Charge Density Waves in ac + dc Electric Fields

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"Shapiro steps" and subharmonic steps in the IV characteristics of sliding charge density waves in ac + dc fields are analyzed in terms of a classical model.

The motion of a charge-density-wave (CDW) in applied ac and dc electric fields may be described by the damped driven pendulum equation[1-2]

$$\varepsilon\ddot{\theta} + G\dot{\theta} + \sin\theta = E + E_{ac}\cos(\omega_o t) \tag{1}$$

The parameters E and E_{ac} represent the dc and ac fields respectively, G/ε is the damping, and the $\sin\theta$ term represents a periodic pinning potential. Although it is usually assumed that random impurities may conspire to produce the sinusoidal potential, I suggest that the origin of the $\sin\theta$ term is simply the contact pinning potential, at least for small samples.

The equation (1) also describes the resistively shunted Josephson junction[3], where E and E_{ac} represent driving currents and the voltage V is proportional to the Josephson frequency, $V \sim \omega_J = \langle\dot{\theta}\rangle$. In a CDW system the current I is proportional to the frequency, $I \sim \omega_{CDW} = \langle\dot{\theta}\rangle$. Hence the roles of currents and voltages are the opposite for the two systems.

The resistively shunted Josephson junction exhibits Shapiro steps where the frequency ω locks into the frequency of the ac current, $\omega = q\omega_o$[4]. In addition, there are subharmonic steps where $p\omega = q\omega_o$[5,6]. The Shapiro steps and the subharmonic steps manifest themselves as plateaus in the IV characteristics. The main "Shapiro" steps have been observed in the CDW system $NbSe_3$[2,7], but it is not clear whether or not the subharmonic steps have been observed.

In $NbSe_3$ it is believe that the CDW is in the overdamped regime, $\varepsilon \ll 1$. In the limit $\varepsilon \to 0$ it can be shown that only the integer steps survive. The problem of solving Eq. 1 can be transformed into that of solving the Schrödinger equation in a periodic potential[8]

$$\ddot{\psi} + V(\tau)\psi = 0 \ , \qquad V(\tau) = V(\tau + \frac{2\pi}{\omega_o}) \ . \tag{2}$$

Note that the time τ plays the role of space x in this equation. If the $\cos\omega_0 t$ in (1) is replaced by a periodic δ-function potential the potential $V(\tau)$ can be calculated explicitly and (2) takes the form of a Kronig-Penney model[9]

$$\ddot{\psi} + (k^2 - \sum_n v\delta(\tau - \frac{2\pi n}{\omega_0}))\psi, = 0 \tag{3}$$

which can be solved rigorously. The parameters v and k are functions of the parameters in Eq. (1).

The solution, with E = 0, that we are interested in may be either a band function

$$\psi_{k'}(\tau) = \exp(\pm ik'\tau)U_{k'}(\tau) \tag{4a}$$

or a gap function

$$\psi_\mu(\tau) = \exp(\pm \mu\tau)U_\mu(\tau) \tag{4b}$$

The wave vector k' of the wave function corresponds to the frequency ω of the CDW, $\omega = 2k'$.

If the relevant wave function is a band function, the frequency ω is a smooth function of the parameters v and k in Eq. (3) and there can be no subharmonic steps. If the wave function is a gap function, the frequency is locked to the wave vector at the band gap, $\omega = 2k'_{gap} = 2q/(2\omega_0) = q\omega_0$. These are the main Shapiro steps.

Waldram and Wu[10] have given an alternative treatment of the problem for $\varepsilon = 0$ in the case of a cosine potential. They calculated the value of the phase θ_{n+1} after n + 1 cycles of the ac field as a function of the phase θ_n after n cycles:

$$\tan^{-1}(\gamma \tan \frac{\theta_{n+1}}{2}) = \pi\delta + \tan^{-1}(\gamma \tan \frac{\theta_n}{2}) \tag{5a}$$

For general pinning potentials the equation takes the form[9]

$$\tan^{-1}(\gamma \tan \frac{\theta_{n+1}}{2} - a) = \pi\delta + \tan^{-1}(\gamma \tan \frac{\theta_n}{2} - a) \ , \tag{5b}$$

where δ is a smooth function of the parameters in (1). Defining $\frac{1}{2}\theta'_n = \tan^{-1}(\gamma \tan \frac{\theta_n}{2} - a)$ one obtains the trivial return map

$$\theta'_{n+1} = 2\pi\delta + \theta'_n \ . \tag{6}$$

The frequency of the CDW is identical to the winding number W of the map (6):

$$\frac{\omega}{\omega_o} = W = \lim_{n\to\infty} \frac{\theta_n - \theta_o}{2\pi n} = \lim_{n\to\infty} \frac{\theta'_n - \theta'_o}{2\pi n} = \delta \tag{7}$$

Again, ω is found to be a smooth function of the parameters so there are no subharmonic steps in the overdamped limit $\varepsilon = 0$.

Now, let us proceed to the much more complicated situation with finite ε. We have considered the situation[9] $\varepsilon \ll 1$ which is relevant for $NbSe_3$.

Our result is that the return map takes the form

$$\tan^{-1}(\gamma \tan \frac{\theta_{n+1}}{2} - a) = \pi\delta + \tan^{-1}(\gamma \tan \frac{\theta_n}{2} - a) + \frac{1}{2}\varepsilon f(\theta_n) , \tag{8a}$$

or

$$\theta'_{n+1} = 2\pi\delta + \theta'_n + \varepsilon f(\theta_n) , \quad f(\theta + 2\pi) = f(\theta) . \tag{8b}$$

The return map (8b) is the so-called circle map which has been studied by several groups[11-12]. In particular the mode locking phenomena have been studied by Jensen et al.[12]

The fact that the return map for the differential equation is a one dimensional map is by no means obvious. Since the differential equation is of second order one would expect the return map to be two dimensional:

$$\theta_{n+1} = g(\theta_n, \dot{\theta}_n) \tag{9}$$

The collapse to one dimensionality expresses the fact that after a transient period $\dot{\theta}_n$ becomes a function of θ_n:

$$\theta_{n+1} = g(\theta_n, h(\theta_n)) = f'(\theta_n) \tag{10}$$

Eq. (8) reveals that the strength of the periodic potential f in the circle map is proportional to the coefficient ε in Eq. (1). Solutions to the map may be generated by iterations. For not too large ε the map exhibits subharmonic steps where ω given by Eq. (7) locks into the applied ac frequency:

$$\frac{\omega}{\omega_o} = \frac{q}{p} \tag{11}$$

In the limit $\varepsilon \ll 1$ which applies to the CDW problem the widths of the subharmonic steps become

$$\Delta(\frac{q}{p}) \sim \varepsilon^p \tag{12}$$

Hence, the widths of high order subharmonic steps are extremely narrow for high p. The fact that ε is small means that it could be very difficult to observe any subharmonic steps.

Even if there are in principle an infinity of subharmonic steps, the frequencies are unlocked (incommensurate) most of the time. At a critical value of ε (which is not relevant to the CDW problem) there is a transition to chaos[11,12]. At the transition line the subharmonic steps "fill up" the whole phase diagram, forming a complete devil's staircase[12] with fractal dimension D ~ 0.87 ... Beyond the transition line the subharmonic steps begin to overlap. Note that the chaos occurring in this regime for large ε is <u>unrelated</u> to the periodic "noise" in $NbSe_3$ where $\varepsilon \ll 1$.

In conclusion, subharmonic steps should in principle occur in charge density wave systems in applied ac + dc fields, but because the motion is near the overdamped limit their widths are expected to be very small.

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