（注２）精度Nビットの計算機上の有限状態力学系をT_Nとしたとき，カオスアトラクタをもつ力学系f_aには，T_Nの無限個の族$\mathscr{J}_a = \{T_{a,N}\}_{N=1,2,\cdots}$が対応する。ここで『計算機のふるまい』といっているのはより厳密には族$\mathscr{J}_a$のふるまいの意味である。

（注３）考えている計算機の精度が大きくなると，このような極限周期軌道は一般には複数個存在する。それら全ての合併が極大不変集合となる。

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３次元トーラス上のロッキングとカオス

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Our objective has been to obtain a global physical picture of dynamical phenomenon on 3-torus and to understand the role of resonances and lockings in the occurrence of irregular dynamics.

Multiple phase dynamics are represented as

$$\frac{d\phi_j}{dt} = \omega_j + \varepsilon\, g_j(\underset{\sim}{\phi})$$

where g_j are coupling functions periodic in each ϕ_i. Although phase locking behaviour of two coupled phases motions is a familiar phenomenon, features of multiple coupled phase dynamics are less well understood. For three or more coupled phases there is complicated multiple locking and irregular dynamics are also possible. Some significant rigorous results are known [1, 2, 3]. However these fall short of describing even qualitatively the full global picture, and their physical implications for typical experimental results (e.g. features of power spectra as one parameter is varied [4, 5]) have been unclear — although the comprehensive theory and numerical studies of quasi-periodic motion and locking on 2-torus (circle map [1, 11]) and the role of resonance overlap in the breakup of 2-torus (contracting annulus map [9], non-invertible circle map [10]) provide some intuitive guide.

Using the 2-torus diffeomorphism

$$\theta_j^{\,n} = F_j(\underset{\sim}{\theta}^{n-1}) \equiv \theta_j^{\,n-1} + \rho_j^{\,0} + \varepsilon\, g_j(\underset{\sim}{\theta}^{n-1})$$

where j=1, 2 and coupling functions g_j are periodic in each θ_i, we have shown [12] the following important features of flow on 3-torus.

(1) A typical one parameter sweep is a complicated sequence of regular and irregular states resulting from the 3-way "pushing" and "pulling".

(2) The sequence can be traced in a space of rotation numbers $\underset{\sim}{\rho}=(\rho_1, \rho_2)$ where lockings occur to a "devil's mesh" of locking lines $n\rho_1+m\rho_2+k=0$ (n, m, k integers).

(3) Irregular dynamics can be seen as competition between lockings to different locking lines; in particular chaos [6, 7, 8] may occur where double-locking domains overlap, and non-chaotic strange dynamics [13] may occur because of locking frustration.

(4) Features of irregular dynamics reflect the lockings involved.

(5) Visibility of chaos roughly scales with the visibility of neighbouring lockings.

To characterize the dynamical states we use in addition to Lyapunov exponents, a multi-dimensional rotation number ρ defined as

$$\rho_j(\underset{\sim}{\theta}^0) = \lim_{N\to\infty} N^{-1}(F_j^{\,N}(\underset{\sim}{\theta}^0) - \theta_j^{\,0}).$$

Although the rotation number is a useful analytic tool and observable, the strong properties of existence, uniqueness and one-to-one correspondence with dynamical type (irrational $\leftrightarrow$ ergodic, rational $\leftrightarrow$ periodic) which hold for the circle diffeomorphism do not necessarily hold in the multi-dimensional case and it must be used with care.

The combination lines $n\rho_1+m\rho_2+k=0$ (n, m, k integers) form a dense web in the ρ space. For zero coupling, $\varepsilon=0$, they correspond to the lines $n\rho_1^0+m\rho_2^0+k=0$ in the $\underset{\sim}{\rho}^0$ space. For finite coupling they can indicate resonances which have resulted in locking. In general as coupling strength increases locking domains of this dense set of lines shift and increase in measure — we refer then to a "devil's mesh" of locking lines. We refer to locking to a combination as a single locking to the mesh and, in particular, to locking to a rational pair of ρ_1, ρ_2 as a double locking to an intersection point of the mesh. In general, a smooth parameter traversal of the mesh will result in a non-smooth, non-monotonic change in $\underset{\sim}{\rho}$ (though in the perhaps non-generic case of the absence of irregular dynamics we expect the variation in ρ to be continuous). The overlap of these locking domains can result in locking frustration and chaos.

We consider resonance domains in parameter space $(\varepsilon,\underset{\sim}{\rho}^0)$, (a generalization of Arnold's "tongues") in terms of conditions that there exist θ^0 such that $G^{N,k}_{n,m}(\varepsilon,\underset{\sim}{\rho}^0,\underset{\sim}{\theta}^0)=0$, where $G^{N,k}_{n,m}(\varepsilon,\underset{\sim}{\rho}^0,\underset{\sim}{\theta}^0)$ $\equiv nF_1+mF_2-n\theta_1^0-m\theta_2^0-k$.

