aligned}$$

and the corresponding ripple term equation. We must notice from Eq. (6) that

$$-\mathbf{M} \cdot \frac{\partial \mathbf{B}}{\partial t} \quad (8)$$

represents the main component of the energy transfer by induction to the magnetic moments μ_i . The ripple term equations must represent various non-Maxwell energy transfers, such as by mechanical stresses, phonons, and thermal conductions. Energy transfer by the mass transportation, however, has been excluded and will be analyzed next.

Now, let us extend our analysis to the electromagnetic momentum-energy density relations. According to the logics developed in the foregoing paper (Iida 1974), the self momentum-energy density four vector of the i -th particle, $\{p_i\}_{\text{self}}$ is represented by

$$\{p_i\}_{\text{self}} = \left\{ \frac{*\rho_i^\circ *a_i}{2c} + r_{vi} \frac{(*j_i^\circ \cdot *a_i^\circ) *v_i}{2c^2}, \frac{i}{c} \left[\frac{*\rho_i^\circ * \phi_i}{2} + r_{vi} \frac{(*j_i^\circ \cdot *a_i^\circ)}{2c} \right] \right\}. \quad (9)$$

Here, $\{*j_i^\circ, i c * \rho_i^\circ\}$ and $\{*a_i^\circ, i * \phi_i^\circ\}$ are the current and potential four vectors in the proper frame of the i -th particle. The physical meaning of the self factor 1/2 is essentially identical to the other well-known cases in electromagnetism. It is noted that, in order to get the total momentum-energy four vector, the integration must be made in the proper space, V° , or, the proper hyperplane, σ° .

From the Maxwell-Lorentz equations for $(*e_i, *h_i)$, having no singularity everywhere, we have derived the following energy-momentum related identities,

$$\begin{aligned}
\iiint_\infty (*e_i \cdot *e_j + *h_i \cdot *h_j) dV &= \iiint_\infty \left[* \phi_i * \rho_j + *a_i \cdot \frac{*j_j}{c} - \frac{\partial * \phi_i}{c \partial t} \frac{\partial * \phi_j}{c \partial t} \right. \\
&\quad \left. + * \phi_j \frac{\partial^2 * \phi_i}{c^2 \partial t^2} + \frac{\partial * a_i}{c \partial t} \cdot \frac{\partial * a_j}{c \partial t} - *a_i \cdot \frac{\partial^2 * a_j}{c^2 \partial t^2} \right] dV, \quad (10)
\end{aligned}$$

$$\iiint_\infty \frac{*e_i \times *h_j}{c} dV = \iiint_\infty \frac{1}{c} \left[* \rho_i * a_j + \nabla * \phi_i \frac{\partial * \phi_j}{c \partial t} - \nabla * a_j \cdot \frac{\partial * a_i}{c \partial t} \right] dV, \quad (11)$$

$$\begin{aligned}
\iiint_\infty \mathbf{r} \times \left(\frac{*e_i \times *h_j}{c} \right) dV &= \iiint_\infty \left\{ \mathbf{r} \times \left[\frac{* \rho_i * a_j}{c} + \frac{\nabla * \phi_i}{c} \frac{\partial * \phi_j}{c \partial t} - \nabla * a_j \cdot \frac{\partial * a_i}{c \partial t} \right] \right. \\
&\quad \left. + *a_j \times \frac{\partial * a_i}{c \partial t} \right\} dV \quad (12)
\end{aligned}$$

Here, $\mathbf{r}^* \mathbf{a}_j$ is a dyadic, and we use only the Lorentz gauge potentials which vanish at infinity. In Eq. (12), any point can be used as the origin of \mathbf{r} , and the last term represents the spin angular momentum of the electromagnetic fields. From these equations the physical meaning of the electromagnetic momentum in the Hamiltonian formalism, $q\mathbf{A}/c$, will be evident. In addition, there are correction terms which are effective only for the time dependent case. Comparing with Eq. (4), we should note that the electromagnetic momentum and the energy flow are different, being analyzed further hereafter.

We regard that these equations are important for analyzing the structure of the electromagnetic momentum-energy densities in the space. When $i \neq j$, these relations are useful without any additions, but when $i = j \neq 0$, careful consideration must be made to all these equations. Comparing with Eq. (9), we should notice that a self-factor $1/2$ is at least needed in order to approximate the self-energy, self-momentum, and self-angular momentum by Eqs. (10), (11), and (12), respectively. In the case, when $(\mathbf{e}_i, \mathbf{h}_i)$ represents the free field component which was radiated from the i -th particle, this self-factor $1/2$ is not necessary, but this complexity is dexterously resolved in our Maxwell-Lorentz electromagnetism, as we see soon.

We get further

$$c \sum_j' \mathbf{r} \cdot (\mathbf{e}_j \times \mathbf{h}_i) + \sum_j' (\mathbf{e}_j \cdot \frac{\partial \mathbf{e}_i}{\partial t} + \mathbf{h}_i \cdot \frac{\partial \mathbf{h}_j}{\partial t}) + \sum_j' \mathbf{e}_j \cdot \mathbf{j}_i = 0, \quad (13)$$

$$c \mathbf{r} \cdot (\mathbf{e}_i \times \mathbf{h}_i) + \frac{\partial}{\partial t} [(\mathbf{e}_i)^2 + (\mathbf{h}_i)^2] / 2 + \mathbf{e}_i \cdot \mathbf{j}_i = 0. \quad (14)$$

Here, \sum_j' indicates that the summation excludes $j = i$. From the principle of work, we can assume

$$\iint_V \mathbf{e}_i \cdot \mathbf{j}_i dV = \frac{\partial}{\partial t} [\text{K. E.}]_i + \frac{\partial}{\partial t} [\text{R. E.}]_i. \quad (15)$$

Here, V_i is the volume where $\mathbf{j}_i \neq 0$,

$$[\text{K. E.}]_i = \frac{m_i c^2}{\sqrt{1 - (v_i/c)^2}}, \quad (16)$$

and $[\text{R. E.}]_i$ indicates the radiated or absorbed electromagnetic energy by the i -th particle. In order to represent the change in the self energy due to the transfer of electromagnetic energy by induction, we regard that m_i is not a constant. But the change is so small that, in usual purpose, it can be neglected. We have to assume further

$$\iint_V \frac{\partial}{\partial t} \left\{ \frac{(\mathbf{e}_i)^2 + (\mathbf{h}_i)^2}{2} \right\} + \iint_S c \mathbf{e}_i \times \mathbf{h}_i \cdot d\mathbf{S} = \frac{\partial}{\partial t} \{ [\text{K. E.}]_i + [\text{R. E.}]_i \}, \quad (17)$$

where, the volume V and its surface S is about the size to include the i -th particle. From Eq. (14), if we have another volume V' , which includes V , then we have the time change equation of

$$\iiint_{V'-V} \frac{\partial}{\partial t} \left\{ \frac{(*e_i)^2 + (*h_i)^2}{2} \right\} dV = - \iint_{S'-S} c *e_i \times *h_i \cdot dS. \quad (18)$$

Therefore, $c *e_i \times *h_i$, when integrated over the surface of a volume where $*j_i = 0$, indicates the flow of $\{(*e_i)^2 + (*h_i)^2\}/2$, so that we need this term in Eq. (17). This fact, however, does not mean that $c *e_i \times *h_i$ itself can represent the energy flow. In the space where $*j_i = 0$, the energy flow $*S_i$ must be

$$*S_i = c *e_i \times *h_i + \nabla \times f, \quad (19)$$

in which f is an arbitrary four-space vector function, but, in the space where $*j_i \neq 0$, even Eq. (19) has no guarantee of existence. Clear recognition of this fact is one of the cruxes of the new frame in electromagnetism.

From Eqs. (17), (14), (15) and (13), we get

$$\begin{aligned} \frac{\partial}{\partial t} \{ * [K.E.]_i + * [R.E.]_i \} &= - \iiint_V *e_i \cdot *j_i dV = \iiint_V \left(\sum_{j \neq i} *e_j \right) \cdot *j_i dV \\ &= - \iiint_V \left[\sum_{j \neq i} \nabla \cdot c *e_j \times *h_i + \sum_{j \neq i} (*e_j \cdot \frac{\partial *e_i}{\partial t} + *h_i \cdot \frac{\partial *h_j}{\partial t}) \right] dV. \end{aligned} \quad (20)$$

From Eqs. (13), (14) and (20), we may be able to assume

$$\nabla \cdot \left\{ c \left(\sum_j *e_j \right) \times *h_i \right\} + \sum_j \left(*e_j \cdot \frac{\partial *e_i}{\partial t} + *h_i \cdot \frac{\partial *h_j}{\partial t} \right) = - \sum_j *e_j \cdot *j_i = 0. \quad (21)$$

Now, Eq. (21) is correct where $*j_i = 0$. Our assumption is that it can hold where $*j_i \neq 0$, for the electrons. In VR, since the $*\rho$ and $*j$ have no mass, the acceleration of $*\rho$ and $*j$ due to the action of $\sum_{j \neq i} *e_j$ and $\sum_{j \neq i} *h_j$ may generate the reaction $\Delta *e_i$ and $\Delta *h_i$, which may just compensate the action. Namely, we may have the relation

$$*\rho_i \sum_j *e_j + \frac{*j_i \times \sum_j *h_j}{c} = 0. \quad (22)$$

Multiplying $*j_i / *\rho_i$ to Eq. (22), we see that Eq. (21) is derivable from Eq. (22).

Then, after summing over i , Eq. (21) can be regarded as the forth component ($\nu = 4$) equation of the energy-momentum density tensor equation of

$$\frac{\partial *T_{\mu\nu}}{\partial x_\mu} = 0. \quad (23)$$

The fact that $\partial^*T_{\mu 4} / \partial x_\mu = 0$ holds indicates that Eq. (23) must hold in general.

The currently accepted basic Lagrangian for deriving the momentum-energy density tensor ${}^*T_{\mu\nu}$ is

$${}^*\mathcal{L} = -\frac{1}{4} ({}^*f_{\mu\nu})^2, \quad (24)$$

$${}^*f_{\mu\nu} = \frac{\partial {}^*a_\nu}{\partial x_\mu} - \frac{\partial {}^*a_\mu}{\partial x_\nu} = \sum_i {}^*f_{\mu\nu}^i, \quad {}^*a_\mu = \sum_i {}^*a_\mu^i, \quad (25)$$

$$\begin{aligned} {}^*T_{\mu\nu} &= {}^*f_{\mu\alpha} {}^*f_{\nu\alpha} - \frac{1}{4} \delta_{\mu\nu} {}^*f_{\alpha\beta}^2 \\ &= \left(\sum_i {}^*f_{\mu\alpha}^i \right) \left(\sum_i {}^*f_{\nu\alpha}^i \right) - \frac{1}{4} \delta_{\mu\nu} \left(\sum_i {}^*f_{\alpha\beta}^i \right) \left(\sum_j {}^*f_{\alpha\beta}^j \right). \end{aligned} \quad (26)$$

Here, the subscripts i, j have been changed to superscripts for convenience, and the standard symmetrization procedure (Landau and Lifshitz 1962) is requested for getting ${}^*T_{\mu\nu}$ from ${}^*\mathcal{L}$. It is easy to see that $\partial^*T_{\mu\nu} / \partial x_\mu = 0$ ($\nu = 1, 2, 3$) gives Eq. (22). But we should notice that, in order to get the Maxwell-Lorentz equations, we should add $(\sum_i {}^*j_\mu^i)(\sum_k {}^*a_\mu^k)/c$ to ${}^*\mathcal{L}$, which will spoil the expression of ${}^*T_{\mu\nu}$ of Eq. (26), if followed the standard procedure of the field theory (e. g., Nishijima 1969). The classical formulation presented by Landau-Lifshitz (1962) is only an elegant tautology, having not resolved the Poincaré paradox (Iida 1974). The currently accepted momentum-energy density tensor has such an essential defect, which necessitates to introduce the complicated renormalization procedure. We are free from this difficulty, because our approximation uses the new momentum-energy density four vector of Eq. (9) for VR. In the new frame, the current frame of the field theory can work only partly.

The crux of these analyses is that, although Eqs. (21)–(26) do not differentiate ${}^*e_i \times {}^*h_i$ and ${}^*e_i \times {}^*h_j$, we must differentiate them clearly, because, otherwise, we can not get a consistent results in Eqs. (9), (11), (20), and (23). The structure must remain when we shift from $({}^*e_i, {}^*h_i)$ to (e_i, h_i) . Therefore, in Eq. (4), we must sum with $j \neq i$, and we must understand the energy relation in the Maxwell electromagnetism of Eq. (7) as the average of Eq. (4). It is noted that the self-energy represented by Eq. (9) can separate clearly the emitting or absorbing electromagnetic free wave energy from the self-energy itself.

It is important for the electromagnetism to confirm the existence of perfectly consistent mathematical frame, in which no mathematical inconsistency is present. We presume that our frame can be such a frame general relativistically. Therefore, in the frame of special relativity, we still have a certain errors in the system, such as in the ratio of $1 : 10^{-365}$,[†] but, we believe that the error is superficial and we can perform differentiation and integration in the whole space without

limitation. Eqs. (10)–(12) were derived under these assumptions.

