物性研究 42－6（1984－9）

The New Frame in Electromagnetism

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（1984年6月25日受理）

はしがき

物性研究 40－5（1983－8）435－473 発表の新体系物理学，“New Frame in Physics”の，電磁気学領域を詳細に解説するものとして，本英文報告を発表させて戴く。重要な概念の変更として，電子の古典像はもはや点ではなく，10^{-2} Å程度の大きさの永久電流であることが述べられる。この新しい考え方は，東京大学理学部物理学教室の人達に対しては，既に周知されていると言える段階にあり，明瞭な反論があれば戴ける状況と考えているが，いまのところない。原子核研究所の平尾泰男教授は明確に，古典的には電子が点でなく，10^{-2}Å位の大きさを持つとすることに対し，実験的な反証は全くないと明言されたことを附記する。なお，高エネルギーの電子－陽電子，電子－電子衝突の実験などにおいて，その衝突断面積から，剛体球の立場で予想される大きさが，10^{-17} m 以下であるという話は，素粒子物理学的な，粒子間反応の確率の問題である。新体系物理学の立場は，こうした実験と異なり，いわば超低エネルギーの物質現象の領域を取り扱うもので，電子は不生不滅の実在と考えるものである。10^{-17} m では電気エネルギーだけでも mc^2 の 10^3 倍にもなり，無矛盾のマクスウェル・ローレンツ電磁気学を構成することが，原理的に不可能である。10^{-2} Å の永久電流としての電子は，もし衝突させると，ソリトンの様に，重畳した上で，元通りの姿で，動くと考えることもできるし，スピンが平行ならパウリの原理が働いて，もともと重畳できないし，もしスピンが反平行であれば，互に回転して，二枚の無限に薄い円盤が，平行にすれ違うというイメージを考えることも可能である。また古典像は，量子物理学的実体である電子の性質のすべてを説明できる筈のないことは，もちろんである。

新体系物理学の立場は，c－数方程式としての古典物理学を完結させるために，電子の最善の古典像を得ることであって，それはマクスウェル・ローレンツの微視的古典電磁気学を無矛盾に成立させると共に，その体系はマクスウェルの巨視的電磁気学の基礎となることが期待される。電子の永久電流モデルは幸いにして，この要求を十分に満足させたのである。

飯田修一（東大・理），「電磁気学の新体系」

なお，電子の持つ，スピン磁気能率の値 $|g|e\hbar/4mc$ は，非常に大きいもので，相対性理論の枠組みを使う以上，光速度 c 以上の速度で動く電荷を考えることは禁止されるが，電荷 $(-e)$ をその極限の速度 c で動かした上でも，上記 10^{-2} Å の大きさが，最小限必要になることを注意する。詳細は本文，及び Appendix A に説明される。さらに本文で重要な点は電磁エネルギーと電磁運動量の移動関係の矛盾のない記述である。この課題は，いわゆる現存の物理学では未完成であったところで，ランダウ・リフシッシ はその「場の古典論」の中で，同語反覆を行って，お茶を濁していることを注意させて戴く。これは電子を点と考え，自己エネルギーの無限大を内蔵した既存の物理学では避けられない矛盾点であったが，新体系物理学は，電子の永久電流モデルをその基礎に取ることによって，その困難を解決したものと考えている。

なお，物性研究誌上での近藤淳氏との紙上討論は終了し，マイスナー効果は永久電流を維持できる体系の古典物理学的性質であることが確立され，新体系物理学はその基礎固めを終了して，発展の段階に入ったことをこゝに宣言させて戴く。

Synopsis

Based on the persistent current model of the electron, a new consistent frame of classical electromagnetism is presented. Different from the present field-theoretical approach, the self-produced electromagnetic field energy of each electron is regarded as a part of the self-energy of the electron, requiring a self-factor 1/2 for the momentum. Clarifying the interrelation between the Maxwell-Lorentz and the Maxwell electromagnetism, a rigorous self-consistent electromagnetic energy-momentum density formulation is presented. In this new frame, Zeemen energy has to be regarded as an effective Hamiltonian of the total system including nonmagnetic energies of the magnetic moment and the source of the external magnetic field involved. It was found that the transfer of electromagnetic energy by induction is a fundamental phenomenon for the magnetic moments, or, the persistent current systems, which is quite implicit in the current frame of the quantum theory.

§1. Introduction

The classical and quantal physics present the two strictly consistent precise mathematical frames, having the accuracy of less than 10^{-8} in each effective field. This fact suggests that there will be an analytical interconnection between the two frames. The effort to clarify this structure leads finally for the author to propose the new frame of physics[1,#,2], which presents another approach for the physics of materials, being more adequate than the present frame of Q.E.D., for the analysis of macroscopic systems. In this effort, the requirement becomes evident that, since the theory of classical electromagnetism for materials was incomplete[4)], in a point of having the self-consistent energy-momentum transfer relation in which electrons are involved, the requirement exists to find out a consistent unified classical electromagnetism for the Maxwell-Lorentz world. This paper presents the essential part of thus newly constructed Maxwell-Lorentz classical electromagnetism, which constitutes a part of the new frame of physics. We use MKS rationalized Gauss unit system for convenience (We call it the MKSP system[5,6)], P: Physical.).

§2. The Persistent Current Model of the Electron

In 1974, we proposed a persistent Vortex Ring current model of the electron, having no adjustable parameter[5)]. We refer it as VR hereafter. Essential features of VR are as follows.

1). It is a tiny ring current with the ring radius of

$$R^{\circ} = g\lambda\!\!\!^{-}_{\mathrm{e}} = 2\left(1+\frac{\alpha}{2\pi}\right)\frac{\hbar}{mc} = 7.73185\times10^{-13}\mathrm{m} \tag{1}$$

and with a very small cross sectional radius, r°.† We should note that there is an old concept of the point charge electron, in which the classical size of the electron is assumed to be less than 10^{-15} m. Since we take the VR as representing the classical electron, the classical size of the electron in the new frame is in the order of 10^{-12} m = 10^{-2} Å, which necessitates to abandon the point electron concept. This must be due in the new frame, because, in the old model, the associated electrostatic self-energy itself already exceeds the rest energy of the electron by a factor of a few orders of magnitude, the sizeless point electron concept, in principle, can not become the basic entity in the proposed strictly self-consistent Maxwell-Lorentz classical electromagnetism. More detailed reasons are explained in Appendix A.

2). The total energy consists of electric and magnetic energies, $U_{\mathrm{E},0}$ and $U_{\mathrm{M},0}$,

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Reference paper 1) was published in English in "Bussei Kenkyu", which is a Japanese journal written mostly in Japanese. There had been many pioneering papers and debates[3)].

† we had a small mistake in the original paper[2)] for the calculated radius of the cross section. The values of (6.33 or 4.75) x 10^{-378} m in p. 1586 should be replaced with (1.241 or 0.967) x 10^{-386} m, for the uniform or surface charge and current distributions. Although we do not believe that the Maxwell electromagnetism can be simply extended to such a tiny distance, the attractive feature of VR model is that its essential characters are independent of the details of these super-microscopic structures.

$$U_{E,0}=\frac{mc^2}{2}+\frac{e^2}{8\pi^2R^\circ}\ ,\quad U_{M,0}=\frac{mc^2}{2}-\frac{e^2}{8\pi^2R^\circ}\ . \tag{2}$$

3). The magnitudes of the angular momentum and the magnetic moment in each single classical state are $\hbar$ and $g\mu_B$, respectively.

