36. BIOLOGICAL PATTERN FORMATION: RESEARCH MANUAL 1.

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It is an important theme to find an underlying law of pattern formation in living system. Our aim lies in answering question whether the law is similar to or quite different from physical laws. If they are similar, there can be an analogy between the both. Further methods in non-equilibrium physics will be available to the study and it is, however, necessary to understand the limitation in biological system. Here we present *Horii's scheme* for the study of biological pattern formation with one-dimension as an example of dental arch in human mandible.

(1) Coarse-graining in complicate pattern of organ.

Patterns of living organs are so complicate and so incomprehensive that the problem has been shelved for a long time. First, we describe the method how to simplify the pattern by extraction of only characteristic feature; so-called, coarse-graining of information of pattern. A longitudinal axis of tooth is represented by a Vector which is directed to the root apex or to the mid-point between the root apecies from the crown center. An axial Vector is denoted by t_j (j: a tooth number from the center to the lateral direction in dental arch sequentially). θ_{ij} denotes an angle of one Vector t_j with another t_i, which is positive in the Vectoral direction. Right handed coordinates (e_1, e_2, e_3) are available to express the position of the Vectors, which can be fixed anywhere in dental arch. In order to investigate the tooth location on either one side, we fix the coordinates at the position of Vector t_6 (at the center of 1st molar crown); e_2-axis is fixed on Vector t_6. The e_1-axis is orthogonal with t_6 in the mesial direction and e_3 is with them in the lingual direction on the right side of arch. The position of Vector t_j is represented by Vector q_{6j} on the coordinates and the summit of t_j is by Vector r_{6j}. The vectors are expressed by

$$\begin{aligned} t_j &= T_{j1}e_1 + T_{j2}e_2 + T_{j3}e_3 \\ q_{6j} &= Q_{j1}e_1 + Q_{j2}e_2 + Q_{j3}e_3 \\ r_{6j} &= R_{j1}e_1 + R_{j2}e_2 + R_{j3}e_3 \end{aligned} \qquad [1]$$

$t_6 = T_{62}e_2$ has been defined and hence it is $T_{61} = T_{63} = 0$ ($e_1 \neq 0$, $e_2 \neq 0$).

For quantitative observation of dental arch, we will report the cephalometric studies in the

two-dimensional projection (1, 2).

(2) Isolation of an observable system.

In some system the behavior of the whole can indeed the sum of its parts. With no specificity in observed region, we can expect that the theory in the whole system will be directed from the results of partial observation. We can propose a model of the maturation of entire dental arch from the results of the locational mode and the behavors of eight lateral teeth (1, 2).

(3) A partial observable process.

DNA is an element to determine states of organ; a fertilized egg corresponds to the absolutely initial state and matured organs do to the final. Development can be simply expressed by a transition of state. In general, transition from one state to another is explained by the theory of dynamical system owing to Newtonian dynamics. However differentiation that gives rise to internal changes in organ would be unlikely to differ from a continuous flow of state transition. Therefore an investigation of development should be limited a process between two fixed states. The whole process can be understood by breaking it down and studying each piece. We choose the process until the maturation of dental arch from a fixed state.

(4) A continuous time-flow.

Physical time t should be replaced by another parameter representing a continuous and constant flow of developmental process because organ development takes place in leaps and bounds. For the purpose we use the component P_{j2} of Vector p_{6j} representing the germ-position t_j.

$$p_{6j} = P_{j1}e_1 + P_{j2}e_2$$

where a value of the component P_{j2} expresses the developmental stage of tooth or its eruptive stage (2).

(5) Determinism and stochastic description.

Determinism is described when the final state is predicted with high precision by the initial values of variables in relevant system. Otherwise the stochastic description is necessary. This concept is crucial in discussing the process of pattern formation. A rich variety of pattern is expressed in living organ, which is derived from a large degree of freedom in the forming process. For observation of such a phenomenon many variables should be projected onto a couple of contrable ones in

relevant system. The selection and the classification of informations are substantial. It is explicit that development is controlled by genetic information but that the process is susceptible to environmental factors. The former is the systematic force and the latter is random variables that generate unpredictable behaviors. The flexibility being adaptable to environmental changes is substantial in living system. Synergentics own to instability and fluctuations in relevant system that cause a temporal evolution (3, 4). Therefore the pattern formation can be described in stochastic methods; a master equation and stochastic differential equations. A change of one variable in complicate and complex system is sorted into two components; that is, the influence of a cluster of constituents in the system as a whole (deterministic) and the other influences of their local neighbourhood (stochastic). Tooth development similarly owns to two kinds of forces. One is generated by mandiblar differentiation itself and the other is caused by the metabolism around the tooth and also by the influence of the local neighbouring teeth. Langevin equation is available in such a system and hence we formulate it in general formula with two cooperative terms;

$$\frac{\partial}{\partial t} x = +\alpha x(t) + \beta x^2(t) - \gamma x^3(t) + R(t) \qquad [2]$$

$$(\alpha, \beta, \gamma > 0)$$

where x denotes an order-parameter and $R(t)$ is random forces in color noise, presumably. When $\beta = 0$, we can obtain the mean value $P(x, t)$ by Suzuki's method. Thus a biological system is cooperative, so that the order-parameter will make an universal temporal evolution (4, 5, 6). The potential will be asymmetric in organ development (2).

(6) An ensemble in system vs. cross-section.

Generally speaking, the most crucial distinction in observation method between physics and biology is an ensemble in the former vs. a cross-section as usual in the latter. Any ensemble in physical system is composed of a numerious number of components. The number is so large as 10^{23} that $R(t)$ is white noise in ideal system. On the contrary, biological system is used to be observed in cross-section. Certainly, a large scale of examination in cross-section would yeild a certain amount of information that provides a discrete pattern of Gaussian distribution. A system of results that is a series of Gaussian distribution patterns on individual stages could provide us an information of the variable associated with development as a Gaussian process. The information can be corresponding to the longitudinal results in time-series of observation. On the other words, with observation made over a small but finite interval of development a discrete Gaussian process will

be given in cross-sectional study, so that we can formulate it to a mathematical relation by the continuum approximation. The relation can describe how the matured state is generated. We should remark that biological organ is thus a cooperative system with asymmetric potential where a pattern is formed with a rich variety owing to color noise. On the other hand, when it is a longitudinal study, the diffusion term $\nabla^2 x$ expressing the coupling of order-parameter in space should be taken into consideration.

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