From these analyses, it will be evident that the energy transfer by induction is a fundamental phenomenon in electrodynamics. Therefore, as explained in detail in Appendix A, the Zeeman energy should be regarded as an effective Hamiltonian of the total system, such as

$$\delta (G_1 + G_2 + U_m) = \delta [- (\boldsymbol{\mu}_2 \cdot \mathbf{H}_1)] , \quad (27)$$

where U_m is the magnetic field energy and G_1 and G_2 are the non-electromagnetic energies of the source of the magnetic field, \mathbf{H}_1 , and the magnetic moment, $\boldsymbol{\mu}_2$, respectively. As shown in Eq. (A5), when we can assume that the magnitude of the persistent current I_2 and the current j_1 of the source do not change appreciably (Iida 1975a, 1975b, 1981c, 1982b. See §4 and Eq. (39).), we get

$$\begin{aligned} \delta G_1 &= \iiint j_1 \cdot \delta \mathbf{e}_2 dt dV = \delta G_2 = -\delta U_{m12} = -\delta U_m \\ &= -\iiint \mathbf{H}_1 \cdot \delta \mathbf{h}_2 dV = \delta [- (\boldsymbol{\mu}_2 \cdot \mathbf{H}_1)] . \end{aligned} \quad (28)$$

We claim in this paper that Eq. (27) must be effective even in quantum physics, although, in the currently accepted frame, the use of γ -matrix makes this structure very implicit.

§3. A brief description on the classical derivation of the Dirac Hamiltonian in Pauli's approximation

In order to clarify the interface between our Maxwell-Lorentz equations and quantum physics, the Dirac Hamiltonian in Pauli's approximation will be derived from our Maxwell-Lorentz fields, $(\mathbf{e}_i, \mathbf{h}_i)$. From Eq. (10), we get easily (Iida 1982b) the total electromagnetic energy of the system as

$$\begin{aligned} U_{em} &= \iiint \frac{(\sum_i \mathbf{e}_i)^2 + (\sum_i \mathbf{h}_i)^2}{2} dV = \sum_i \sum_{j \neq i} \iiint \frac{1}{2} (\mathbf{e}_i \cdot \mathbf{e}_j + \mathbf{h}_i \cdot \mathbf{h}_j) dV + \sum_i \iiint \frac{1}{2} (\mathbf{e}_i^2 + \mathbf{h}_i^2) dV \\ &= \sum_{j>i} \iiint \left\{ \phi_i^* \rho_j + \mathbf{a}_i \cdot \frac{\mathbf{j}_j}{c} - \frac{1}{c} \frac{\partial^* \phi_i}{\partial t} \frac{1}{c} \frac{\partial^* \phi_j}{\partial t} + \phi_j \frac{1}{c^2} \frac{\partial^2 \phi_i}{\partial t^2} + \frac{1}{c} \frac{\partial^* \mathbf{a}_i}{\partial t} \cdot \frac{1}{c} \frac{\partial^* \mathbf{a}_j}{\partial t} - \mathbf{a}_i \cdot \frac{1}{c^2} \frac{\partial^2 \mathbf{a}_j}{\partial t^2} \right\} dV \\ &\quad + \sum_i \iiint \frac{1}{2} \left\{ \phi_i^* \rho_i + \mathbf{a}_i \cdot \frac{\mathbf{j}_i}{c} - \left(\frac{1}{c} \frac{\partial^* \phi_i}{\partial t} \right)^2 + \phi_i \frac{1}{c^2} \frac{\partial^2 \phi_i}{\partial t^2} + \left(\frac{1}{c} \frac{\partial^* \mathbf{a}_i}{\partial t} \right)^2 - \mathbf{a}_i \cdot \frac{1}{c^2} \frac{\partial^2 \mathbf{a}_i}{\partial t^2} \right\} dV \\ &= \sum_{j>i} \iiint \left\{ \phi_i^C \rho_j + \mathbf{a}_i^C \cdot \frac{\mathbf{j}_j}{c} + \frac{1}{c} \frac{\partial^* \mathbf{a}_i^C}{\partial t} \cdot \frac{1}{c} \frac{\partial^* \mathbf{a}_j^C}{\partial t} - \mathbf{a}_i^C \cdot \frac{1}{c^2} \frac{\partial^2 \mathbf{a}_j^C}{\partial t^2} \right\} dV \\ &\quad + \sum_i \iiint \frac{1}{2} \left\{ \phi_i^C \rho_i + \mathbf{a}_i^C \cdot \frac{\mathbf{j}_i}{c} + \left(\frac{1}{c} \frac{\partial^* \mathbf{a}_i^C}{\partial t} \right)^2 - \mathbf{a}_i^C \cdot \frac{1}{c^2} \frac{\partial^2 \mathbf{a}_i^C}{\partial t^2} \right\} dV . \end{aligned} \quad (29)$$

†In the VR, the Lorentz electric force of repulsion is almost exactly cancelled by the Lorentz magnetic force of attraction and the magnitude of the next term is in the order of 10^{-365} of the main term. The gravitational force expected is in the order of 10^{-43} of the main terms.

In the new frame of physics, the four electromagnetic potentials in the Lorentz gauge, $\{\mathbf{a}_i, \mathbf{a}_i^C\}$, are regarded as the physical entities and those in the Coulomb gauge, $(\mathbf{a}_i^C, \phi_i^C)$, as a convenient mathematical tool. In order to get the Dirac Hamiltonian, we have to assume that the system is so small that the retardation of the electromagnetic signal inside of the system can be neglected. Then, we get (Darwin 1920)

$$\{^*a_i^C(\mathbf{r}_a), ^*\phi_i^C(\mathbf{r}_a)\} = \left\{ \frac{q_i v_i}{4\pi c r_{ia}} + \frac{q_i \hat{\mathbf{r}}_{ai} \times (\hat{\mathbf{r}}_{ai} \times \mathbf{v}_i)}{8\pi c r_{ia}} + \frac{^*\mu_i \times \hat{\mathbf{r}}_{ai}}{4\pi r_{ia}^2}, \left(\frac{q_i}{4\pi r_{ia}} + \frac{^*\mathbf{p}_i \cdot \hat{\mathbf{r}}_{ai}}{4\pi r_{ia}^2} \right) \right\}, \quad (30)$$

as the Coulomb gauge electromagnetic potentials of the i -th particle located at \mathbf{r}_i with the charge q_i , the velocity \mathbf{v}_i , the magnetic moment μ_i , and the electric dipole moment \mathbf{p}_i ($\mathbf{p}_i = (\mathbf{v}_i/c) \times \mu_i$ for electron.). Eq. (30) is correct up to $(v_i/c)^2$, and, in this degree of the approximation, the difference between retarded and advanced potentials does not appear, i. e., free radiation having been completely neglected. Different from the Lorentz gauge potentials, the acceleration, \mathbf{a}_i , does not appear in Eq. (30). Then, for particles 1 and 2,

$$U_{em}^{12} = \iiint_{\infty} \frac{^*e_1^2 + ^*h_1^2}{2} dV + \iiint_{\infty} \frac{^*e_2^2 + ^*h_2^2}{2} dV + \iiint_{\infty} (^*e_1 \cdot ^*e_2 + ^*h_1 \cdot ^*h_2) dV, \quad (31)$$

$$U^{12} = \frac{m_1 c^2}{\sqrt{1 - (v_1/c)^2}} + \frac{m_2 c^2}{\sqrt{1 - (v_2/c)^2}} + \iiint_{\infty} [^*\phi_1^C \cdot ^*\rho_2 + ^*a_1^C \cdot \frac{^*j_2}{c}] dV. \quad (32)$$

In deriving Eq. (32) from (31), we have amalgamated each electromagnetic self-energy into the kinetic energy of the particle, and neglected higher order terms. By calculating the integral of Eq. (32) analytically in detail, we get

$$\begin{aligned} U^{12} = & m_1 c^2 + m_1 \frac{v_1^2}{2} + m_1 \frac{3}{8} \frac{v_1^4}{c^2} + m_2 c^2 + m_2 \frac{v_2^2}{2} + m_2 \frac{3}{8} \frac{v_2^4}{c^2} \\ & + \frac{q_1 q_2}{4\pi r_{12}} + \frac{q_1 \mathbf{v}_1 \cdot q_2 \mathbf{v}_2}{4\pi r_{12} c^2} + \frac{q_1 \mathbf{v}_1 \times \hat{\mathbf{r}}_{12} \cdot q_2 \mathbf{v}_2 \times \hat{\mathbf{r}}_{21}}{8\pi r_{12}^2 c^2} \\ & - \frac{\mathbf{v}_1 \times \mu_1 \cdot q_2 \hat{\mathbf{r}}_{12}}{4\pi r_{12}^2 c} - \frac{\mathbf{v}_2 \times \mu_2 \cdot q_1 \hat{\mathbf{r}}_{21}}{4\pi r_{12}^2 c} + \frac{\mu_1 \cdot q_2 \mathbf{v}_2 \times \hat{\mathbf{r}}_{12}}{4\pi r_{12}^2 c} + \frac{\mu_2 \cdot q_1 \mathbf{v}_1 \times \hat{\mathbf{r}}_{21}}{4\pi r_{12}^2 c} \\ & - \frac{3(\mu_1 \cdot \hat{\mathbf{r}}_{12})(\mu_2 \cdot \hat{\mathbf{r}}_{21}) + \mu_1 \cdot \mu_2}{4\pi r_{12}^3} \\ & - q_1 q_2 K_{12} \delta^{(3)}(\mathbf{r}_{12}) + \frac{2}{3} (\mu_1 \cdot \mu_2) \delta^{(3)}(\mathbf{r}_{12}) \end{aligned} \quad (33)$$

Further, taking into account the energy transfer by induction rigorously, i. e., adding δG_1 or δG_2 of Eq. (27) or of Appendix A, being understood such as $\delta m_1 c^2$ or $\delta m_2 c^2$, to the magnetic interaction terms accurately, we get finally (Iida 1982c),

$$U^{12} = m_1^{\circ} c^2 + m_2^{\circ} c^2 + \frac{m_1^{\circ}}{2} v_1^2 + \frac{m_2^{\circ}}{2} v_2^2 + \frac{3m_1^{\circ}}{8c^2} v_1^4 + \frac{3m_2^{\circ}}{8c^2} v_2^4$$

$$\begin{aligned}
& + \frac{q_1 q_2}{4\pi r_{12}} + \frac{q_1 q_2 \mathbf{v}_1 \cdot \mathbf{v}_2}{4\pi r_{12} c^2} + \frac{q_1 q_2 \mathbf{v}_1 \times \hat{\mathbf{r}}_{12} \cdot \mathbf{v}_2 \times \hat{\mathbf{r}}_{21}}{8\pi r_{12}^2 c^2} \\
& + \frac{3(\boldsymbol{\mu}_1 \cdot \hat{\mathbf{r}}_{12})(\boldsymbol{\mu}_2 \cdot \hat{\mathbf{r}}_{21}) + \boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2}{4\pi r_{12}^3} - q_1 q_2 K_{12} \delta^{(3)}(\mathbf{r}_{12}) - \frac{2}{3}(\boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2) \delta^{(3)}(\mathbf{r}_{12}), \quad (34)
\end{aligned}$$

where the term with K_{12} is the Darwin electrostatic contact term and the last term is the Fermi magnetic contact term (Breit 1929, 1930, 1932; Bethe and Salpeter 1957, Messiah). We should note that the magnetic interaction terms, which are linear in $\boldsymbol{\mu}$, have dropped, and, those, which are quadratic in $\boldsymbol{\mu}$, have changed the signs. Thence, for the total system, we get

$$\begin{aligned}
U = & \sum_i m_i c^2 + \sum_i \frac{m_i}{2} \mathbf{v}_i^2 + \sum_i \frac{3m_i^2 \mathbf{v}_i^4}{8c^2} + \sum_{i>j} \frac{q_i q_j}{4\pi r_{ij}} \\
& + \sum_{i>j} \frac{q_i q_j \mathbf{v}_i \cdot \mathbf{v}_j}{4\pi r_{ij} c^2} + \sum_{i>j} \frac{q_i q_j (\mathbf{v}_i \times \hat{\mathbf{r}}_{ij}) \cdot (\mathbf{v}_j \times \hat{\mathbf{r}}_{ji})}{8\pi r_{ij} c^2} \\
& + \sum_{i>j} \frac{\boldsymbol{\mu}_i \cdot \boldsymbol{\mu}_j + 3(\boldsymbol{\mu}_i \cdot \hat{\mathbf{r}}_{ij})(\boldsymbol{\mu}_j \cdot \hat{\mathbf{r}}_{ji})}{4\pi r_{ij}^3} \\
& - \sum_{i>j} q_i q_j K_{ij} \delta^{(3)}(\mathbf{r}_{ij}) - \sum_{i>j} \frac{2}{3}(\boldsymbol{\mu}_i \cdot \boldsymbol{\mu}_j) \delta^{(3)}(\mathbf{r}_{ij}) \quad (35)
\end{aligned}$$