4). Ensemble concept is employed in order to bridge the model with the quantum characters of the electron, in which the state α, i.e., the one with the spin directed along $+z$ axis, is represented by an ensemble in which the top of the angular momentum vector distributes uniformly over the upper hemisphere[5),7)]. Hence, the averaged charge distribution becomes spherical with the angular momentum and the magnetic moment components of $\hbar/2$ and $g\mu_B/2$, resepectively. In the new frame, ensemble concept is regarded important for bridging the physical concepts in classical and quantal physics.

5). Different from the old understanding[8),9)], we have proposed to introduce a self-factor 1/2 for the calculation of its self-produced electromagnetic momentum. Therefore, when VR is at rest, the momentum density in the proper frame is either

$$\frac{{}^*\rho^\circ\,{}^*\boldsymbol{a}^\circ}{2c}\quad \text{or}\quad \frac{{}^*\boldsymbol{e}^\circ\times{}^*\boldsymbol{h}^\circ}{2c}\ , \tag{3}$$

where ${}^*\rho^\circ$, ${}^*\boldsymbol{a}^\circ$, ${}^*\boldsymbol{e}^\circ$ and ${}^*\boldsymbol{h}^\circ$ are the original Maxwell-Lorentz's charge density, vector potential, and electric and magnetic fields, respectively. Here * indicates that it is the quantity before the ensemble average procedure.

6). VR is electromagnetically stable. In the Maxwell electromagnetism, the Lorentz electric force of repulsion is almost exactly cancelled by the Lorentz magnetic force of attraction and the magnitude of the next term is extremely small, such as 10^{-365} of the main term. Since the gravitational force expected is quite large, such as 10^{-43} of the main term, we expect that the model is stable in terms of general relativity.

7). The model keeps a quantized flux of hc/e[5)], which is twice of the fluxoid of superconducting circuits.

8). As shown in Eq. (1), VR has anomalous magnetic moment intrinsically, or, the g-factor of VR agrees with the result of quantum electrodynamics, up to the second order perturbation, i.e., less than 2 ppm.

In the present field theory[8)], the electron is described by the electron field with γ-matrix and associated quantized electromagnetic field, leaving the divergence difficulty unsolved. Since the mathematical frame in the field theory is so simple and, also, since the VR model can represent the real electron so precisely[23], we assume in the new frame that the momentum and energy of the electron are mostly electromagnetic, or, at least, they will be approximated by the analytical extension of the frame of electromagnetism. There will be no doubt that the momentum and energy of the self-produced electromagnetic fields of an electron should form important and inseparable components of the momentum and energy of the electron.

§3. The Maxwell-Lorentz and the Maxwell Equations

The real quantal electron is approximated by the following two steps. In the first step, the electron is represented by the classical VR located in the space with a certain velocity. In the next step, if the state is α, the state is represented by an ensemble in which the top of the angular momentum vector distributes uniformly over the upper hemisphere, and centers of the VR's are distributed according to the quantal probability $\psi^*(\boldsymbol{r}, t)\,\psi(\boldsymbol{r}, t)$, moving in accordance with the probable orbital electric current density,

$$\psi^*(-e\boldsymbol{v})\psi = \left(\frac{-e}{2m}\right)\left[\psi^*\left\{\left(\frac{\hbar}{\mathrm{i}}\right)\nabla - \left(\frac{-e}{c}\right)\boldsymbol{a}\right\}\psi + \psi\left\{\left(\frac{\hbar}{-\mathrm{i}}\right)\nabla - \left(\frac{-e}{c}\right)\boldsymbol{a}\right\}\psi^*\right] . \tag{4}$$

Here $\boldsymbol{a}$ is a microscopic vector potential in the Lorentz gauge. Thus, we get the electric current density four vector, $\{ j_i, \underline{i}c\rho_i \}$, for the electron i, j_i being composed of spin and orbital currents and $\underline{i}$ being i of the four space[6]. In the new frame, we regard the Schrödinger representation most basic, as only this representation can afford the direct correspondence between the classical and quantal physics.

Let us denote the electromagnetic fields of the i-th electron in the first step in the Maxwell-Lorentz world as $(^*\boldsymbol{e}_i, {}^*\boldsymbol{h}_i)$. For the nucleus, although we do not know the detail, we assume that we know its $(^*\boldsymbol{e}_j, {}^*\boldsymbol{h}_j)$. Then, by applying the superposition principle for the following step, we obtain $(\boldsymbol{e}_i, \boldsymbol{h}_i)$ as the total ensemble average. We regard this $(\boldsymbol{e}_i, \boldsymbol{h}_i)$ as the $(\boldsymbol{e}_i, \boldsymbol{h}_i)$ of our Maxwell-Lorentz world. Then, we have

$$\left.\begin{aligned} &\nabla\times\boldsymbol{e}_i = -\frac{1}{c}\frac{\partial \boldsymbol{h}_i}{\partial t} , && \nabla\cdot\boldsymbol{h}_i = 0 , \\ &\nabla\times\boldsymbol{h}_i = \frac{1}{c}\frac{\partial \boldsymbol{e}_i}{\partial t} + \frac{\rho_i \boldsymbol{v}_i}{c} + \frac{\boldsymbol{I}_i}{c} , && \nabla\cdot\boldsymbol{e}_i = \rho_i . \end{aligned}\right\} \tag{5}$$

Here, the microscopic currents, j_i, is intentionally decomposed as $j_i = \rho_i \boldsymbol{v}_i + \boldsymbol{I}_i$, where $\boldsymbol{I}_i$ represents the microscopically closed persistent currents, due to spin and/or orbital magnetic moment[10,11]. In addition, there will be free electromagnetic waves, $\boldsymbol{e}_0$ and $\boldsymbol{h}_0$, which have similar equations to Eq. (5) with $\rho_i = j_i = 0$.

Then, after being superposed, we have our Maxwell-Lorenz equations of

$$\left.\begin{aligned} &\nabla\times\boldsymbol{e} = -\frac{1}{c}\frac{\partial \boldsymbol{h}}{\partial t}, && \nabla\cdot\boldsymbol{h}=0, \\ &\nabla\times\boldsymbol{h} = \frac{1}{c}\frac{\partial \boldsymbol{e}}{\partial t} + \frac{\rho\boldsymbol{v}}{c} + \frac{\boldsymbol{I}}{c}, && \nabla\cdot\boldsymbol{e}=\rho, \end{aligned}\right\} \tag{6}$$

in which

$$\boldsymbol{e}=\sum_i \boldsymbol{e}_i, \quad \boldsymbol{h}=\sum_i \boldsymbol{h}_i, \quad \rho(\boldsymbol{r},t)=\sum_i \rho_i(\boldsymbol{r},t), \quad \rho\boldsymbol{v}=\sum_i \rho_i\boldsymbol{v}_i, \quad \boldsymbol{I}=\sum_i \boldsymbol{I}_i. \tag{7}$$