In particular, the condition for a period N fixed point with $\rho_1=p_1/q_1$, $\rho_2=p_2/q_2$, ($N=\mathrm{l.c.m.}(q_1,q_2)$):

$$G^{N,N\rho_1}_{1,0} = G^{N,N\rho_2}_{0,1} = 0$$

defines a double locking set whose projection onto parameter space gives the double locking domain. Unlike the circle map where the orbits are unique, the possibility of coexistence of fixed points allows crossing points and cusps in single domain boundaries and the overlap of domains for fixed points of different periods.

If chaotic dynamics are associated with non-degenerate homoclinic or heteroclinic intersection then a dimensional argument shows that, on N-torus, at least two lockings are necessary. Hence chaos in a two-torus map may be associated with the coexistence of fixed points, and the overlap of double-locking domains. We expect that when two double-locking domains overlap, an infinite number of other locking domains squeezed between them also overlap. Numerical evidence suggests features of chaos (orbit, range of coexistence of ρ, magnitude of λ) reflect the neighbourhood of lockings. This strongly suggests that the global ordering of locking domains is in some sense preserved. The proof of this remains as an important challenge.

The connection between our results and the Ruelle-Takens-Newhouse idea of chaos arbitrarily close to unlocked states is seen in the perturbational regime of the $(\varepsilon,\underset{\sim}{\rho}^{0})$ parameter space.

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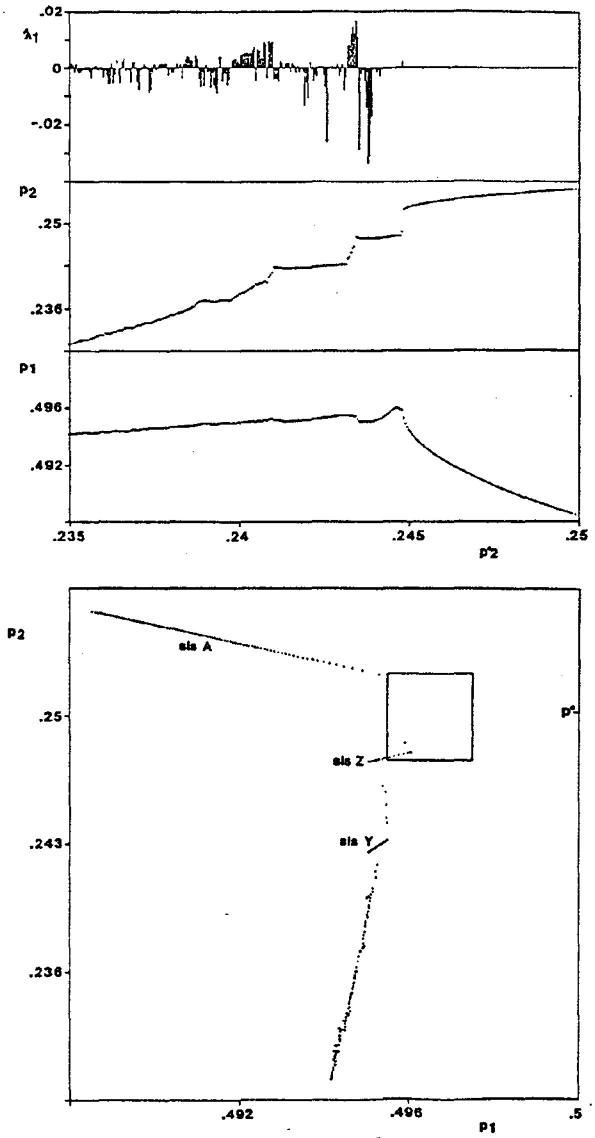