The Lagrangian of the system can be derivable from Eq. (35) almost uniquely (see §5). It is

$$\begin{aligned}
L = & - \sum_i m_i c^2 + \sum_i \frac{m_i}{2} \mathbf{v}_i^2 + \sum_i \frac{1}{8} \frac{m_i}{c^2} \mathbf{v}_i^4 - \sum_{i>j} \frac{q_i q_j}{4\pi r_{ij}} + \sum_{i>j} \frac{q_i q_j \mathbf{v}_i \cdot \mathbf{v}_j}{4\pi r_{ij} c^2} \\
& + \sum_{i>j} \frac{q_i q_j (\mathbf{v}_i \times \hat{\mathbf{r}}_{ij}) \cdot (\mathbf{v}_j \times \hat{\mathbf{r}}_{ji})}{8\pi r_{ij} c^2} + \frac{1}{2} \sum_{i \neq j} \sum_{i \neq j} \frac{q_j \hat{\mathbf{r}}_{ij} \times \mathbf{v}_i}{4\pi r_{ij}^2 c} \cdot \boldsymbol{\mu}_i + \sum_{i \neq j} \sum_{i \neq j} \frac{q_j \mathbf{v}_j \times \hat{\mathbf{r}}_{ij} \cdot \boldsymbol{\mu}_i}{4\pi r_{ij}^2 c} \\
& - \sum_{i>j} \frac{\boldsymbol{\mu}_i \cdot \boldsymbol{\mu}_j + 3(\boldsymbol{\mu}_i \cdot \hat{\mathbf{r}}_{ij})(\boldsymbol{\mu}_j \cdot \hat{\mathbf{r}}_{ji})}{4\pi r_{ij}^3} \\
& + \sum_{i>j} q_i q_j K_{ij} \delta^{(3)}(\mathbf{r}_{ij}) + \sum_{i>j} \frac{2}{3}(\boldsymbol{\mu}_i \cdot \boldsymbol{\mu}_j) \delta^{(3)}(\mathbf{r}_{ij}). \quad (36)
\end{aligned}$$

Here, a certain freedom is present for the factors of the two spin-orbit coupling terms, being not explained in this paper (Thomas factor, Iida 1982c). From Eq. (36), the standard procedures can derive the momenta and the Hamiltonian of the system as

$$\mathbf{p}_i = \frac{\partial L}{\partial \mathbf{v}_i}, \quad \mathcal{H}(\mathbf{p}_i, \mathbf{r}_i) = \sum_i \mathbf{p}_i \cdot \mathbf{v}_i - L. \quad (37)$$

By regarding \mathbf{p}_i and $\boldsymbol{\mu}_i = g_i \mu_B \mathbf{s}_i$ as the well-known operators, Eqs. (37) represent the Dirac Hamiltonian in Pauli's approximation exactly, which is the most fundamental Hamiltonian for solid state physics. We have to, however, note that the Hamiltonian is only derivable under the assumption that the system is so small that the retardation of the electromagnetic signal can be entirely neglected. Unfortunately this assumption can not work for the Meissner effect, in which the system must include the source of the magnetic field applied, and, even the specimen itself should be regarded

large, exhibiting a macroscopically inhomogeneous final equilibrium state. It is noted that, if the final equilibrium state is homogeneous, the system can be regarded as the sum of many small parts in each of which the retardation of the signal is unimportant.

§4. Travel of electromagnetic signals from one system to the other

Let us analyze the case where the system is large and Eq. (30) can not work. Let us assume that at $t \leq t^\circ$, the two electromagnetic systems, C_1 and C_2 , are in a meta-stationary state with the Maxwell quantities $\rho_1^\circ(r)$, $j_1^\circ(r)$, $E_1^\circ(r)$, and $H_1^\circ(r)$ for C_1 , and $\rho_2^\circ(r)$, $j_2^\circ(r)$, $E_2^\circ(r)$, and $H_2^\circ(r)$ for C_2 (Iida 1977, 1982b). We assume that system C_2 had started variation at $t = t^\circ$ and the variation has continued up to $t = t^\circ + \delta t^\circ$, where C_2 has arrived at a new meta-stationary state, but the electromagnetic signals which contain the information that C_2 was changing are still travelling in the vacuum space between C_2 and C_1 . As shown in Fig. 1, we define the wave front and tail surfaces of the transient signal at $t = t^\circ + \delta t^\circ + \delta t^\alpha$ as S_β and S_α , respectively.

$$\begin{aligned}
 & \text{The energy transferred through the surface } S_\alpha \text{ during } t^\circ < t < t^\circ + \delta t^\circ + \delta t^\alpha \text{ is} \\
 & \int_{t^\circ}^{t^\circ + \delta t^\circ + \delta t^\alpha} \iint_{S_\alpha} c(\mathbf{E}_1^\circ + \mathbf{E}_2^\circ + \delta \mathbf{E}_2) \times (\mathbf{H}_1^\circ + \mathbf{H}_2^\circ + \delta \mathbf{H}_2) \cdot d\mathbf{S} dt \\
 & = \int_{t^\circ}^{t^\circ + \delta t^\circ + \delta t^\alpha} \iint_{S_\alpha} c[(\mathbf{E}_1^\circ + \mathbf{E}_2^\circ) \times \delta \mathbf{H}_2 + \delta \mathbf{E}_2 \times (\mathbf{H}_1^\circ + \mathbf{H}_2^\circ) + \delta \mathbf{E}_2 \times \delta \mathbf{H}_2] \cdot d\mathbf{S} dt \\
 & = I_e^\alpha + I_m^\alpha + I_R^\alpha \\
 & = \iiint_{V_\alpha} \left\{ -\left[\frac{1}{2} \{ (\mathbf{E}_1^\circ + \mathbf{E}_2^\circ + \delta \mathbf{E}_2)^2 + (\mathbf{H}_1^\circ + \mathbf{H}_2^\circ + \delta \mathbf{H}_2)^2 \} \right]_{t^\circ}^{t^\circ + \delta t^\circ + \delta t^\alpha} \right. \\
 & \quad \left. - \int_{t^\circ}^{t^\circ + \delta t^\circ + \delta t^\alpha} (\mathbf{E}_1^\circ + \mathbf{E}_2^\circ + \delta \mathbf{E}_2) \cdot (\mathbf{j}_2^\circ + \delta \mathbf{j}_2) dt \right\} dV \\
 & = \iiint_{V_\beta - V_\alpha} \left[\frac{1}{2} \{ \mathbf{E}_1^\circ + \mathbf{E}_2^\circ + \delta \mathbf{E}_2 \}^2 + (\mathbf{H}_1^\circ + \mathbf{H}_2^\circ + \delta \mathbf{H}_2)^2 \right]_{t^\circ}^{t^\circ + \delta t^\circ + \delta t^\alpha} dV \tag{38}
 \end{aligned}$$

where δA_2^f and δj_2^f indicate the final values of the variation of A_2 and j_2 . Eq. (39) is an important relation, by which the usual conventional Hamiltonian (Appendix B), which neglects both the energy transfer by induction from C_2 to C_1 and the change in the magnetostatic mutual interaction energy of the system, U_{m12} , can be selfconsistent in terms of the conservation of the energy of the system. It is noted, however, that the energy transfer by induction is definitely an important energy transfer, being utilized daily in the civil life as the transformers. Therefore, in neglecting these important phenomena, justification must be needed for the conventional Hamiltonian to be used as the thermodynamical energy to be minimized, in cooperation with the entropy term (as shown later this has turned out to be not possible).

Another important relation we can get is

$$\begin{aligned} -\iiint_{V_2} \int_{t^\circ}^{t^\circ + \delta t^\circ} (\mathbf{E}_1^\circ + \mathbf{E}_2^\circ + \delta \mathbf{E}_2) \cdot (\mathbf{j}_2^\circ + \delta \mathbf{j}_2) dt dV \\ = -\mathbf{E}_1^\circ \cdot \delta \mathbf{p}_2^f + \delta U_{e22} + \delta U_{m22} + \delta U_R \end{aligned} \quad (40)$$

Here, the electric field, \mathbf{E}_1° , from C_1 to C_2 is assumed to be uniform, and δp_2^f is the variation in the dipole moment of C_2 . U_{e22} and U_{m22} are the Maxwell's electric and magnetic field self energies of C_2 and δU_R , which is the main component of I_R^α , is the electromagnetic radiation energy emitted during the process. If $\mathbf{E}_1^\circ = 0$ at C_2 , then we have a rigorous balancing of the self-energies of C_2 , because the left side equation of Eq. (40) should represent the negative of the change of all the other energies of C_2 (such as the kinetic energies of the charges), which is not included in the Maxwell's electromagnetic field energies.

Since the Maxwell equations are passive equations, they can describe the phenomena which may never happen in nature. For instance, the sign of the energy, δW_1 , is arbitrary, at least, in the foregoing description. Since the Maxwell equations are invariant for the time reversal operation, we can put $t \rightarrow -t$ in the description, then it describes an idealized absorption of electromagnetic radiation energy, δU_R , by C_2 , coming from infinity in the form of the advanced potentials. It is noted that $\mathbf{H} \rightarrow -\mathbf{H}$, $\mathbf{j} \rightarrow -\mathbf{j}$ by this operation.

It will be added that, when we regard the electromagnetic quantities of C_2 , i. e., $\rho_2(\mathbf{r})$, $\mathbf{j}_2(\mathbf{r})$, $\mathbf{E}_2(\mathbf{r})$, and $\mathbf{H}_2(\mathbf{r})$, as the Maxwell-Lorentz quantities, such as $\rho_2(\mathbf{r})$, $\mathbf{i}_2(\mathbf{r})$, $\mathbf{e}_2(\mathbf{r})$, and $\mathbf{h}_2(\mathbf{r})$ (see Eq. (1)), our equations can describe the electromagnetic action from or to an atom, ion, or molecule. We shall prove later that these descriptions have physical significances even in quantum physics, in regarding all the electromagnetic quantities as representing the averaged quantal expectation values of the system. But, there is still a long way left, because we have only studied the quantum theory in

Hamiltonian formalism in §3, where the retardation of the signal has been completely neglected. It is, however, noted that, although the existing quantum theory can describe δU_R , i. e., the emission or absorption of free electromagnetic radiation, no quantum theory exists for the action of I_m^α or δW_1 , where the new frame plays an essential role as shown later.

As a result of the principle of action through medium, in Fig. 1, when the signal from C_2 is travelling in the vacuum space, the state of C_1 has no influence from this signal, classically, as well as quantally. We insist that the conventional method of only superposing the normal modes of electromagnetic waves (Heitler 1954, Messiah) faces a serious difficulty for the adequate description of this physical situation, especially for the action of I_m^α .

§5. On the quantal equivalence and inequivalence of the Lagrangian and Hamiltonian formalisms and the principle of classification

When the retardation is not important and the Hamiltonian is effective, the standard quantum theory with the Schrödinger equation can work. Our problem is what quantum theory can work when the Hamiltonian formalism can not be effective.

Let us show first the effectiveness of the Lagrangian formalism in quantum theory for the simplest small system in which the retardation of the electromagnetic signal can be neglected and the Hamiltonian formalism can work.

Then, as in Eqs. (36) and (37), we have the Lagrangian function, the momenta, and the Hamiltonian as

$$L(q_r, \dot{q}_r), \quad p_r = \frac{\partial L}{\partial \dot{q}_r}, \quad H(q_r, p_r) = \sum_r p_r \dot{q}_r - L(q_r, \dot{q}_r) \quad (41)$$

By the nature of the Legendre transformation, we get algebraically

$$\delta H(q_r, p_r) = \frac{\partial H}{\partial q_r} \delta q_r + \frac{\partial H}{\partial p_r} \delta p_r = - \frac{\partial L}{\partial q_r} \delta q_r + \delta p_r \dot{q}_r, \quad (42)$$

$$\frac{\partial H}{\partial q_r} = - \frac{\partial L}{\partial q_r}, \quad \frac{\partial H}{\partial p_r} = \dot{q}_r. \quad (43)$$

Since Eqs. (41)–(43) are algebraical relations, they must be valid also as operator equations, only after paying caution for the orders of the symbols.

Now, the action principle leads the Lagrangian equation (e. g., Nishijima 1969),

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_r} \right) = \frac{dp_r}{dt} = \frac{\partial L}{\partial q_r} = - \frac{\partial H}{\partial q_r}, \quad (44)$$

which, together with an Eq. (43), constructs the Hamiltonian equations.

Now, we state that, the classical equations (41)–(44) transit directly to quantum equations after regarding the physical quantities, q_r , \dot{q}_r , p_r and $H = E$, as operators and placing the requirement

$$[q_r, p_s] = i\hbar \delta_{rs}, \quad [t, E] = -i\hbar. \quad (45)$$

The proof is as follows. In the new frame of physics, we take the Schrödinger representation as most fundamental and regard the other representations, such as those by the Heisenberg representation, as the mathematical modifications from them. We regard the state function, $\psi(q_r, t)$, in the Schrödinger representation as representing “the maximum information on the system, in which the real material wave and the statistical probability or mathematically appearing wave are intrinsically inseparable”.* In the Schrödinger representation, q_r ’s and t are regarded c-number quantities and put the operator relation of the Fourier transformation

$$p_r = \frac{\hbar}{i} \frac{\partial}{\partial q_r}, \quad E = -\frac{\hbar}{i} \frac{\partial}{\partial t}. \quad (46)$$

The Schrödinger equation is

$$-\frac{\hbar}{i} \frac{\partial}{\partial t} \psi(q_r, t) = H(q_r, \frac{\hbar}{i} \frac{\partial}{\partial q_r}) \psi(q_r, t), \quad (47)$$

with its formal solution of

$$\psi(q_r, t) = e^{-\frac{i}{\hbar} H_{\text{SR}}(t-t_0)} \psi(q_r, t_0). \quad (48)$$

Here H_{SR} is the H in Eq. (47) and $\psi(q_r, t_0)$ is the initial arbitrary state, which can be a Heisenberg representation of the state $\psi(q_r, t)$.