Let us define fictitious magnetic shell functions $\boldsymbol{m}_i(r, t)$[12]. The function $\boldsymbol{m}_i(r, t)$ has finite values only in the magnetic shell bounded by $\boldsymbol{I}_i(r, t)$ and has a meaning of the microscopic fictitious magnetization. The fictitious $\boldsymbol{H}$ field, $\boldsymbol{h}_i^{\mathrm{m}}(r, t)$, that the magnetic shell $\boldsymbol{m}_i$ would produce is $\boldsymbol{h}_i^{\mathrm{m}}(r, t) = -\nabla\phi_i^{\mathrm{m}}(r, t)$, and

$$\nabla\times\boldsymbol{m}_i = \frac{\boldsymbol{I}_i}{c} \tag{8}$$

where ϕ_i^{m} is a magnetic potential due to $\boldsymbol{m}_i$. Then we get

$$\left.\begin{aligned} &\overline{\overline{\boldsymbol{e}}}=\boldsymbol{E}, \quad \overline{\overline{\boldsymbol{h}}}=\boldsymbol{B}, \quad \overline{\overline{\boldsymbol{m}}}=\overline{\overline{\sum_i \boldsymbol{m}_i}}=\boldsymbol{M}, \\ &\frac{\overline{\overline{\rho\boldsymbol{v}}}}{c} = \frac{\overline{\overline{(\rho\boldsymbol{v})_{\mathrm{free}}}}}{c} + \frac{\overline{\overline{(\rho\boldsymbol{v})_{\mathrm{bound}}}}}{c} = \frac{\boldsymbol{j}}{c} + \frac{1}{c}\frac{\partial \boldsymbol{P}}{\partial t}, \\ &\frac{\overline{\overline{\boldsymbol{I}}}}{c} = \nabla\times\boldsymbol{M}, \quad \overline{\overline{\rho}} = \overline{\overline{\rho}}_{\mathrm{free}} + \overline{\overline{\rho}}_{\mathrm{bound}} = \rho^{\mathrm{F}} - \nabla\cdot\boldsymbol{P}. \end{aligned}\right\} \tag{9}$$

Therefore, from Eq. (6), we get easily the Maxwell equations and the equation for ripple electromagnetic fields.

§4. Energy Transfer Relations

From Eqs. (5), we get

$$-\nabla\cdot c\boldsymbol{e}_j\times\boldsymbol{h}_i=\boldsymbol{e}_j\cdot\frac{\partial\boldsymbol{e}_i}{\partial t}+\boldsymbol{h}_i\cdot\frac{\partial\boldsymbol{h}_j}{\partial t}+\boldsymbol{e}_j\cdot(\rho_i\boldsymbol{v}_i+\boldsymbol{I}_i), \tag{10}$$

so that, from Eq. (6),

$$-\nabla\cdot c\boldsymbol{e}_j\times(\boldsymbol{h}_i-\boldsymbol{m}_i)=\boldsymbol{e}_j\cdot\frac{\partial\boldsymbol{e}_i}{\partial t}+\boldsymbol{h}_i\cdot\frac{\partial\boldsymbol{h}_j}{\partial t}+\boldsymbol{e}_j\cdot\rho_i\boldsymbol{v}_i-\boldsymbol{m}_i\cdot\frac{\partial\boldsymbol{h}_j}{\partial t}. \tag{11}$$

Integrating Eq. (11) over an arbitrary volume V with the surface S, we get

$$\begin{aligned}&-\iint_S c\sum_{j\neq i}\boldsymbol{e}_j\times(\boldsymbol{h}_i-\boldsymbol{m}_i)\cdot\mathrm{d}\boldsymbol{S}\\&=\iiint_V\sum_{j\neq i}\left[\left(\boldsymbol{e}_j\cdot\frac{\partial\boldsymbol{e}_i}{\partial t}+\boldsymbol{h}_i\cdot\frac{\partial\boldsymbol{h}_j}{\partial t}\right)+\boldsymbol{e}_j\cdot\rho_i\boldsymbol{v}_i+(-\boldsymbol{m}_i)\cdot\frac{\partial\boldsymbol{h}_j}{\partial t}\right]\mathrm{d}V\end{aligned} \tag{12}$$

Here, i can be either fixed or summed. Further, from Eq. (10) for $j=i$,

$$-\iint_S c\boldsymbol{e}_i\times\boldsymbol{h}_i\cdot\mathrm{d}\boldsymbol{S}=\iiint_V\left[\frac{\partial}{\partial t}\frac{(\boldsymbol{e}_i^2+\boldsymbol{h}_i^2)}{2}+\boldsymbol{e}_i\cdot(\rho_i\boldsymbol{v}_i+\boldsymbol{I}_i)\right]\mathrm{d}V. \tag{13}$$

Using Eqs. (10), and (12), we have an identity of

$$\iiint_V\sum_{j\neq i}(-\boldsymbol{m}_i)\cdot\frac{\partial\boldsymbol{h}_j}{\partial t}\mathrm{d}V=\sum_{j\neq i}\left[\iiint_V\boldsymbol{e}_j\cdot\boldsymbol{I}_i\,\mathrm{d}V+\iint_S c\boldsymbol{e}_j\times\boldsymbol{m}_i\cdot\mathrm{d}\boldsymbol{S}\right]. \tag{14}$$

In Eq. (14), since the left side is a volume integral, which will increase monotonously with the volume, the surface integral of the last term must be compensated by a part of the first term of the right side, which is only effective when S cut $\boldsymbol{I}_i$'s or $\boldsymbol{m}_i$'s. When S cuts $\boldsymbol{m}_i$, then the principal term of $\boldsymbol{e}_j$, having the form of $-\nabla\phi_j$, works to $\boldsymbol{I}_i$, but, we have no interest to this work, because mathematically it has no contribution when integrated over whole $\boldsymbol{I}_i$. Therefore, Eq. (14) represents the action of $-\partial\boldsymbol{a}_j/c\partial t$, i.e., the rate of work done by induction to the magnetic moments. Thus the right side terms of Eq. (12) represent the time change of the elctromagnetic interaction energies, the rate of work done to $\rho_i \boldsymbol{v}_i$, and the rate of work done to $\boldsymbol{I}_i$ by induction. Accordingly, the left side term should show the flow rate of electromagnetic energy through the surface S.

In the new frame, besides the free radiation component, the electromagnetic self-energy of particle i, $\iiint [(e_i^2 + \boldsymbol{h}_i^2)/2]\,\mathrm{d}V$, can not be separated from the relativistic self energy of the particle. This view is different from the usual starting view of the field theory[8], and, therefore, the renormalization procedure becomes not necessary. Eq. (13) includes the time change of this part of the self energy of particle i. The essential significance of this term will be explained in §5. We can disregard Eq. (13) and consider only Eq. (12) after summing over i, because, in a usual material, the ratio of the weight of Eqs. (13) to (12) is N:N^2, where N is the number of particle i in a small volume ΔV. From Eqs. (12) and (13), we get

$$
\begin{aligned}
&-\iint_S c\boldsymbol{E}\times(\boldsymbol{B}-\boldsymbol{M})\cdot \mathrm{d}\boldsymbol{S} \\
&=\iiint_V \Big[\boldsymbol{E}\cdot\frac{\partial \boldsymbol{E}}{\partial t}+\boldsymbol{B}\cdot\frac{\partial \boldsymbol{B}}{\partial t}+\boldsymbol{E}\cdot\boldsymbol{j}+\boldsymbol{E}\cdot\frac{\partial \boldsymbol{P}}{\partial t}-\boldsymbol{M}\cdot\frac{\partial \boldsymbol{B}}{\partial t}\Big]\,\mathrm{d}V
\end{aligned}
\tag{15}
$$

and a corresponding ripple term equation. We must notice that

$$
-\boldsymbol{M}\cdot\frac{\partial \boldsymbol{B}}{\partial t} \tag{16}
$$

represents the main component of the energy transfer by induction to the magnetic moments μ_i. In the new frame, the energy transfer by induction to the persistent currents is regarded as an essential phenomenon, being also effective quantum mechanically. Relation of this phenomenon to the Zeeman energy is given in Apeendix B.