Figure 1

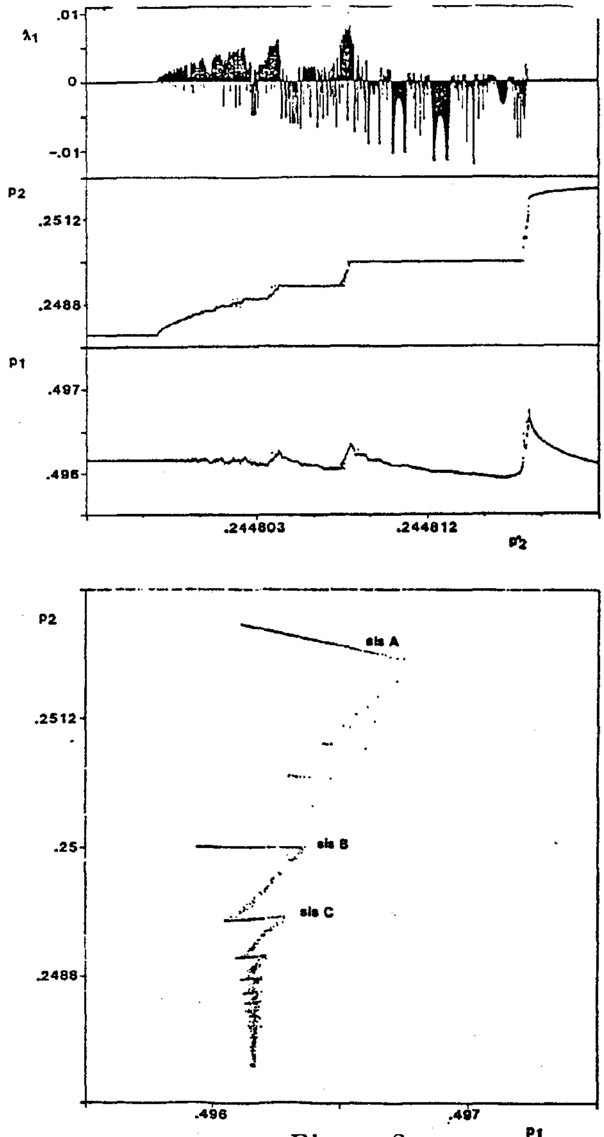


Figure 2

Figure 1	A typical one parameter scenario on 3-torus.
Figure 1a	Maximum Lyapunov exponent and rotation numbers ρ_1 and ρ_2 for a fine scale sequence of values of parameter $\rho_2^{\,0}$ near $\underset{\sim}{\rho}^*=(1/2,\ 1/4)$.
Figure 1b	ρ_1 versus ρ_2 for the sequence of Fig. 1a.
Figures 2a,2b	A subsequence of Fig. 1 corresponding to the insert region of Fig. 1b.

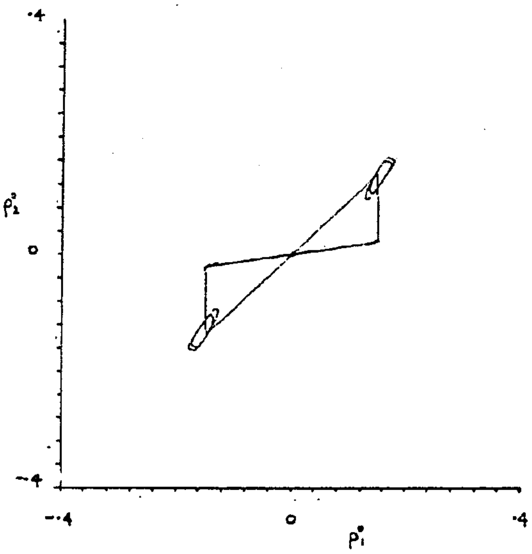
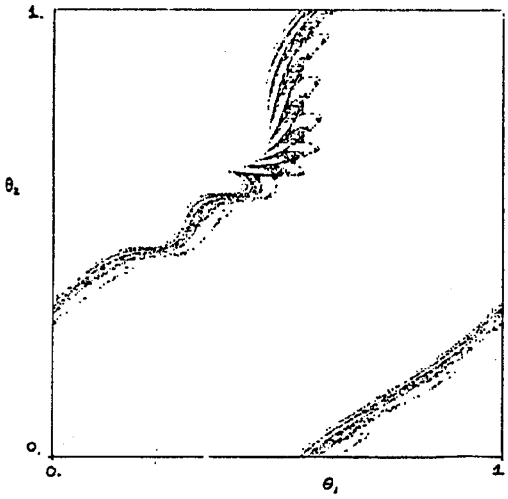


Fig. 3



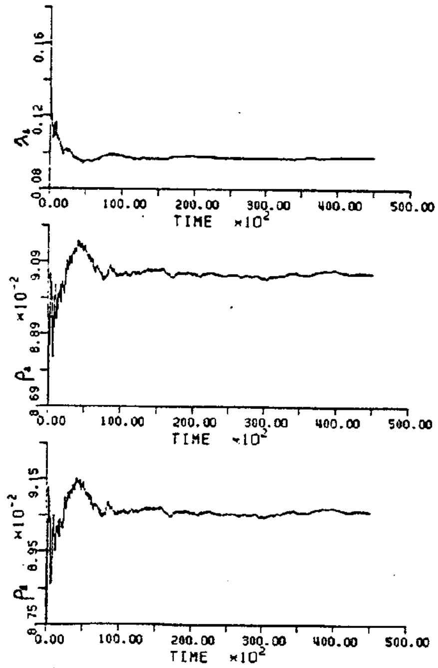


Fig. 4

Figure 3 Domains of existence of low order periodic orbits of 3 different periods: $\rho=(0,0)$, $\rho=(1/10,\ 1/10)$, $\rho=(1/9,\ 1/9)$ on the single locking line $\rho_2=\rho_1$.

Figure 4 Chaos of "type 1" (on low order single locking line) in region of overlap of domains in Fig. 3.