Let us consider the expectation value of an arbitrary physical quantity A . It is

$$\begin{aligned} \langle \psi(q_r, t) | A_{\text{SR}} | \psi(q_r, t) \rangle &= \langle \psi(q_r, t_0) | e^{\frac{i}{\hbar} H_{\text{SR}}(t-t_0)} A_{\text{SR}} \\ &\quad e^{-\frac{i}{\hbar} H_{\text{SR}}(t-t_0)} | \psi(q_r, t_0) \rangle, \end{aligned} \quad (49)$$

so that its time derivative is

$$\begin{aligned} \frac{d}{dt} \langle \psi(q_r, t) | A_{\text{SR}} | \psi(q_r, t) \rangle &= \langle \psi(q_r, t_0) | e^{+\frac{i}{\hbar} (H_{\text{SR}} A_{\text{SR}} - A_{\text{SR}} H_{\text{SR}})} e^{-} | \psi(q_r, t_0) \rangle \\ &= \langle \psi_0 | \frac{i}{\hbar} [(e^{+} H_{\text{SR}} e^{-})(e^{+} A_{\text{SR}} e^{-}) - (e^{+} A_{\text{SR}} e^{-})(e^{+} H_{\text{SR}} e^{-})] | \psi_0 \rangle. \end{aligned} \quad (50)$$

*We define the entity, which is most convenient to be regarded as the objective existence, for the self-consistent description of the physical phenomena, as the physical existence. In physics, there will be no further decisive word than the “simple, convenient, and selfconsistent”.

Here

$$e^{+\frac{i}{\hbar} H_{SR}(t-t_0)} A_{SR} e^{-\frac{i}{\hbar} H_{SR}(t-t_0)} = U A_{SR} U^{-1} = A_{HR} \quad (51)$$

defines a unitary transformation and the resultant representation is called the Heisenberg representation, in which the state function is

$$\psi(q_r, t_0) = U \psi(q_r, t), \quad U = e^{+\frac{i}{\hbar} H_{SR}(t-t_0)}, \quad (52)$$

and $H_{HR} = H_{SR}$. Then Eq. (50) indicates

$$\frac{d}{dt} \langle A \rangle = \langle \frac{\partial}{\partial t} (A_{HR}) \rangle = \langle \frac{i}{\hbar} [H_{HR}, A_{HR}] \rangle. \quad (53)$$

Therefore, as the crux of this section, by defining the operator dA/dt as

$$\langle \frac{dA}{dt} \rangle = \frac{d}{dt} \langle A \rangle, \quad (54)$$

we get the Heisenberg equation

$$\frac{\partial (A_{HR})}{\partial t} = \left(\frac{dA}{dt} \right)_{HR} = \frac{i}{\hbar} [H_{HR}, A_{HR}], \quad (55)$$

since, when $\langle A \rangle = \langle B \rangle$ for any arbitrary state, $A \equiv B^*$. Hence, we have, in general,

$$\frac{dA}{dt} = \frac{i}{\hbar} [H, A], \quad (56)$$

which is the generalized Heisenberg equation. From Eq. (56), dA/dt is not a fictitious operator, and, from Eq. (55), it is convenient to use the Heisenberg representation for obtaining the operator elements of dA/dt from A . Since, for the polynomial P of q_r and p_r , Eqs. (45) leads to

$$\frac{\partial P}{\partial p_r} = \frac{i}{\hbar} [P, q_r], \quad \frac{\partial P}{\partial q_r} = -\frac{i}{\hbar} [P, p_r] \quad (57)$$

we get

$$\frac{dq_r}{dt} = \frac{i}{\hbar} [H, q_r] = \frac{\partial H}{\partial p_r}, \quad \frac{dp_r}{dt} = \frac{i}{\hbar} [H, p_r] = -\frac{\partial H}{\partial q_r}, \quad (58)$$

as the operator equations, which is independent of the representation. Therefore, under the postulate of Eqs. (45) and (54), we have proved Eqs. (43) and (44) as operator equations. Since Eqs. (41)–

*When $\langle A \rangle = \langle B \rangle$ for any arbitrary state, $A \equiv B$. A short proof is: putting $A - B = C$, $4 \langle x | C | y \rangle = \langle x + y | C | x + y \rangle - \langle x - y | C | x - y \rangle - i \langle x + iy | C | x + iy \rangle + i \langle x - iy | C | x - iy \rangle = 0$.

(43) are algebraical relations, this completes the consistency of Eqs. (41)–(45), (54) and (56), as the operator equations in quantum physics for the assumed specialized system.

Now, when the retardation of the signals should be taken into account, we propose that the Lagrangian formalism becomes more fundamental and convenient than the Hamiltonian formalism, classically as well as quantally. We should note first that the description of a transient phenomenon in §4 is made by using entirely Lagrangian variables, r_i , $\dot{r}_i = v_i$, and $j = \sum_i q_i v_i$. When Eq. (30) is not effective, we can not get Eq. (36), so that, Eqs. (37). As shown in Appendix B, when magnetism is related, there happens another mathematical difficulty in the Hamiltonian formalism, in which the simple definition of the momentum, p_i , becomes ineffective. Of course Lagrangian description is easy to understand, as the frame is identical to our classical experiences. Therefore, in several complicated systems, the Lagrangian formalism, at least, can give the better insight of the object. Although it is not easy to solve the Lagrangian operator equations for many particle systems, but, at least, exact quantal Lagrangian solutions are obtainable in the case of the dynamics of a spin system.

In the Lagrangian description, once we obtained some classical equation, such as

$$f(q_r, \dot{q}_r, \ddot{q}_r) = g(q_r, \dot{q}_r, \ddot{q}_r), \quad (59)$$

then we can expect

$$\langle f(q_r, \dot{q}_r, \ddot{q}_r) \rangle = \langle g(q_r, \dot{q}_r, \ddot{q}_r) \rangle \quad (60)$$

for any arbitrary quantum state or ensemble of quantum states. Since the thermodynamics usually concerns with a large system in which the expectation value and the classical value are not different, we regard this expectation very important in thermodynamics.

Now, since most of the classical equations are valid in quantum physics as the q-number equations and additional requirements will be only both the commutation requirement of Eq. (45), where \hbar appears, and the Pauli principle requiring the symmetry of the states for the permutation of the involved same particles, we propose “the principle of classification”, stating that almost all physical phenomena, whose characteristic constants do not contain \hbar and has no relation with the symmetry requirement of the quantum states, will be describable classically, without using the operator frame of the quantum theory (Iida 1981b, 1981c). Almost all terms of the Dirac Hamiltonian in Pauli’s approximation (Eqs. (37) and (36)) are subjected to this category. We claim that $g = 2$ of the electron belongs to this category (Iida 1974), but the anomalous part, as it contains \hbar , is not. The Meissner effect, again, becomes a delicate typical example, as being analyzed later.

§6. Dual structure of the physical world, ensemble concept, and a new definition of “classical”.

Now, although there remains a slight freedom for the q-number representation of travelling electromagnetic fields, we have almost established that our quantal physical world of concern is composed of q-number* algebraical Lagrangian equations and the state functions, or, the solution of these operator equations. This dual structure is essential in the physical world and classical electrodynamics of charged particles comes about by regarding the corresponding q-number equations as c-number* equations. The classical electromagnetism, in a sense, comes about by picking up the electromagnetic c-number side-view of the corresponding quantal state functions. The superposition principle is adoptable for the latter, but is not for the former, in agreement with the corresponding quantum systems. Now, even if the total system is macroscopic, the retardation may be neglected in its local part, so that, locally, the existing frame will work. Therefore, it happens in the new frame that we know either the classical or/and quantal solutions of many parts of the system, but neither both nor the whole system. In these situations, in order to understand the whole system intuitively, without obtaining the complete mathematical solutions, we propose that the concept of ensemble or adequate set of classical or quantum states is useful in order to bridge the classical and quantum physics.

The concept of ensemble is well-known in statistical thermodynamics, and, in the existing quantum theory, also, after the old proposal by Born, we can figure out, e. g., a quantum state of a hydrogen atom by an electronic cloud having the density of $\psi^*(r, t) \psi(r, t)$, which is equivalent to an ensemble of millions of classical states of the hydrogen atom. Of course, we know from Eq-(46) that the spacial change of the phase of $\psi(r, t)$ represents the expected momentum of the electron, and, it is possible to introduce this additional knowledge in this classical ensemble. Accordingly, the concept must also be quite useful for bridging the classical and quantum states in the new frame. A classical equation in the new frame, when represented by algebraical notations, represents not only a behavior of a simple classical state, but, an ensemble of classical states, allowing for a quantum state to be approximated by the ensemble. Usually, a quantum state will have a corresponding ensemble of classical states, and, even though each classical state may emit radiation classically, when the ensemble of states represented is stationary, no electromagnetic action is expected. Since we know already the quantum states for atoms, molecules, and typical solids, it will be also possible for an arbitrary quantum system, to construct the corresponding classical ensembles having the expected quantum characters. Although the classical particle dynamics can not accept the superposition, by using the ensemble concept, we can accept the superposition and can extract

c : classical, q : quantal.

certain collective quantum characters of the system. From this view, the procedure of the approximation employed in §2 can be reinterpreted as an example of the extended mixed application of these ensemble concepts.

In turn, a classical state can be regarded as a superposition of an ensemble of quantum states. Then, we can regard our classical Lagrangian physics of a macroscopic system as depicting these collective characters of the corresponding quantum ensembles. We can regard the classical physics, such as mechanics, electromagnetism, etc., as describing such collective classical characters of certain corresponding microscopic quantum ensembles. Then we propose a new terminology of “classical”, if there is a c-number equation system for any side-view of a quantum system. Even the electron clouds of atoms can be regarded as “classical” in this sense in a certain study, so far as we don’t manipulate the operator frame of the system in that study. In this way, in the new frame, we can use classical frame, if it is effective and is convenient. If the result obtained agrees with the experiences, we claim that it is already sufficient for physics, regarding the clearing out of its quantal significance, as another problem, which is not always simple. At the same time, since the physical world has the dual structure, and our experiences cover many side views of that structure, i.e., classical experience both for the q-number equation side [e.g., electrodynamics] and the state function side [e.g., electromagnetism], and a plenty of quantum knowledges for small systems where the Hamiltonian formalism are effective, we can synthesize these knowledge by means of the mentioned ensemble concept and the mathematical knowledge of the Fourier transformation to understand the total system.

After accepting these concepts, the actual physical experiment may be understood by the new frame in the following simple way. After making many simplifications, the experimental apparatus will correspond to the q-number Lagrangian equation side and the experiment corresponds to the investigation of the detail of the state function, after giving an initial condition of the state function. A typical case will be seen when an electron beam is injected to an electron diffraction apparatus. The physical understanding of the growth of a single crystal in a crucible in a furnace will present another example, but the interpretation will be much more complicated. In the new frame, however, since we are dealing with Lagrangian frame, which is identical to the classical experience, we may say that the first order quantum approximation of our experiments has been obtained by the new frame.

In the following description of this paper, we shall present an application of the new frame, in which we shall follow the procedure of classical physics faithfully, with keeping the ensemble concept in mind, but, without explaining its quantum significance in detail.

§7. Derivation of the London equation for the classical Coulomb gas by regular thermodynamics

In order to show that the Meissner effect is a classical property of a system which can sustain persistent current, we have studied the magnetism of a classical electron gas. Since the classical electron gas will solidify at below a certain temperature (Pollock and Hansen 1973), if limited strictly to the classical, the analysis has to be limited at above this temperature, but in the new frame, this limitation is not necessary. We know that there were many pioneering works on the diamagnetism of electron gas both classically (van Leeuwen 1921, Van Vleck 1932) and quantally (Landau 1930, Van Vleck 1932, Ham 1953, Peierls 1955, Heuser and Hajdu 1974). From the view point of the new frame of physics, these works are not acceptable, because, for the dense Coulomb gas, under the control of the principle of the action through medium, incorporating with the retarded energy transfer by induction, the leading elementary irreversible process should occur quite locally without knowing its consequence over the total system, and, the conventional Hamiltonian formalism, which locates the source of the magnetic field applied outside of the consideration and disregards the retardation of the electromagnetic signal generated, can not depict these most important factors of this process. Since the final equilibrium state might become macroscopically inhomogeneous, the difference between the thermodynamical energy and the effective Hamiltonian for the approximate description of the kinematical motion of a part of the total system must be strictly distinguished, which was not the case in the old treatments. As shown in Appendix B, the effective Hamiltonian can not become the thermodynamical energy, in addition to the included essential mathematical difficulties for rigorous treatment.