The ripple term equation must represent various non-Maxwell energy transfers, such as by the mechanical stresses, phonons, thermal conductions, and thermal radiations. The energy transfer by mass transportation, however, will be analyzed in the next section.

It is noted that up to this point the only assumption needed is that the magnetic moments are composed of persistent currents and the details of VR model is not necessary.

§5. Electromagnetic Momentum-Energy Density Relations

We show first that a consistent electromagnetic momentum-energy density relation can be obtained in terms of $({}^*e_i, {}^*\boldsymbol{h}_i)$. According to the logics developed in the foregoing paper[5], the self momentum-energy density four vector of the i-th particle, $\{p_i\}_{\mathrm{self}}$, is represented by

$$\{p_i\}_{\text{self}}=\{\frac{{}^*\rho_i^\circ\,{}^*\boldsymbol{a}_i}{2c}+r_{\boldsymbol{v}_i}\frac{({}^*\boldsymbol{j}_i^\circ\cdot{}^*\boldsymbol{a}_i^\circ)}{2c^2}\frac{{}^*\boldsymbol{v}_i}{c},\ \frac{i}{c}[\frac{{}^*\rho_i^\circ\,{}^*\phi_i}{2}+r_{\boldsymbol{v}_i}\frac{({}^*\boldsymbol{j}_i^\circ\cdot{}^*\boldsymbol{a}_i^\circ)}{2c}]\}\tag{17}$$

Here, $\{{}^*j_i^\circ, \underline{i}c{}^*\rho_i^\circ\}$ and $\{{}^*a_i^\circ, \underline{i}{}^*\phi_i^\circ\}$ are the current and potential four vectors in the proper frame of the i-th particle. It is noted that, in order to get the total momentum-energy four vector, the integration must be made in the proper space V°, or, in the proper hyper plane σ°. We assume that $({}^*\boldsymbol{e}_i, {}^*\boldsymbol{h}_i)$ satisfies also the Maxwell-Lorentz equations which are similar to Eq. (5).

Now, from the Maxwell-Lorentz equations for $({}^*\boldsymbol{e}_i, {}^*\boldsymbol{h}_i)$, we have the following identities,

$$\iiint_\infty({}^*\boldsymbol{e}_i\cdot{}^*\boldsymbol{e}_j+{}^*\boldsymbol{h}_i\cdot{}^*\boldsymbol{h}_j)\,\mathrm{d}V=\iiint_\infty[{}^*\phi_i{}^*\rho_j+{}^*\boldsymbol{a}_i\cdot\frac{{}^*\boldsymbol{j}_j}{c}-\frac{\partial^*\phi_i}{c\partial t}\frac{\partial^*\phi_j}{c\partial t}$$
$$+{}^*\phi_j\frac{\partial^{2*}\phi_i}{c^2\partial t^2}+\frac{\partial^*\boldsymbol{a}_i}{c\partial t}\cdot\frac{\partial^*\boldsymbol{a}_j}{c\partial t}-{}^*\boldsymbol{a}_i\cdot\frac{\partial^{2*}\boldsymbol{a}_j}{c^2\partial t^2}]\,\mathrm{d}V\tag{18}$$

$$\iiint_\infty\frac{{}^*\boldsymbol{e}_i\times{}^*\boldsymbol{h}_j}{c}\,\mathrm{d}V=\iiint_\infty\frac{1}{c}[{}^*\rho_i{}^*\boldsymbol{a}_j+\nabla{}^*\phi_i\frac{\partial^*\phi_j}{c\partial t}-\nabla{}^*\boldsymbol{a}_j\cdot\frac{\partial^*\boldsymbol{a}_i}{c\partial t}]\,\mathrm{d}V,\tag{19}$$

$$\iiint_\infty\boldsymbol{r}\times(\frac{{}^*\boldsymbol{e}_i\times{}^*\boldsymbol{h}_j}{c})\,\mathrm{d}V=\iiint_\infty\{\boldsymbol{r}\times[\frac{{}^*\rho_i{}^*\boldsymbol{a}_j}{c}+\frac{\nabla{}^*\phi_i}{c}\frac{\partial^*\phi_j}{c\partial t}-\nabla{}^*\boldsymbol{a}_j\cdot\frac{\partial^*\boldsymbol{a}_i}{c\partial t}]$$
$$+{}^*\boldsymbol{a}_j\times\frac{\partial^*\boldsymbol{a}_i}{c\partial t}\}\,\mathrm{d}V.\tag{20}$$

Here, $\nabla{}^*a_j$ is a dyadics, and we use only the Lorentz gauge potentials which vanish at infinity. In Eq. (20), any point can be used as the origin of $\boldsymbol{r}$, and the last term represents the spin angular momentum of the electromagnetic fields.

It is to be noted that, in order to get these identities, many differentiations and integrations over the whole space have to be made. Since, in the new frame, we can assume essentially regular functions for all the electromagnetic quantities in the four space, all the mathematical operations can be made without having caution to singularities. This is an essential feature of the new frame of electromagnetism, which was not present in the old frame where the divergence difficulties are always present for the point charge electrons with spin.

We regard that these equations are important for analyzing the structure of the electromagnetic momentum-energy densities in the space. When $i\neq j$, these relations are useful without any additionals, but when $i=j\neq 0$, careful consideration must be made to all these equations. Comparing with Eq. (17), we should notice that a self-factor 1/2 is at least needed in order to approximate the self-energy, self-momentum, and self-angular momentum by Eqs. (18), (19), and (20), respectively. In the case when ($^*\boldsymbol{e}_i$, $^*\boldsymbol{h}_i$) represents the free field component which was radiated from the i-th particle, this self-factor 1/2 is not necessary, but this complexity is dexterously resolved in our Maxwell-Lorentz electromagnetism, as we see soon.

We get further

$$c\sum_{j\neq i}\nabla\cdot({}^*\boldsymbol{e}_j\times{}^*\boldsymbol{h}_i)+\sum_{j\neq i}\left({}^*\boldsymbol{e}_j\cdot\frac{\partial^*\boldsymbol{e}_i}{\partial t}+{}^*\boldsymbol{h}_i\cdot\frac{\partial^*\boldsymbol{h}_j}{\partial t}\right)+\sum_{j\neq i}{}^*\boldsymbol{e}_j\cdot{}^*\boldsymbol{j}_i=0, \tag{21}$$

$$c\nabla\cdot({}^*\boldsymbol{e}_i\times{}^*\boldsymbol{h}_i)+\frac{\partial}{\partial t}\left[({}^*\boldsymbol{e}_i)^2+({}^*\boldsymbol{h}_i)^2\right]/2+{}^*\boldsymbol{e}_i\cdot{}^*\boldsymbol{j}_i=0. \tag{22}$$

From the principle of work, we can assume

$$\iiint_{{}^*V_i}\left(\sum_{j\neq i}{}^*\boldsymbol{e}_j\right)\cdot{}^*\boldsymbol{j}_i\,\mathrm{d}V=\frac{\partial}{\partial t}{}^*[\mathrm{K.E.}]_i+\frac{\partial}{\partial t}{}^*[\mathrm{R.E.}]_i\,. \tag{23}$$

Here, $^*V_i$ is the volume where $^*\boldsymbol{j}_i\neq 0$,

$${}^*[\mathrm{K.E.}]_i=\frac{m_i c^2}{\sqrt{1-(v_i/c)^2}} \tag{24}$$

and $^*[\mathrm{R.E.}]_i$ indicates the electromagnetic energy being radiated or absorbed by the i-th particle. In order to represent the change in the self energy due to the transfer of electromagnetic energy by induction, we regard that m_i is not a constant[13]. But the change is so small that, in usual purpose, it can be neglected. We have to assume further

$$\iiint_V\frac{\partial}{\partial t}\left\{\frac{({}^*\boldsymbol{e}_i)^2+({}^*\boldsymbol{h}_i)^2}{2}\right\}\mathrm{d}V+\iint_S c\,{}^*\boldsymbol{e}_i\times{}^*\boldsymbol{h}_i\cdot\mathrm{d}\boldsymbol{S}=\frac{\partial}{\partial t}\left\{{}^*[\mathrm{K.E.}]_i+{}^*[\mathrm{R.E.}]_i\right\}, \tag{25}$$

where, the volume V and its surface S is about the size to include the i-th particle. From Eq. (22), if we have another volume V', which includes V, we have the time change equation of

$$\iiint_{V'-V} \frac{\partial}{\partial t}\left\{\frac{({}^*\boldsymbol{e}_i)^2+({}^*\boldsymbol{h}_i)^2}{2}\right\} \mathrm{d}V = -\iint_{S'-S} c^*\boldsymbol{e}_i \times {}^*\boldsymbol{h}_i \cdot \mathrm{d}\boldsymbol{S}. \tag{26}$$

Therefore, $c^*\boldsymbol{e}_i \times {}^*\boldsymbol{h}_i$, when integrated over the surface of a volume where ${}^*\boldsymbol{j}_i = 0$, indicates the flow of $\{({}^*\boldsymbol{e}_i)^2 + ({}^*\boldsymbol{h}_i)^2\}/2$, so that we need this term in Eq. (25). This fact, however, does not mean that $c^*\boldsymbol{e}_i \times {}^*\boldsymbol{h}_i$ can represent the energy flow. In the space where ${}^*\boldsymbol{j}_i = 0$, the energy flow ${}^*\boldsymbol{S}_i$ must be

$${}^*\boldsymbol{S}_i = c^*\boldsymbol{e}_i \times {}^*\boldsymbol{h}_i + \nabla \times \boldsymbol{f}, \tag{27}$$

but, in the space where ${}^*\boldsymbol{j}_i \neq 0$, even Eq. (27) has no guarantee of existence. Clear recognition of this fact is one of the cruxes of the new frame in electromagnetism.

From Eqs. (25), (22), (23) and (21), we get

$$\frac{\partial}{\partial t}\{{}^*[\mathrm{K.\,E.}]_i + {}^*[\mathrm{R.\,E.}]_i\} = -\iiint_V {}^*\boldsymbol{e}_i \cdot {}^*\boldsymbol{j}_i \mathrm{d}V = \iiint_V \Big(\sum_{j\neq i} {}^*\boldsymbol{e}_j\Big) \cdot {}^*\boldsymbol{j}_i \mathrm{d}V$$

$$= -\iiint_V \Big(\sum_{j\neq i} \nabla \cdot c^*\boldsymbol{e}_j \times {}^*\boldsymbol{h}_i$$

$$+ \sum_{j\neq i} \Big({}^*\boldsymbol{e}_j \cdot \frac{\partial^* \boldsymbol{e}_i}{\partial t} + {}^*\boldsymbol{h}_i \cdot \frac{\partial^* \boldsymbol{h}_j}{\partial t}\Big)\Big] \mathrm{d}V. \tag{28}$$

From Eqs. (21), (22) and (28), we may be able to assume

$$\nabla \cdot \{c\,(\sum_j {}^*\boldsymbol{e}_j) \times {}^*\boldsymbol{h}_i\} + \sum_j \Big({}^*\boldsymbol{e}_j \cdot \frac{\partial^* \boldsymbol{e}_i}{\partial t} + {}^*\boldsymbol{h}_i \cdot \frac{\partial^* \boldsymbol{h}_j}{\partial t}\Big) = -\sum_j {}^*\boldsymbol{e}_j \cdot {}^*\boldsymbol{j}_i = 0. \tag{29}$$

Now, Eq. (29) is correct where ${}^*\boldsymbol{j}_i = 0$. Our assumption is that it can hold where ${}^*\boldsymbol{j}_i \neq 0$, for the electrons. In VR, since the ${}^*\rho$ and ${}^*\boldsymbol{j}$ have no mass, the acceleration of ${}^*\rho$ and ${}^*\boldsymbol{j}$ due to the action of $\sum_{j\neq i} {}^*\boldsymbol{e}_j$ and $\sum_{j\neq i} {}^*\boldsymbol{h}_j$ may generate the reaction $\Delta {}^*\boldsymbol{e}_i$ and $\Delta {}^*\boldsymbol{h}_i$, which may just compensate the action. Since the original ${}^*\boldsymbol{e}_i$ and ${}^*\boldsymbol{h}_i$, which have Eq. (29), are so huge, that there is a reason to believe the presence of balance both in a usual mathematical meaning and general relativistically. Therefore,

$$ {}^*\rho_i \sum_j {}^*\boldsymbol{e}_j + \frac{{}^*\boldsymbol{j}_i \times \sum_j {}^*\boldsymbol{h}_j}{c} = 0. \tag{30} $$

Multiplying ${}^*j_i/{}^*\rho_i$ to Eq. (30), we see that Eq. (29) is derivable from Eq. (30).