Now, our Coulomb gas system is composed of a large ring-like container C_1 and a small simply connected container C_2 located at the center of C_1 (Fig. 1). In order to represent the total energy U of the system, we assume initially that the system is in a stationary state. The total system can be located inside of a perfect reflector, but the thermal radiation energy can be out of the consideration, because its total amount is so small (such as $1 : 10^{-20}$). Then, we can use the form of Eq. (35). Assuming the point charge model of the electron and neglecting the higher order terms of $(v/c)^4$, we get effectively

$$\begin{aligned}
 U = & \sum_{\lambda} \frac{m_{\lambda}}{2} \dot{\mathbf{v}}_{\lambda}^2 + \sum_i \frac{m_i}{2} \dot{\mathbf{v}}_i^2 + \sum_{\lambda > \mu} \frac{q_{\lambda} q_{\mu}}{4 \pi r_{\lambda \mu}} + \sum_{i > j} \frac{q_i q_j}{4 \pi r_{ij}} \\
 & + \sum_{\lambda > \mu} \frac{q_{\lambda} q_{\mu} \mathbf{v}_{\lambda} \cdot \mathbf{v}_{\mu}}{4 \pi r_{\lambda \mu} c^2} + \sum_{\lambda, j} \frac{q_{\lambda} q_j \mathbf{v}_{\lambda} \cdot \mathbf{v}_j}{4 \pi r_{\lambda j} c^2} + \sum_{i, j} \frac{q_i q_j \mathbf{v}_i \cdot \mathbf{v}_j}{4 \pi r_{ij} c^2} .
 \end{aligned} \tag{61}$$

It is noted that the sixth term of Eq. (35) is almost zero, so far as the macroscopic currents, $\mathbf{j}_1(\mathbf{r})$ and $\mathbf{j}_2(\mathbf{r})$ in C_1 and C_2 , are stationary, or, circulating ($\nabla \cdot \mathbf{j}_{\xi}(\mathbf{r}) = 0$). In this approximation, since

the electrostatic interaction is much stronger than the magnetic interaction, we assume that the system is already in an equilibrium with respect to this interaction. In the Coulomb gas approximation, the positive charges are assumed to be completely continuous, distributing uniformly over the volumes of C_1 and C_2 , and, the electrostatic mutual interaction makes the point electrons spreading uniformly over C_1 and C_2 , defining the electron densities, $n_1(\mathbf{r})$ and $n_2(\mathbf{r})$, as being almost constant. The electrons are making very fast thermal motions, under the strong action of the electrostatic mutual forces operating between every two electrons and between the positive uniform charge density and each electron. We decompose the velocity as

$$\mathbf{v}_\lambda = \mathbf{v}_{D1}(\mathbf{r}) + (\mathbf{v}_\lambda - \mathbf{v}_{D1}), \quad \mathbf{v}_i = \mathbf{v}_{D2}(\mathbf{r}) + (\mathbf{v}_i - \mathbf{v}_{D2}). \quad (62)$$

Here, $\mathbf{v}_{D1}(\mathbf{r})$ and $\mathbf{v}_{D2}(\mathbf{r})$ are the drift velocities in C_1 and C_2 , respectively, with

$$\mathbf{j}_1(\mathbf{r}) = -en_1 \mathbf{v}_{D1}(\mathbf{r}), \quad \mathbf{j}_2(\mathbf{r}) = -en_2 \mathbf{v}_{D2}(\mathbf{r}). \quad (63)$$

Thence we get

$$\begin{aligned} U = & \iiint_{C_1+C_2} \frac{(\mathbf{j}_1 + \mathbf{j}_2) \cdot (\mathbf{A}_1 + \mathbf{A}_2)}{2c} dV + \sum_\lambda \frac{m_\lambda}{2} \mathbf{v}_{D1}^2 + \sum_i \frac{m_i}{2} \mathbf{v}_{D2}^2 \\ & + \sum_\lambda \frac{m_\lambda}{2} (\mathbf{v}_\lambda - \mathbf{v}_{D1})^2 + \sum_i \frac{m_i}{2} (\mathbf{v}_i - \mathbf{v}_{D2})^2 \\ = & U_{m11} + U_{m12} + U_{m22} + U_{D1} + U_{D2} + U_{T1} + U_{T2} = U_m + U_D + U_T. \end{aligned} \quad (64)$$

U_m , U_D and U_T indicate magnetic energy, and, drift and thermal parts of the kinetic energies of the electrons. U_T can include the additional electrostatic mutual interaction energies.

Now, we assume that, so far consistent in electromagnetism, any $\mathbf{H}(\mathbf{r})$ and $\mathbf{j}_\xi(\mathbf{r})$ with $\nabla \cdot \mathbf{j}_\xi(\mathbf{r}) = 0$ is realizable in the system, at least temporarily, after having a small adjustment in terms of the corresponding small shifts of the charges to generate $E(\mathbf{r})$, which is extremely small (Iida 1977, see §9).

Let us represent the system in a thermodynamical configuration space, Ω , in which the current $\mathbf{j}(\mathbf{r})$ takes the role of the internal parameters (X_1, X_2, \dots), and U^c and S^c are the additional coordinates (Callen 1960). Here, S represents the entropy and c indicates that it is the quantity in Ω . In this system, C_2 (or C_1) has a character of $S_2^c = S_2^c(U_{T2}^c, \mathbf{j}(\mathbf{r}))$ and, $\mathbf{j}(\mathbf{r}) = \mathbf{j}_1(\mathbf{r}) + \mathbf{j}_2(\mathbf{r})$ determines $\mathbf{H}(\mathbf{r}) = \mathbf{H}_1(\mathbf{r}) + \mathbf{H}_2(\mathbf{r})$. The temperature can be a function of the location, \mathbf{r} , but initially we assume uniform temperatures, T_1 and T_2 , for C_1 and C_2 , respectively. Then, thermodynamics gives

$$\delta S_2^c = \frac{1}{T_2} \delta U_{T2}^c + (\delta S_2^c)_{U_{T2}^c}^{\delta \mathbf{j}}, \quad (65)$$

so that

$$(\delta U_{T_2}^c)_{S_2^c}^{\delta j} = -T_2 (\delta S_2^c)_{U_T^c}^{\delta j}, \quad (66)$$

$(\delta S_2^c)_{U_{T_2}^c}^{\delta j}$ being the variation of S_2^c due to δj in C_1 or C_2 with keeping $U_{T_2}^c$ constant. With the use of Eq. (66), the formal thermodynamics for the equilibrium states, i. e., the entropy maximum, the energy minimum, and the free energy minimum principles, request uniquely for δj

$$\delta U^c - T \delta S^c = -T [(\delta S_1^c)_{U_{T_1}^c}^{\delta j} + (\delta S_2^c)_{U_{T_2}^c}^{\delta j}] + \delta (U_m^c + U_D^c) = 0, \quad (67)$$

so that

$$(\delta S_1^c)_{U_{T_1}^c}^{\delta j} + (\delta S_2^c)_{U_{T_2}^c}^{\delta j} = \frac{[\delta (U_m + U_D)]^{\delta j}}{T} = \frac{(\delta U^c)_{U_T^c}^{\delta j}}{T}, \quad (=0). \quad (68)$$

Then, we propose as a new principle for the thermodynamics with long range interactions that the variation δj must simulate the real possible thermal fluctuation of the system, requiring for δj to be confined in a macroscopically point-like volume ΔV with $\nabla \cdot \delta j = 0$ and keeping the other $j(r)$'s outside ΔV , so that $A(r)$'s due to them, constant. Eq. (68) indicates that, at near the equilibrium state, both sides of Eq. (68) should be zero, otherwise the change of U^c at constant U_T^c produces the maximum change of the entropy for the quasi-static process, which is physically unacceptable in the present case, because as $\delta U_{T_2}^c$ will always have the main contribution to S_2^c in this system. The logics in this paragraph is one of the cruxes of the study. Thence, for C_2

$$\begin{aligned} \delta (U_m^c + U_D^c) &= \iiint_{\Delta V} \left(\mathbf{A} + \frac{mc}{n_2 e^2} \mathbf{j}_2 \right) \cdot \frac{\delta \mathbf{j}_2}{c} dV = \sum_{\lambda} \frac{\Delta I_{\lambda}}{c} \int_{C_{\lambda}} \left(\mathbf{A} + \frac{mc}{n_2 e^2} \mathbf{j}_2 \right) \cdot d\boldsymbol{\ell} \\ &= \sum_{\lambda} \frac{\Delta I_{\lambda}}{c} \iint_{S_{\lambda}} \nabla \times \left(\mathbf{A} + \frac{mc}{n_2 e^2} \mathbf{j} \right) \cdot d\mathbf{S} = \sum_{\lambda} \left(\mathbf{H} + \frac{mc}{n_2 e^2} \nabla \times \mathbf{j}_2 \right) \cdot \Delta \boldsymbol{\mu}_{2\lambda} = \mathbf{H}^* \cdot \delta \boldsymbol{\mu}_2 = 0. \end{aligned} \quad (69)$$

Here $\delta \mathbf{j}_2$ is subdivided into many microcircuits C_{λ} with the current ΔI_{λ} and the line of element $d\boldsymbol{\ell}$. $\delta \boldsymbol{\mu}_2$ is the magnetic moment generated by $\delta \mathbf{j}_2$. Hence, we get a London equation

$$\mathbf{A} + \frac{mc}{n_2 e^2} \mathbf{j}_2 = \nabla \psi_2(\mathbf{r}), \quad \mathbf{A}^L = \mathbf{A} - \nabla \psi_2 \quad (70)$$

for C_2 (as well as for C_1).

Although a part of this derivation procedure was known (Casimir 1966, de Gennes 1966), no rigorous thermodynamical foundation of the procedure had been given before, because the physical, classical as well as quantal, significance of the Lagrangian formalism had not been understood well. In order to justify this procedure further, we have studied the relevant irreversible process directly in detail next. It is noted that, mathematically Eq. (67) can be extended to at $T = 0$, leading Eqs. (69) and (70); indicating the delicate quantal and dynamical significance of the result.

§8. Derivation of the transient energy principle and its application to the Meissner effect

In Ω , from the view point of the irreversible process, the entropy maximum, the energy minimum and the Helmholtz free energy minimum principles (e. g., Callen 1960) are unified as that

$$\left. \begin{aligned} -\delta U^c + T\delta S^c &= T \sum_{\ell} \frac{\partial \psi}{\partial X_{\ell}} \delta X_{\ell} = - \sum_{\ell} \frac{\partial \phi}{\partial X_{\ell}} \delta X_{\ell} > 0, \\ S^c &= \psi(U^c, X_{\ell}), \quad U^c = \phi(S^c, X_{\ell}) \end{aligned} \right\} \quad (71)$$

is a necessary condition for the process $\{\delta X_{\ell}\}$ to be irreversible.

Callen (1960) emphasized the role of the energy minimum principle, as representing the interface between thermodynamics and classical non-thermal dynamics. Let us extend the Callen's idea slightly more. If the system is mechanical, such as a pendulum, and is supported at rest in a nonequilibrium state, then, when the support is released, it starts a motion with which U^c decreases. Then, after the start, the released configurational energy, $-\delta U^c$, becomes a macroscopic kinetic energy of the system, which is not representable in Ω , since, in Ω , the system is assumed to be static or in equilibrium under the given values of the internal parameters. We call this kind of extra energy as the transient energy, $[TE] = [-\delta U^c]$. Since the macroscopic motion always contains an irreversible part, the equation, as an irreversible process, must be

$$[TE] = [-\delta U^c] > 0,$$

being in agreement with the Lagrangian equation (44) and the proposed thermal equation (71). The intentional discharge of a charged capacitor may present another example. The principle may be depicted that a macroscopic energy, once released in a transient form, can never completely be returned. Then, our problem is that what happens when the process of producing a transient energy competes with the change of the entropy of the system. We conclude that this is the case for the Meissner effect, being explained hereafter.

Let us consider δj_2 as the real nucleus of an irreversible process. The size must be quite small, located at just between a macroscopic point and a volume. When δj_2 in C_2 has just happened, the electromagnetic signal of δj_2 must be near the location of δj_2 . Since the total energy must be conserved, the extra energy, $[-\delta U^c]$, is in a transient state at this instant. The explicit form of $[-\delta U^c]$, is given in §4 (Iida 1977, 1982c). Therefore, Eq. (71) in this case takes the form of

$$[-\delta U^c]_{\text{local}} + T[\delta S^c]_{\text{local}} = [TE] + T[\delta S^c]_{\text{local}} > 0, \quad (72)$$

where $[TE]$ indicates the transient state energy. How to calculate $[\delta S^c]_{\text{local}}$ will be a problem.