Then, after summing over i, Eq. (29) can be regarded as the forth component ($\nu = 4$) equation of the energy-momentum density tensor equation of

$$ \frac{\partial^* T_{\mu\nu}}{\partial x_\mu} = 0. \tag{31} $$

The fact that $\partial^* T_{\mu 4}/\partial x_\mu = 0$ holds in any frames indicates that Eq. (31) must hold in general. The basic Lagrangian for obtaining the electromagnetic energy-momentum density tensor ${}^*T_{\mu\nu}$ is[14]

$$ {}^*\mathscr{L} = -\frac{1}{4}\left({}^*f_{\mu\nu}\right)^2, \tag{32} $$

$$ {}^*f_{\mu\nu} = \frac{\partial^* a_\nu}{\partial x_\mu} - \frac{\partial^* a_\mu}{\partial x_\nu} = \Sigma\, {}^*f^i_{\mu\nu}\,, \quad {}^*a_\mu = \sum_i {}^*a^i_\mu\,, \tag{33} $$

$$ \begin{aligned} {}^*T_{\mu\nu} &= {}^*f_{\mu\alpha}{}^*f_{\nu\alpha} - \frac{1}{4}\,\delta_{\mu\nu}\,{}^*f_{\alpha\beta}{}^2 \\ &= \left(\sum_i {}^*f^i_{\mu\alpha}\right)\left(\sum_j {}^*f^j_{\nu\alpha}\right) - \frac{1}{4}\,\delta_{\mu\nu}\left(\sum_i {}^*f^i_{\alpha\beta}\right)\left(\sum_j {}^*f^j_{\alpha\beta}\right). \end{aligned} \tag{34} $$

Here, the subscript, i, j are changed to superscript for convenience, and the standard symmetrization procedure[9] is requested for getting ${}^*T_{\mu\nu}$ from ${}^*\mathcal{L}$. It is easy to see that $\partial^* T_{\mu\nu}/\partial x_\mu = 0$ ($\nu = 1,2,3$) gives Eq. (30). But we should notice that, in order to get the Maxwell-Lorentz equations, we should add $(\sum_i {}^*j^i_\mu)\,(\sum_k {}^*a^k_\mu)/c$ to ${}^*\mathcal{L}$, which will spoil the expression of ${}^*T_{\mu\nu}$ of Eq. (34), if simply followed the standard procedure of the field theory[15]. So far as we use the new momentum-energy density four vector of Eq. (17) for VR, the frame of the present field theory can work only partly.

The crux of these analyses is that, although Eqs. (29) – (34) do not differentiate ${}^*\boldsymbol{e}_i \times {}^*\boldsymbol{h}_i$ and ${}^*\boldsymbol{e}_i \times {}^*\boldsymbol{h}_j$, we must differentiate them clearly, because, otherwise, we can not get a consistent results in Eqs. (17), (19), (28) and (31). The structure must remain when we shift from (${}^*\boldsymbol{e}_i$, ${}^*\boldsymbol{h}_i$) to ($\boldsymbol{e}_i$, $\boldsymbol{h}_i$). Therefore, in Eq. (12), we must sum with $j \neq i$, and we must understand the energy relation in the Maxwell electromagnetism of Eq. (15) as the average of Eq. (12).

It is noted that the Maxwell equations are passive equations and ${}^*\rho_i(r_i, t)$ and ${}^*j_i(r_i, t)$ do not determine ($*e_i$, $*h_i$) precisely. We should notice the fact that even a classical hydrogen atom in which a point charge electron is making a closed orbital motion can be stationary, when the electromagnetic potentials are the average of the retarded and advanced potentials of the motion. In this sense, (e_i, h_i) should be determined according to the expectation of quantum theory in each specific case. It will be added in advance that, in the new frame[1)], there is a quantum reason that the emission or the absorption of an electromagnetic wave is made in average, following after the (e_i, h_i) of our Maxwell-Lorentz equations.

It is important for the electromagnetism to confirm the existence of perfectly consistent mathematical frame, in which no mathematical inconsistency is present. We believe that our frame can be such a frame general relativistically. Although, in the frame of special relativity, we still have a certain ambiguities in the system, such as in the ratio of $1:10^{-365}$, the ambiguities are superficial and can be neglected by postulating a certain adjusting function which can be effective only in such anomalous regions.

It is easy to prove that, in the existing classical theory of electromagnetism with charged particles, no consistent energy-momentum density tensor, with no renormalization difficulty, had been presented. The logics presented by Landau and Lifshitz[9)] is only an elegant tautology, because they did not solve the Poincarè paradox[5)].

§6. A Few Additionals

In addition to the short explanation given in Appendix B, we have derived in this paper already that the Zeeman energy is a kind of effective Hamiltonian of the total system, including the source of the external magnetic field.

It will be noted in advance that the extension of the new frame of classical electromagnetism has enabled to derive the c-number form of the Dirac Hamiltonian in Pauli's approximation entirely classically[13)]. In the derivation procedure, as in the case of the Zeeman energy, transfers of electromagnetic energy by induction play a crucial role; the phenomena are entirely implicit in Q.E.D. In a sense, we can regard the γ-matrix in Q.E.D. as a mathematical device which has replaced this important electromagnetic phenomena in classical physics.

The dissipation of energy in the macroscopic physics is a transfer of energy from the macroscopic system to the unspecified microscopic system. In the Maxwell-Lorentz world, however, there is no further unspecified super-microscopic system. Therefore, in the new frame, in conformity with the quantum theory, unspecified dissipation of energy is not emphasized in the Maxwell-Lorentz world.

Our understanding on the Maxwell tensor and the electromagnetic forces will be shown in Appendix C.

Appendix A. The classical size of the electron

Although the electron is definitely a quantal existence, in the new frame of physics, in order to get the analytical continuation between quantal and classical physics, a requirement exists to find out the best self-consistent classical representation of the electron, which can be used as the basic element in the classical frame of the new physics, especially in its Maxwell-Lorentz electromagnetism. It has turned out that our VR model satisfies this requirement. With VR, we have to accept that the classical size of the electron is in the range of the Compton wave length, which, in terms of the radius, be $g\lambda\!\!\!^{-}_c = g\hbar/mc \sim 10^{-2}$ Å $= 10^{-12}$m. We know that there is an old concept, in which the electron is assumed to be a point, less than 10^{-15}m, and the enormous electrostatic self-energy associated with this point charge has been just, without reason, disregarded. This concept can not be used in the new frame, because we look for a strictly selfconsistent electromagnetism and, in this old representation, the problem of the electrostatic selfenergy itself affords definite selfinconsistency. Therefore, the only choice for the new frame is VR, and, the question is whether VR can represent the classical electron adequately or not.

Let us compare the old and new concepts. For the electron, the minimum Heisenberg uncertainty in the location, Δx, and the de Broglie wave length, λ_B, in the proper frame are

$$\Delta x \sim \frac{\hbar}{mc} = \lambda\!\!\!^{-}_C, \quad \lambda_B = \frac{h}{mc} = \lambda_C \ , \tag{A1}$$

i.e., its two Compton wave lengthes. Therefore, in the old concept, the point electron is assumed to make an iteneration in the range of the Compton wave length, being called the "Zitterbewegung", and the intrinsic spin magnetic moment of the electron is ascribed to the rotational orbital motion of this Zitterbewegung, leaving the g-factor problem (Why $g = 2$) unsolved. In the new concept, the electron itself is a persistent current, having the radial extension of $g\lambda\!\!\!^{-}_c \sim 10^{-2}$ Å and the spin angular momentum, $\hbar$, as its intrinsic virtue of the model. Since the model has $g = 2(1 + \alpha/2\pi)$, the accuracy of the model is in the range of 10^{-6}. It is noted that, since the electric charge has relativistic invariance, although the ring charge is assumed to rotate with the speed of light, c, (being identical to the velocity of the Zitterbewegung in Q.E.D.) this order comes about decisively as the minimum size, being supported also by the Q.E.D. through its Darwin term[16].

Now since Eqs. (A1) exist, the question of whether a person takes the new or old model in his brain may be a matter of taste, but, in the new frame, we have to take VR model, because only this model gives a classical electron with its all non-wave virtues, enabling to construct a consistent Maxwell-Lorentz electromagnetism, and to establish an analytical continuation between quantal and classical physics[1].

It is noted that, different from the α-particle, no classically explainable quantitative Rutherford scattering data exists for the electron. Although the quantum mechanical calculation of the scattering of the electron by an idealized electrostatic Coulomb potential gives a formulus whose leading term is identical to the Rutherford classical scattering formulus[17], no such idealized Coulomb potential is available in nature for the electron scattering, so that, the observed scattering data of the electron by materials are rather utilized quantally to determine the electric charge distribution of the nuclei in the target[18]. The electron beams of less than 10 MeV give only the electron diffraction by the target, and the beam of very high energy, having the velocity of light, e.g., 0.5 BeV[18),19], still gives diffraction by the nuclei of the target, since its de Broglie wave length of 2.47×10^{-15}m is yet in the range of the size of the nuclei.

It is further noted that although the high energy electron-positron or electron-electron collision experiments have given the cross sectional data of the electron, from which the size is said to be less than 10^{-17}m, the experiments are of essentially quantum mechanical, and, although the theory had assumed the point charge electron, the divergence problem was left unsolved, and, under the allowance of the super-position and Pauli's exclusion principles, the obtained cross sectional data do not necessarily be related directly to the classical size of the electron. Classically, the two particles in these experiments, and also two VR's, may behave like as two electromagnetic solitons, which can penetrate or overlap mutually, without introducing any particle reactions. In this case, therefore, the word "size" may be the replacement of the probability of the quantum mechanical reactions, for which, the classical frame has nothing to do. Of course, in the new frame, we are mostly interested in the physics of materials, in which the relevant energies are very low, and electrons are regarded eternal. In conclusion, we state that no classically explainable Rutherford type simple data, which appoints for the size of the electron to be less than 10^{-2}Å, has been present.

The charge distribution range of 10^{-2}Å, given by VR model can not only describe most of the classical properties of the electron but also describe the hyperfine field to the nucleus precisely[2]. The instability of high Z number nuclei, which is known to be partly due to the capturing of their $1s$ electrons, might also be explained by the fact that the mean radii of $1s$ orbital in these nuclei approach to the range of 10^{-2}Å.

Appendix B[6]. A Derivation of the Zeeman Energy

Let us assume two persistent current circuits C_1 and C_2; C_1 being learge and C_2 being small and located in the center of C_1. We assume further that C_1 and C_2 keep fluxes Φ_1 and Φ_2, respectively, and C_2 is regarded as a magnetic moment $\boldsymbol{\mu}_2$ located in a magnetic field, $\boldsymbol{H}_1$, from C_1. Then, the total magnetic energy of the system, $C_1 + C_2$, is

$$U_{\mathrm{m}} = \frac{1}{2} L_{11} I_1^2 + L_{12} I_1 I_2 + \frac{1}{2} L_{22} I_2^2 . \tag{B1}$$

Here I_1 and I_2 are the total currents in C_1 and C_2, and L_{11}, L_{22}, and L_{12} are the averaged self- and mutual-inductances. We have for the magnetic fluxes, Φ_1 and Φ_2, such as,

$$\phi_1 = c\,(L_{11} I_1 + L_{12} I_2), \tag{B2}$$

so that, if we changed the mutual configurations of C_1 and C_2 slowly, we get

$$\begin{aligned} \delta U_{\mathrm{m}} &= L_{11} I_1 \delta I_1 + L_{12} I_2 \delta I_1 + I_1 I_2 \delta L_{12} + L_{12} I_1 \delta I_2 + L_{22} I_2 \delta I_2 \\ &= I_1 \frac{\delta \Phi_1}{c} + I_2 \frac{\delta \Phi_2}{c} - I_1 I_2 \delta L_{12} \\ &= -\delta G_1 - \delta G_2 + \delta^* \left(-\boldsymbol{\mu}_2 \cdot \boldsymbol{H}_{21}\right) . \end{aligned} \tag{B3}$$

Here, G_1 or G_2 is a nonmagnetic energy of C_1 or C_2 which couples to the persistent current I_1 or I_2 inseparably, e.g., when C_1 or C_2 is a superconducting circuit, it is the kinetic energy of the drift component of the superconducting electrons. $\boldsymbol{H}_{21}$ is the $\boldsymbol{H}_1$ at $\boldsymbol{\mu}_2$, and δ^* indicates the variation with respect to L_{12}, or, due to the change in the mutual configuration between $\boldsymbol{\mu}_2$ and $\boldsymbol{H}_1$. Therefore, we get

$$\delta^* \left(-\boldsymbol{\mu}_2 \cdot \boldsymbol{H}_{21}\right) = \delta \left[U_{\mathrm{m}} + G_1 + G_2 \right] . \tag{B4}$$

More rigorous treatments by the same logics will be seen in Ref. 20).

Hence, the Zeeman energy expression is a kind of effective Hamiltonian of the total system. In Eq. (B4), we have disregarded a few secondary processes, because they are usually very small. For instance, if $\boldsymbol{\mu}_2$ has been rotated quickly, a free electromagnetic radiation, δU_{R}, starts outwards from the location of μ_2, together with the electromagnetic action, which should realize δG_1, but the former is not large. A detailed analytical explanation of these processes in terms of electromagnetism will be seen in another paper[13].

It will be noted that, by Q.E.D., there is no doubt that the magnetic moment of the electron is due to the electric current[8].

Appendix C. The Maxwell Tensor and Electromagnetic Forces[21]

We know that, quantum mechanically, the Pauli principle requests for all the electrons to be indistinguishable, so that $\rho(r, t)$, rather than $\sum_i \rho_i(r, t)$, may be more closer to the truth. In this modified Maxwell-Lorentz world of Eqs. (5) and (6), since each single electron spreads over a large macroscopic volume, $e_i \ll e$, or, it is not necessary to distinguish $\sum_{j\neq i} e_j$ from e, but, at the same time, the kinetic energy of electron i, $m_i c^2/\sqrt{1-(v_i/c)^2}$, must be considered as an entity which is independent from e.