Since the entropy is an extensive quantity and is calculatable in terms of the number of possible configurations, it should be calculatable at and near the location of δj_2 , at least in principle. Eq. (72) represents the explicit form of the proposed new thermodynamic principle called the transient energy principle. Very localized heat can be regarded either $[TE]$ or $T[\delta S^c]_{\text{local}}$, which does not disturb the inequality of Eq. (72), being an important necessary condition for the representation. It is to be noted that Eq. (72) indicates the possibility that the entropy may decrease temporarily, if a very extremum initial condition can be set up. Then with the aid of Eqs. (69) and (65), we get

$$\begin{aligned} & [-\delta(U_m^c + U_D^c) - \delta U_{T2}^c]_{\text{local}} + T_2 [\delta S_2^c]_{\text{local}} \\ & = -H^* \cdot \delta \mu_2 + T_2 (\delta S_2^c)_{U_{T2}^c}^{\delta j_2} > 0. \end{aligned} \quad (73)$$

From Eq. (68), we get at near the equilibrium,

$$(\delta S_1^c)_{U_{T1}^c}^{\delta j} = -(\delta S_2^c)_{U_{T2}^c}^{\delta j}, \quad (= 0) \quad (74)$$

irrespectively on the shape, size, and location of C_1, C_2 and δj , which can be understood only when each term of Eq. (74) is zero. Then, if $H^* \neq 0$, it is always possible to have a $\delta \mu_2$ with $-H^* \cdot \delta \mu_2 > 0$, indicating that an irreversible process must happen by Eq. (73). Therefore, we need Eq. (69) as a necessary condition for the equilibrium state.

The entropy term of Eq. (73) will be evaluated as follows. When the magnetic field H_1 has penetrated into C_2 substantially, as the cyclotron motion has a radius of $r_c = mc v / eH$, the orbit converges to a point, when $H_1 \rightarrow \infty$. Then the entropy of that state must be smaller, because the orientational freedom of the velocity becomes ineffective. Hence, if δj_2 is created in C_2 diamagnetically to reduce the effect of H_1 , we should get with Eq. (66)

$$-\frac{(\delta U_{T2}^c)_{S_2^c}^{\delta j_2}}{T_2} = (\delta S_2^c)_{U_{T2}^c}^{\delta j_2} > 0, \quad (75)$$

which supports our conclusion. Quantum treatment seems to give the same result. Introduction of a magnetic field creates the bunching of the free electron levels, (Van Vleck 1932, Peierls 1955) and, since the characteristic length $\Lambda_H = \sqrt{\hbar c / eH}$ decreases with the increase of H , the number of states in a single bunched level increases, together with the raise of all the levels, including that of the zero point motion. As a result the expected kinetic energy for the lowest energy state increases in average. Because of the presence of electrostatic interactions, we can expect that the lowest energy state has no degeneracy. Then, the initial part of Eq. (75) tells that, when the state

changes from the van Leeuwen type state with $H \neq 0$ to the state of the Meissner-Ochsenfeld effect, with $H = 0$ in most of the volume,

$$0 > \frac{(\delta U_{T_2}^C)_{S_2}^{\delta j_2}}{T} = -(\delta S_2^C)_{U_{T_2}^C}^{\delta j_2}. \quad (76)$$

Namely, the entropy must increase in the change.

We should note that, when we put $T = 0$ in Eq. (73), the requirement coincides with the law of classical dynamics, i. e., diamagnetic response of the orbital motion of the electrons in a magnetic field.

We state that the transient energy principle is an important finding of the new frame which is located at just the interface between the new and old frames, and also between the classical and quantal physics. The most delicate physical structure of the system will be further explained in §10.

§9. Classical stability of the Meissner effect, Lagrangian kinematics, and the idealized magnetic shielding.

The Lagrangian equation of the motion of a charge q_i in general in a Coulomb gas is (Appendix C)

$$\frac{d\mathbf{p}_i}{dt} = \nabla_i L_i = q_i \nabla_i \left\{ \frac{\mathbf{v}_i \cdot \mathbf{A}(\mathbf{r}_i, t)}{c} - \phi(\mathbf{r}_i, t) \right\} + [\mathbf{S} \cdot \mathbf{I}]_i \quad (77)$$

where $[\mathbf{S} \cdot \mathbf{I}]_i$ means the short range electric and magnetic mutual interactions of the charges. Let us prove that the state of the Meissner effect in C_2 is kinematically stable. Let us assume that $\{\mathbf{A}, \Phi\}$ are the London gauge potentials, $\{\mathbf{A}^L, \Phi^L\}$, which are stationary. Let us have an attention to the surface region and assume that \mathbf{A}^L , the surface normal, and the magnetic field $\mathbf{H}(\mathbf{r})$ are along x -, y -, and z -axes, respectively. Since the first and second terms are $\nabla_i [q_i v_{ix} A_x^L(y_i)/c]$ and $-\nabla_i \Phi(y_i)$, the component of p_i^L in x - z plane, being denoted by \parallel , will have the relation

$$\frac{dp_{i\parallel}^L}{dt} = [\mathbf{S} \cdot \mathbf{I}]_{i\parallel}, \quad (78)$$

so that, after integrating between t° and $t^\circ + \Delta t$, we get

$$[\Delta v_i]_{\parallel} = -\frac{q_i}{m_i c} \Delta A^L + \frac{1}{m_i} \int_{t^\circ}^{t^\circ + \Delta t} [\mathbf{S} \cdot \mathbf{I}]_{i\parallel} dt. \quad (79)$$

Let us fix the change of the y -variable of the particle, $\Delta y_i \geq 0$, and consider the possible different cases for electron i . Since, in $[\mathbf{S} \cdot \mathbf{I}]_{i\parallel}$, action and reaction will cancel with each other, we expect the last term of Eq. (79) as zero in average, which makes Eq. (79) identical to a London equation. Hence, we conclude that the Meissner effect of the Coulomb gas is kinematically stationary.

For the general non-stationary case, we found that there is a unique gauge \mathbf{I} in which $\mathbf{A}^I(\mathbf{r}, t)$ has no normal component at the surface of C_2 . With the use of this gauge, the right side of Eq. (77) gives a transient force which can not circulate and should be quickly reduced to zero by the

depolarizing electric field introduced by the response $\delta\phi^I$ due to very small instant shifts, δr_{iP} 's, of all the charges in C_2 . The time of response will be $\tau \sim 1/\omega_p = \sqrt{m/ne^2} \sim 10^{-16}$ sec., ω_p being the plasma frequency of the Coulomb gas. Separating this part, we get in average

$$\overline{\frac{d}{dt} (p_i^I - m_i v_{iP})} = 0 \quad (80)$$

$$\overline{\Delta (v_i - v_{iP})} = - \frac{q_i}{m_i c} \Delta A^I, \quad (81)$$

which shows the idealized magnetic shielding effect. If the change of the potentials has ceased, then v_{iP} will become zero quickly, indicating that the state is approximately identical to the state of the Meissner effect.

The predicted idealized magnetic shielding effect is very similar to the case of the Larmor diamagnetism of atoms, ions, and molecules. We note that even the Larmor precession of the electron spin can be regarded diamagnetic and similar in the character in the new frame of physics. In these cases, the application of a magnetic field creates acceleration of the kinematical motion of the charges or the currents and the resultant motions are maintained, so far as the field is maintained, being understood by the action of the Lorentz magnetic force in the stationary states (Iida 1982d). In the Larmar diamagnetism, the system rotates as a whole with the angular velocity $\omega = eH/2mc$, keeping the original kinematical motion almost unchanged. When the system becomes large, then (see Eq. (B8)), the magnetic field produced by the Larmor Precession of the circumferential region supresses the penetration of the magnetic field applied and the system has to transit to the state of the Meissner effect. The old treatments, both classical and quantal (van Leeuwen 1921, Landau 1930, Darwin 1930, Van Vleck 1932) have made misunderstandings on both the essential defect of the effective Hamiltonian formalism in the thermostistical treatments and also the recognition of the role of the principle of action through medium for such a macroscopic electromagnetic system. (see §10).

Let us extend our analyses to the Edwards' (1981) case. In his case, the subject is not C_2 nor C_1 . The charged particles are supposed to be located in an open space initially, could be under the action of gravity. As shown in Appendix C, there is a covariant guage in which the averaged $\bar{L}_i = 0$ in Eq. (77).

In this gauge, we have

$$\overline{\frac{d p_i^E}{dt}} = 0, \quad (82)$$

where E indicates Edwards, as Eq. (77) in this case becomes almost equivalent to Eq. (7) of Edwards

(1981). Quantally, this indicates that each particle is in constant momentum state. Namely

$$\overline{v_i(r, t)} = -\frac{q}{mc} A^E(r, t) + \bar{v}_0. \quad (83)$$

Assuming $\overline{v_0(r)} = 0$, we get the Edwards' equation

$$\bar{v}_i = -\frac{q}{mc} A^E, \quad \nabla \times \bar{v}_i = -\frac{q}{mc} H \neq 0. \quad (84)$$

In this case, since no boundary surface has been explicitly assumed and no discussion is made on the thermal stability of the effect, the situation is certainly different from the London equation in the Meissner effect. In the case of $\{A^I, \phi^I\}$ of Eq. (81), we have excluded v_{ip} , but this is not the case for $\{A^E, \phi^E\}$. There is, however, a terse way to relate this formulation with the Meissner-Ochsenfeld effect, as will be shown in the next section.

§10. A few discussions

In our system δj_2 is created with the thermal requirement $\delta(\dot{U}_m^c + U_D^c) < 0$ in Eq. (73), but, when the effect of δj_2 has finished in the system, we should have $\delta(U_m^c + U_D^c) > 0$ (Casimir 1966). Physically, each electron is to exhibit a diamagnetic cyclotron motion, according to the magnetic field H present, irrespectively of the results that this motion would produce in future. This is another crux of the Meissner effect and the action through medium operates there. The mentioned many pioneering works on the diamagnetism of electron gases, both classical and quantal, are not acceptable in the new frame, not only from the thermodynamically incorrect consideration on the role of the magnetic field energies but also from the neglect of the requirement from the action through medium, being inevitable with their use of the Hamiltonian formalism (Appendix B).

The essential mechanism is already present for the system of C_1 , without C_2 . For C_1 , the problem might look similar to the so-called flux quantization in one dimensional model (e. g., Schick 1968). But, the past treatment on this model can not represent the essential part of the problem. The assumption of the presence of uniform vector potential denies the cooperative action of the current and the magnetic field, and, further, the construction of the free energy by using the conventional Hamiltonian has no support from the new frame of physics. We insist that the Maxwell equations are rigidly correct macroscopically, requesting that the effect of electromagnetic induction is unavoidable, so that the change of the magnetic flux confined in C_1 , once created, has to be made only step by step, such as pushing out the outside boundary of the flux outwards, while keeping the total flux confined unchanged. We have proposed in this paper a few new thermodynamic operations. This must be due, because, for superconductors, we know experimentally

that the magnetic flux confined in C_1 is an additional independent parameter of the system, which is not describable by the conventional thermodynamics, both quantal and classical, which have only one equilibrium state. For the superconductors, our proposal is that quantum theory (Bardeen et al. 1957) is needed for obtaining the superconducting electrons, but, once the material can sustain persistent currents, the Meissner effect will be its classical consequence.

Edwards used the action principle with a Lagrangian, having partly appreciated our conclusion. His variation of Eq. (6) is very similar to our variation of Eq. (69). Our classical derivation of the Meissner effect uses the point charge picture of the electron, thereby enables to discuss thermal stability of the effect. If, however, we could make an artificial situation at $t = 0$ where H_1 has already penetrated completely in C_2 , each electron, in the following instant $t > 0$, may start cyclotron motion, together with the pushing out of the flux from the inside of C_2 . But, as shown in Eq. (40), the Maxwell equations request that a thermal kinetic energy of U_{T2} must be consumed in such a way as $\delta U_{T2}^* = -\delta U_{D2} - \delta U_{m22} - \delta U_R$, indicating the conservation of the total self-related energy of C_2 in this process. Here, we have another crux of the Meissner effect. Therefore, if the electrons initially has no such kinetic energy $-\delta U_{T2}^* > 0$, the flux has to be locked, or there is a limitation for the Meissner effect, in the point charge classical electrons. Quantally, since there are zero point motions and the wave function of each particle spreads substantially, the flux locking problem is not present. We may regard Edwards' work as partly filling this classical gap. Although, for the purpose of applying to plasma and astronomical physics, he emphasized the low density of the carriers and the times of interest which is shorter than the mean collision times, in the foot light of the new frame of physics, the theory may be regarded as picking up classically the persistent current character of the state function of the system. In the new frame of physics, two different classical aspects will be proposed for charged particle gas; the usual particle picture and the persistent current picture of the state function. By taking persistent current picture, the thermal part of the motion, $(\nu_i - \nu_D)$, has to be disregarded, being acceptable quantally at very low temperatures and/or density. His Eq. (6) may be reasonable for the system which can maintain persistent currents, because it is simple, covariant, and can derive a desired relation (Appendix C). At the last, we must note the many objections directed to this simple mathematical frame (Taylor 1982, Henyey 1982, Segall et al. 1982, de Vegvar 1982, Edwards 1982). It is an interesting problem to analyze these objections in the foot light of the new frame in physics.