We define the Maxwell-Lorentz tensor $t_{\alpha\beta}$ as

$$t_{\alpha\beta} = e_\alpha e_\beta - \frac{1}{2}\delta_{\alpha\beta}\sum_\lambda e_\lambda^2 + h_\alpha h_\beta - \frac{1}{2}\delta_{\alpha\beta}\sum_\lambda h_\lambda^2 \qquad (\alpha, \beta = 1, 2, 3) \tag{C1}$$

From Eq. (31), we see that $t_{\alpha\beta}$ is the spacial part of $T_{\alpha\beta}$, such as $t_{\alpha\beta} = -T_{\alpha\beta}$. Then we get an identity[3] of

$$\iiint_{V^{\mathrm{Mat}}} \left(\rho \boldsymbol{e} + \frac{\rho \boldsymbol{v}}{c} \times \boldsymbol{h}\right) \mathrm{d}V + \iiint_{V^{\mathrm{Mat}}} \frac{\partial}{\partial t}\left(\frac{\boldsymbol{e}\times \boldsymbol{h}}{c}\right) \mathrm{d}V = \iiint_{S^{\mathrm{Mat}}} t \cdot \mathrm{d}\boldsymbol{S} , \tag{C2}$$

where, V^{Mat} has the surface S^{Mat}, which is so constructed as to not cut the microscopic persistent current $\boldsymbol{I}$ or electric dipole moment $\boldsymbol{p}$. The first integral represents the total sum of the Lorentz force inside the S^{Mat}, which must be identical to the rate of change of the total mechanical momentum, $\boldsymbol{P}^{\mathrm{K}}$,

$$\frac{\mathrm{d}\boldsymbol{P}^{\mathrm{K}}}{\mathrm{d}t} = \sum_{i \text{ in } V^{\mathrm{Mat}}} \frac{\mathrm{d}\boldsymbol{p}_i^{\mathrm{K}}}{\mathrm{d}t} , \tag{C3}$$

and, from Eq. (28), the second integral represents the total sum of the rate of change of the electromagnetic momentum, $\boldsymbol{P}^{\mathrm{e.m.}}$,

$$\frac{\partial}{\partial t}\Big(\iiint_{V^{\rm Mat}} \sum_{\substack{i=j=0\\ \&\\ j\neq i\neq 0}} \frac{\boldsymbol{e}_i\times\boldsymbol{h}_j}{c}\,\mathrm{d}V\Big)=\frac{\partial}{\partial t}\Big(\iiint_{V^{\rm Mat}}\Big\{\sum_{j\neq i\neq 0}\frac{1}{c}\rho_i\boldsymbol{a}_j$$

$$+\sum_{\substack{i=j=0\\ \&\\ j\neq i\neq 0}}\Big[\nabla\phi_i\frac{\partial\phi_j}{c\,\partial t}-\nabla\boldsymbol{a}_j\cdot\frac{\partial\boldsymbol{a}_i}{c\,\partial t}\Big]\Big\}\,\mathrm{d}V\Big). \qquad \text{(C4)}$$

The first term of the right side of Eq. (C4) is the rate of change of the electromagnetic momenta due to the presence of charge, ρ_i, in a vector potential, $\boldsymbol{a}_j$, and, the second and third terms will represent those of the electromagnetic momenta of the free radiations as well as the transient corrections to the first term. Therefore, if the surface integral of Eq. (C2) is not zero, this term should represent the total action of the electromagnetic force at the surface $S^{\rm Mat}$.

In classical mechanics, we are usually interested in the mechanical momentum. Then we get

$$\frac{\mathrm{d}\boldsymbol{P}^{\rm K}}{\mathrm{d}t}=\sum_i \boldsymbol{f}_i=\iint_{S^{\rm Mat}} t\cdot\mathrm{d}\boldsymbol{S}-\iiint_{V^{\rm Mat}}\frac{\partial}{\partial t}\Big(\frac{\boldsymbol{e}\times\boldsymbol{h}}{c}\Big)\,\mathrm{d}V. \qquad \text{(C5)}$$

When we drill a very thin but macroscopic shell volume $V^{\rm S}$ with $S^{\rm Mat}$ inside, the $\boldsymbol{e}$ and $\boldsymbol{h}$ fields, $\boldsymbol{e}^{\rm S}$ and $\boldsymbol{h}^{\rm S}$ in $V^{\rm S}$, will be $\boldsymbol{e}^{\rm S}=\boldsymbol{E}^{\rm S}=\boldsymbol{E}+P_n\boldsymbol{n}$ and $\boldsymbol{h}^{\rm S}=\boldsymbol{H}^{\rm S}=\boldsymbol{H}+M_n\boldsymbol{n}$, where $\boldsymbol{n}$ is the outwards directed unit normal vector on $S^{\rm Mat}$. Then, from Eqs. (C1), (C2) and (C5), the total Maxwell-Lorentz electromagnetic force acting on the material inside of $V^{\rm S}$ or $S^{\rm Mat}$ is

$$\boldsymbol{F}=\iint_{S^{\rm Mat}}\Big[\boldsymbol{E}^{\rm S}\boldsymbol{E}^{\rm S}-\frac{1}{2}(\boldsymbol{E}^{\rm S})^2\boldsymbol{I}+\boldsymbol{H}^{\rm S}\boldsymbol{H}^{\rm S}-\frac{1}{2}(\boldsymbol{H}^{\rm S})^2\boldsymbol{I}\Big]\cdot\mathrm{d}\boldsymbol{S}$$

$$-\iiint_{V^{\rm Mat}}\frac{\partial}{\partial t}\frac{\boldsymbol{E}\times\boldsymbol{B}}{c}\,\mathrm{d}V-\iiint_{V}\frac{\partial}{\partial t}\frac{\boldsymbol{e}'\times\boldsymbol{h}'}{c}\,\mathrm{d}V, \qquad \text{(C6)}$$

which can be transformed into[21]

$$\boldsymbol{F}=\iiint_{V^{\rm Mat}}\Big[\rho\boldsymbol{E}+(\boldsymbol{P}\cdot\nabla)\boldsymbol{E}+\nabla^{\rm H}(\boldsymbol{M}\cdot\boldsymbol{H})+\frac{1}{c}\Big(\boldsymbol{j}+\frac{\partial\boldsymbol{P}}{\partial t}\Big)\times\boldsymbol{B}\Big]\,\mathrm{d}V$$

$$-\iiint_{V^{\rm Mat}}\frac{\partial}{\partial t}\Big(\frac{\boldsymbol{e}'\times\boldsymbol{h}'}{c}\Big)\,\mathrm{d}V+\iint_{S}\frac{P_n^{\,2}+M_n^2}{2}\,\mathrm{d}\boldsymbol{S}, \qquad \text{(C7)}$$

where $\boldsymbol{e}'$ and $\boldsymbol{h}'$ are the ripple fields. A few different but equivalent representations are possible for Eq. (C7).[21] The last term of Eq. (C7) represents the inversed surface tension term which comes from the introduction of artificial drilling of V^{S}, and is not present in the normal situation. A typical example of the action of this term will be seen in the thorny surface of ferromagnetic liquid in a magnetic field. ∇^{H} indicates that the operation will be only on $\boldsymbol{H}(r, t)$. It is noted that, in this expression, every term has a simple explanation and the main component of the change in the electromagnetic momentum, i.e., the middle term of Eq. (C6), has just been cancelled out.

We know that there are different opinions,[22] being not supported by the new frame of physics.

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訂正

物性研究 42 巻 2 号 (1984 年 5 月号) pp. 160 − 203

Errata

	Line	Original	Corrected
p.161	−1	Syuichi	Shuichi
p.165	+6	$\ell_2 = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$	$\ell_z = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$
	+10	$p_r = \frac{\hbar}{i}\frac{1}{r}\frac{\partial}{\partial r}$	$p_r = \frac{\hbar}{i}\frac{1}{r}\frac{\partial}{\partial r}r$
p.168	−1	(3-36)	(3-41)
p.169	+6		
p.170	+16		
	+17		