Physically the results by non-thermal dynamics must continue rigorously to those by thermodynamics at low temperature. But, at the same time, we know there are many critical problems at $T = 0$, because it is a kind of singular point in physics. More detailed analyses on the mutual

relation among action principle, Lagrangian, Hamiltonian, entropy, thermodynamics, and quantization of fields in the new frame will be made elsewhere.

We propose that the new frame in physics with the joint-use of Lagrangian and Hamiltonian formalism and the ensemble concept, both classically and quantally, is effective not only for the Meissner effect, plasma physics, and astronomy, but also for most of the physics of macroscopically inhomogeneous systems, such as the Josephson junction (Josephsen 1962), biophysics. logical circuits for high speed computations and optical communication systems. We may add that a very general elegant analysis has been made by the new frame on the Stern-Gerlach experiment in relation to the quantum theory of the measurement.

The author would like to present his sincere appreciation to all the people in the world who have participated in the discussion with him on this extraordinary subject.

Appendix A.

A derivation of the Zeeman energy

Let us assume two persistent current rings C_1 and C_2 ; C_1 being large and C_2 being small and located in the center of C_1 with keeping the fluxes ϕ_1 and ϕ_2 , respectively. C_2 is regarded as the magnetic moment μ_2 located in the magnetic field, H_1 . The total magnetic energy of the system is

$$U_m = \frac{1}{2} L_{11} I_1^2 + L_{12} I_1 I_2 + \frac{1}{2} L_{22} I_2^2, \quad (A1)$$

$$\Phi_1 = c(L_{11} I_1 + L_{12} I_2), \quad \Phi_2 = c(L_{21} I_1 + L_{22} I_2). \quad (A2)$$

Here I_1 and I_2 are the total currents in C_1 and C_2 , ϕ_1 and ϕ_2 are the averaged magnetic fluxes, and L_{11} , L_{22} , and L_{12} are the averaged self- and mutual-inductances. Assuming L_{11} and L_{22} unchanged, when we have changed the mutual configurations of C_1 and C_2 slowly, we get

$$\begin{aligned} \delta U_m &= I_1 \frac{\delta \Phi_1}{c} + I_2 \frac{\delta \Phi_2}{c} - I_1 I_2 \delta L_{12} \\ &= -\delta G_1 - \delta G_2 + \delta^* (-\mu_2 \cdot H_{21}). \end{aligned} \quad (A3)$$

Here, G_1 and G_2 are the non-magnetic energies of the persistent current systems C_1 and C_2 , respectively. H_{21} is the H_1 at μ_2 and δ^* indicates the variation with respect to L_{12} , or due to the change in the mutual configuration between μ_2 and H_1 . Therefore, we get

$$\delta^* (-\mu_2 \cdot H_{21}) = \delta [U_m + G_1 + G_2]. \quad (A4)$$

Hence, the Zeeman expression is a kind of effective Hamiltonian of the total system. Further, when we can assume the I_1 and I_2 unchanged, we get

$$-I_1 I_2 \delta L_{12} = \delta G_1 = \delta G_2 = -\delta U_m = \delta^* (-\boldsymbol{\mu}_2 \cdot \mathbf{H}_{21}). \quad (\text{A5})$$

If $\boldsymbol{\mu}_2$ has been rotated quickly, the electromagnetic action, which should realize δG_1 , starts from the location of $\boldsymbol{\mu}_2$ towards C_1 , together with a free electromagnetic radiation, δU_R , which is not considered in Eq. (A4), because it is usually very small. The situation is partly (but not fully) described by the analysis of §4. It is noted that relativistic consideration is essential for obtaining δG_2 from the Maxwell-Lorentz field and source quantities.

Appendix B.

Advantage of the Lagrangian formalism over the Hamiltonian formalism for the magnetism of the classical Coulomb gas and the thermodynamical significance of the falling off of the magnetic field interaction energy in the effective Hamiltonian of the system.

From Eq. (36) and separating C_1 from C_2 by denoting the charged particles in C_1 with λ , μ, \dots , we get the Lagrangian of the system, $C_1 + C_2$, as

$$\begin{aligned} L = & -\sum_i m_i c^2 + \sum_i \frac{m_i}{2} \mathbf{v}_i^2 - \sum_{i>j} \frac{q_i q_j}{4\pi r_{ij}} + \sum_{i>j} \frac{q_i q_j (\mathbf{v}_i \cdot \mathbf{v}_j)}{4\pi r_{ij} c^2} \\ & - \sum_{i,\lambda} \frac{q_i q_\lambda}{4\pi r_{i\lambda}} + \sum_{i,\lambda} \frac{q_i q_\lambda \mathbf{v}_i \cdot \mathbf{v}_\lambda}{4\pi r_{i\lambda} c^2} \\ & - \sum_\lambda m_\lambda c^2 + \sum_\lambda \frac{m_\lambda}{2} \mathbf{v}_\lambda^2 - \sum_{\lambda>\mu} \frac{q_\lambda q_\mu}{4\pi r_{\lambda\mu}} + \sum_{\lambda>\mu} \frac{q_\lambda q_\mu (\mathbf{v}_\lambda \cdot \mathbf{v}_\mu)}{4\pi r_{\lambda\mu} c^2}, \end{aligned} \quad (\text{B1})$$

and

$$\mathbf{p}_i = \frac{\partial L}{\partial \mathbf{v}_i} = m_i \mathbf{v}_i + \frac{q_i}{c} \sum_{j \neq i} \frac{q_j \mathbf{v}_j}{4\pi r_{ij} c} + \frac{q_i}{c} \sum_\lambda \frac{q_\lambda \mathbf{v}_\lambda}{4\pi r_{i\lambda} c}, \quad (\text{B2})$$

$$\mathbf{p}_\lambda = \frac{\partial L}{\partial \mathbf{v}_\lambda} = m_\lambda \mathbf{v}_\lambda + \frac{q_\lambda}{c} \sum_{\mu \neq \lambda} \frac{q_\mu \mathbf{v}_\mu}{4\pi r_{\lambda\mu} c} + \frac{q_\lambda}{c} \sum_i \frac{q_i \mathbf{v}_i}{4\pi r_{\lambda i} c}. \quad (\text{B3})$$

Here, we have neglected many terms of Eq. (36) as being small. The sixth term has been neglected by assuming that the macroscopic currents are circulating.

The total energy U and the total Hamiltonian \mathcal{H} are

$$\begin{aligned} U(\mathbf{r}_i, \mathbf{r}_\lambda; \mathbf{v}_i, \mathbf{v}_\lambda) &= \sum_i \mathbf{p}_i \cdot \mathbf{v}_i + \sum_\lambda \mathbf{p}_\lambda \cdot \mathbf{v}_\lambda - L \\ &= U_{22}(\mathbf{r}_i, \mathbf{v}_i) + U_{21}(\mathbf{r}_i, \mathbf{v}_i; \mathbf{r}_\lambda, \mathbf{v}_\lambda) + U_{11}(\mathbf{r}_\lambda, \mathbf{v}_\lambda) \\ &= + \sum_i m_i c^2 + \sum_i \frac{m_i}{2} \mathbf{v}_i^2 + \sum_{i>j} \frac{q_i q_j}{4\pi r_{ij}} + \sum_{i>j} \frac{q_i q_j (\mathbf{v}_i \cdot \mathbf{v}_j)}{4\pi r_{ij} c^2} \end{aligned}$$

$$\begin{aligned}
 & + \sum_{i, \lambda} \frac{q_i q_\lambda}{4\pi r_{i\lambda}} + \sum_{i, \lambda} \frac{q_i q_\lambda \mathbf{v}_i \cdot \mathbf{v}_\lambda}{4\pi r_{i\lambda} c^2} \\
 & + \sum_{\lambda} m_\lambda c^2 + \sum_{\lambda} \frac{m_\lambda}{2} v_\lambda^2 + \sum_{\lambda > \mu} \frac{q_\lambda q_\mu}{4\pi r_{\lambda\mu}} + \sum_{\lambda > \mu} \frac{q_\lambda q_\mu (\mathbf{v}_\lambda \cdot \mathbf{v}_\mu)}{4\pi r_{\lambda\mu} c^2} ,
 \end{aligned} \tag{B4}$$

$$\mathcal{H}(\mathbf{r}_i, \mathbf{r}_\lambda; \mathbf{p}_i, \mathbf{p}_\lambda) = U(\mathbf{r}_i, \mathbf{r}_\lambda; \mathbf{v}_i(\mathbf{p}_j, \mathbf{p}_\lambda; \mathbf{r}_j, \mathbf{r}_\lambda), \mathbf{v}_\lambda(\mathbf{p}_\mu, \mathbf{p}_j; \mathbf{r}_\mu, \mathbf{r}_j)) . \tag{B5}$$

As has been explained in detail in §'s, 4 and 5, in order to make these Lagrangian and Hamiltonian effective, we have to assume that the retardation of the electromagnetic signal can be neglected. This can be approximately allowed, if the rate of the change of the macroscopic quantities of the systems is sufficiently slow. This situation is, however, only for the description of the approximate kinematical motion of the system, and is not applicable for the elementary irreversible process of the system, which is very small but still macroscopic, being motivated instantly by the stochastic thermal motion of the system, for which the retardation may become essential, as is described in the main text.

Let us make the effective Hamiltonian for C_2 , by regarding the magnetic field H_1 and the vector potential $A_1(r)$ at C_2 from C_1 constant. Then

$$\mathbf{p}_i = m_i \mathbf{v}_i + \frac{q_i}{c} \sum_{j \neq i} \frac{q_j \mathbf{v}_j}{4\pi r_{ij} c} + \frac{q_i}{c} \mathbf{A}_1(\mathbf{r}_i) , \tag{B6}$$

so that

$$\mathbf{v}_i(\mathbf{p}_j, \mathbf{p}_\lambda; \mathbf{r}_j, \mathbf{r}_\lambda) = \mathbf{v}_i(\mathbf{p}_j, \mathbf{A}_1(\mathbf{r}_i); \mathbf{r}_j) . \tag{B7}$$

In Eq. (B2) or (B6), different from the microscopic system of atoms and molecules, in a macroscopic system, the last two terms are not always smaller than the other terms. The ratio of

$$m_i \mathbf{v}_i : \frac{q_i}{c} \mathbf{A}_1(\mathbf{r}_i) \sim \mathbf{v}_i : \frac{e}{mc} \frac{H \cdot \mathbf{r}}{2} \sim \mathbf{v}_i : 5 \times 10^{6 \sim 7} \text{ m/sec} \tag{B8}$$

is present for $r \sim 5\text{mm}$ and at $H = 10^{2 \sim 3}$ Oe. Even at 400 K, the average v_i may be 10^5 m/s, hence, in Eq. (B6), the approximation

$$\sum_{j \neq i} \frac{q_j \mathbf{v}_j}{4\pi r_{ij} c} \doteq \sum_{j \neq i} \frac{q_j \mathbf{p}_j}{4\pi r_{ij} m_j c} \tag{B9}$$

can not be justified, so far as we assume the normal magnitude for the H_1 applied. Therefore, the explicit representation of Eq. (B5) requests to solve Eq. (B6) for \mathbf{v}_i , which is mathematically almost impossible. This situation indicates the essential disadvantage of the Hamiltonian formalism over

the Lagrangian formalism, for this macroscopic system.

The effective Lagrangian and Hamiltonian for C_2 are

$$L_2 = -\sum_i m_i c^2 + \sum_i \frac{m_i}{2} v_i^2 - \sum_{i>j} \frac{q_i q_j}{4\pi r_{ij}} + \sum_{i>j} \frac{q_i q_j (\mathbf{v}_i \cdot \mathbf{v}_j)}{4\pi r_{ij} c^2} \\ - \sum_{i,\lambda} \frac{q_i q_\lambda}{4\pi r_{i\lambda}} + \sum_i \frac{q_i \mathbf{v}_i}{c} \cdot \mathbf{A}_1(\mathbf{r}_i), \quad (\text{B10})$$

and

$$\mathcal{H}_2(\mathbf{r}_i, \mathbf{A}_1(\mathbf{r}_i); \mathbf{p}_i) = \sum_i \mathbf{p}_i \cdot \mathbf{v}_i - L_2 \\ = \sum_i m_i c^2 + \sum_i \frac{m_i}{2} v_i^2 + \sum_{i>j} \frac{q_i q_j}{4\pi r_{ij}} + \sum_{i>j} \frac{q_i q_j (\mathbf{v}_i \cdot \mathbf{v}_j)}{4\pi r_{ij} c^2} \\ + \sum_{i,\lambda} \frac{q_i q_\lambda}{4\pi r_{i\lambda}} \quad (\text{B11})$$

with Eq. (B7).

Different from the case of the total Hamiltonian of Eqs. (B4) and (B5), the magnetic interaction term has fallen off in Eq. (B11). This is due, because, in the transformation from L_2 to \mathcal{H}_2 , the terms in L_2 , which are linear in v_i , should fall off. Of course, the fallen off term

$$\sum_{i,\lambda} \frac{q_i q_\lambda \mathbf{v}_i \cdot \mathbf{v}_\lambda}{4\pi r_{i\lambda} c^2} = \iiint_{V_1} \iiint_{V_2} \frac{\mathbf{j}(\mathbf{r}_1) \cdot \mathbf{j}(\mathbf{r}_2)}{4\pi r_{12} c^2} dV_1 dV_2 = \iiint_{V_1} \frac{\mathbf{j}(\mathbf{r}_1) \cdot \mathbf{A}_2(\mathbf{r}_1)}{c} dV_1 \\ = \iiint_{V_2} \frac{\mathbf{j}(\mathbf{r}_2) \cdot \mathbf{A}_1(\mathbf{r}_2)}{c} dV_2 = \iiint_{\infty} \mathbf{H}_1 \cdot \mathbf{H}_2 dV \quad (\text{B12})$$

is the magnetic mutual interaction energy between C_1 and C_2

\mathcal{H}_2 , however, is an approximate constant for the kinematical motion of the Coulomb gas, because, from Eq. (39), the change in the magnetic interaction energy between C_1 and C_2 is just cancelled by the energy transfer by induction to C_1 . But, \mathcal{H}_2 or any of its equivalence must be used most carefully for the thermodynamical energy of the system, so far as no rigorous justification for omitting both the magnetic field interaction energy and the energy transfer by induction has been found. As shown in the main text and, also later in this Appendix, there is no such general justification, and the old treatments (van Leeuwen 1921, Van Vleck 1932) are physically not acceptable.

There are other approximations in which the magnetic field energy may be approximated by a superposition of the normal modes of free electromagnetic fields (Heitler 1954). In addition to the problem relating to the representation of a static field by normal modes, the role of the

principle of action through medium becomes unclear and no thermostistical treatment has been made with this approximation.

We know that not \mathcal{H}_2 but U_2 of Eq. (B4),

$$U_2 = U_{22} + U_{21} , \quad (\text{B13})$$

becomes important thermodynamically, being different from \mathcal{H}_2 by just the term of Eq. (B12). It looks that the magnetic interaction energy corresponds thermally to the potential energy of a mechanical system and the travelling electromagnetic energy for the induction corresponds to the kinetic energy of the same system. Although the total amount is unchanged, action always starts towards the lower potential energy. Here, we show very briefly the thermodynamical significance of Eq. (B13). In Ω (§7), let us take a function

$$F_2(A_1(r), j_2(r), S_2) = U_2 - TS_2 \quad (\text{B14})$$

as the possible free energy of the system C_2 under the applied magnetic field H_1 or $A_1(r)$. Then we have mathematically and physically

$$\begin{aligned} \delta F_2 &= \left(\frac{\partial U_2}{\partial S_2} \right)_{A_1(r), j_2(r)} \delta S_2 + (\delta U_2)_{S_2} - S_2 \delta T - T \delta S_2 \\ &= (\delta U_2)_{S_2} - S_2 \delta T . \end{aligned} \quad (\text{B15})$$

Obviously

$$(\delta U_2)_{S_2} = (\delta U_2)_{S_2}^{\delta j_2} + (\delta U_2)_{S_2, j_2(r)}^{\delta A_1} . \quad (\text{B16})$$

Let us apply the equilibrium condition to $\delta j_2(r)$. Then, in general for $C_1 + C_2$, from our equilibrium requirement expressed in Eqs. (67), (74), and (66), we have

$$(\delta U_2)_{S_2, A_1(r)}^{\delta j_2} = 0 . \quad (\text{B17})$$

Therefore, we get

$$\delta F_2 = (\delta U_2)_{S_2, j_2(r)}^{\delta A_1} - S_2 \delta T , \quad (\text{B18})$$

which indicates that, in an equilibrium under the constant temperature and applied field, F_2 should be minimized for the possible variation of δj_2 . Therefore, not U_{22} but U_2 should be used for the thermodynamical energy of the partial system, C_2 .

We should note quantally as well as classically that denial of the paramagnetic boundary

electrons in C_2 is expected in the new frame, because they will raise the magnetic field energy significantly, in contrast to the case of the old frame, where the conventional effective Hamiltonian is used as the thermodynamical energy.

Appendix C

Derivation of covariant special gauges for the kinematics of a charged particle in a Coulomb gas

The action and the proper time differential of i -th charged particle are

$$dS_i = -m_i c^2 d\tau_i + \frac{q_i}{c} a_\mu(\mathbf{r}_i, t) dx_i^\mu \quad (C1)$$

$$d\tau_i = \sqrt{1 - (v_i/c)^2} dt = r_i^{-1} dt. \quad (C2)$$

The Lagrangian and the momentum are

$$L_i = \frac{dS_i}{dt} = -m_i c^2 r_i^{-1} + \frac{q_i}{c} \mathbf{a}(\mathbf{r}_i, t) \cdot \mathbf{v}_i - q_i \phi(\mathbf{r}_i, t) \quad (C3)$$

$$\mathbf{p}_i = \frac{\partial L_i}{\partial \mathbf{v}_i} = m_i r_i \mathbf{v}_i + \frac{q_i}{c} \mathbf{a}(\mathbf{r}_i, t). \quad (C4)$$

The equation of motion is

$$\frac{d\mathbf{p}_i}{dt} = \nabla_i L_i = q_i \nabla_i \left[\frac{\mathbf{v}_i \cdot \mathbf{A}(\mathbf{r}_i, t)}{c} - \Phi(\mathbf{r}_i, t) \right] + [\text{S.I.}]_i \quad (C5)$$

where

$$\mathbf{A}(\mathbf{r}_i, t) = \overline{\mathbf{a}(\mathbf{r}_i, t)}, \quad \Phi(\mathbf{r}_i, t) = \overline{\phi(\mathbf{r}_i, t)} \quad (C6)$$

are the averaged macroscopic potentials and $[\text{S.I.}]_i$ indicates the short range mutual interactions. Now, assuming $\{A^\mu\} = \{\mathbf{A}, \phi\}$ as the Lorentz gauge potentials, we define the Edwards' gauge potentials covariantly as

$$\mathbf{A}^E = \mathbf{A} - \nabla \psi^E, \quad \Phi^E = \Phi + \frac{1}{c} \frac{\partial \psi^E}{\partial t} \quad (C7)$$

where ψ^E is a scalar defined by

$$\begin{aligned} \psi^E(\mathbf{r}, t) &= \int_0^t [\overline{\mathbf{v}}(\mathbf{r}, t) \cdot \mathbf{A}(\mathbf{r}, t) - c \Phi(\mathbf{r}, t)] dt \\ &= \int_0^\tau c v_\mu A^\mu d\tau, \end{aligned} \quad (C8)$$

in which

$$u^\mu = \left\{ r \frac{\bar{v}}{c}, r \right\} \quad (C9)$$

is the four velocity and the integration should be made along the averaged drift line of the particles.

Then, in the average, we get covariantly

$$\frac{d\bar{p}_i^E}{dt} = q_i \bar{v}_i \left[\frac{\bar{v}_i \cdot A^E}{c} - \phi^E \right] = 0 \quad (C10)$$

Because we get from Eq. (C8)

$$\frac{d\psi^E}{dt} = \bar{v}_i \cdot A - c\phi \quad (C11)$$

and, since

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\bar{v}_i \cdot \nabla) \quad (C12)$$

we have

$$\begin{aligned} \frac{\bar{v}_i \cdot A^E}{c} - \phi^E &= \frac{\bar{v}_i \cdot (A - \nabla \psi^E)}{c} - \phi - \frac{1}{c} \frac{\partial \psi^E}{\partial t} \\ &= \frac{\bar{v}_i \cdot A - c\phi}{c} - \frac{d\psi^E}{cdt} = 0 \end{aligned} \quad (C13)$$

It is noted that, in the classical dense Coulomb gas, the averaged drift direction and the path of each particle can be completely different, but, in the new frame, this structure might not be emphasized strongly.

References

- Aharonov, Y., and Bohm, D., 1959, Phys. Rev., **115**, 485.
- Bardeen, J., Cooper, L. N., and Schrieffer, J. R., 1957, Phys. Rev., **108**, 1175.
- Bethe, H. A., and Salpeter, E. E., 1957, "*Quantum Mechanics of one-and two-Electron Atoms*", Springer-Verlag, Berlin et al., 181.
- Breit, G., 1929, Phys. Rev., **34**, 553; 1930, Ibid., **36**, 383; 1932, Ibid., **39**, 616.
- Callen, H. B., 1960, "*Thermodynamics*", John Willey & Sons Inc., Chap. 5.
- Casimir, H. B. G., 1966, "*Preludes in Theoretical Physics*", Ed. A. De-Shalit et al., North-Holland Co., p. 287.
- Darwin, C. G., 1920, Phil. Mag. **39**, 537.
- Darwin, C. G. 1930, Proc. Camb. Phil. Soc., **27**, 86.
- de Gennes, P. G., 1966, "*Superconductivity of Metals and Alloys*" Eng. Ed., W. A. Benjamin Inc.,

- New York et al., p. 6.
- de Vegrar, P. G. N., 1982, Phys. Rev. Lett., **49**, 418.
- Edwards, W. Farrell, 1981, Phys. Rev. Letts., **47**, 1863.
- Edwards, W. Farrell, 1982, Phys. Rev. Lett., **49**, 419.
- Feynman, R. P., Leighton, R. B., and Sands, M., 1964, "*The Feynman Lectures on Physics*", II, Addison-Wesley Co., Sec. 28.
- Ham, F. S., 1953, Phys. Rev., **92**, 1113.
- Heitler, W., "*The Quantum Theory of Radiation*", 1954, 3rd Ed. Oxford, at the Clarendon Press, I, §6.
- Henry, F. S., 1982, Phys. Rev. Lett. **49**, 416.
- Heuser, M., and Hajdu, J., 1974, Z. Phys., **270**, 289.
- Iida, S., 1974, J. Phys. Soc. Jpn., **37**, 1183.
- Iida, S., 1975a, "*New Frame of Electromagnetism*", Vols. 1 & 2, Maruzen Book Co. In Japanese.
- Iida, S., 1975b, Bussei Kenkyu*, **24**, 1, 207. In English.
- Iida, S., 1977, Bussei Kenkyu*, **28**, 45. In English.
- Iida, S., 1978, Bussei Kenkyu*, **31**, 87; 1979, Ibid., **32**, 14; 1980, Ibid., **33**, 223; 1981a, Ibid., **35**, 187; 1981b, **36**, 159; 1982a, Ibid., **37**, 189. In Japanese.
- Iida, S., 1981c, Proc. 3rd Int. Conf. Ferrites, 141.
- Iida, S., 1982b, "*Unified Frame of Electromagnetism*", submitted to J. Phys. Soc. Jpn.
- Iida, S., 1982c, "*Classical Derivation of the Dirac Hamiltonian*", submitted to J. Phys. Soc. Jpn.
- Iida, S., 1982d, "*Transient Energy Principle and the Meissner Effect*", submitted to J. Phys. Soc. Jpn.
- Iida, S., 1982e, "*Basic Concepts in the Thermodynamics of Magnetizable Materials*", submitted to J. Phys. Soc. Jpn.
- Josephson, B. D., 1962, Physics Letters **1**, 251
- Kamerlingh Onnes, H., 1911, Leiden Comm., 120b, 122b, 124c.
- Kondo, J., 1978, Bussei Kenkyu*, **31**, 71; **32**, 27, 521; 1979, Ibid., **34**, 439, 446; 1981, Ibid., **36**, 153; 1982, Ibid., **37**, 18. In Japanese.
- Landau, L. D., 1930, Z. Phys., **64**, 629.
- Landau, L. D., and Lifshitz, E. M., 1962, "*The Classical Theory of Fields*", Revised 2nd Ed., Translated, Pergamon Press, Oxford, London et al. §33.
- London, F., and London, H., 1935, Proc. Roy. Soc. (London), **A149**, 72.

*Bussei Kenkyu is a mostly Japanese Journal issued from Publication Office, Bussei Kenkyu, c/o Yukawa Hall, Kyoto University, requesting to be regarded as "private communications".

- Meissner, W., and Ochsenfeld, R., 1933, *Naturwissenschaften*, **21**, 787.
- Messiah, A., "*Quantum Mechanics*", John Wiley & Sons, Inc., New York et al., vol. II, 947.
- Nishijima, K., 1969, "*Fields and Particles*", W. A. Benjamin, Inc., Chap. 1.
- Peierls, R. E., 1955, "*Quantum Theory of Solid*", Oxford at the Clarendon Press, 144.
- Pollock, F. L. and Hansen, J. P., 1973, *Phys. Rev.*, **A8**, 3110.
- Schick, M., 1968, *Phys. Rev.*, **166**, 166.
- Segall, B., Foldy, L. L., and Brown, R. W., 1982, *Phys. Rev. Lett.*, **49**, 417.
- Taylor, J. B., 1982, *Nature* **299**, 681.
- Tonomura, A., Matsuda, T., Suzuki, R., Fukuhara, A., Osakabe, N., Umezaki, H., Endo, J., Shinagawa, K., Sugita, Y., and Fujiwara, H., 1982, *Phys. Rev. Lett.*, **48**, 1443.
- van Leeuwen, J. H., 1921, *J. de Phys.*, (6) **2**, 361.
- Van Vleck, J. H., 1932, "*The Theory of Electric and Magnetic Susceptibilities*", Oxford, Chap. IV and §81.