研究会報告

DYNAMICAL EVOLUTION OF SELF-GRAVITATING SYSTEMS

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1. Introduction

The aim of this paper is to present astrophysical phenomena which may be interpreteted in terms of nonlinear-nonequilibrium physics. I do not attempt to use the terminologies and concepts used in nonlinear-nonequilibrium physics rather I would like to ask people in this field to interpret in their terminologies and concepts and to find the way to develop the research.

Stellar systems have some similar properties to electrostatic plasms. Both obey inverse square force law. Therefore the same kinetic equations, e.g. the Klimontovich equation, BBGKY equations, collisionless Boltzmann equation, the Fokker-Planck equation, etc., are available. On the other hand, all the force in stellar systems is attractive. This yields no charge neutrality and no Debye shielding. The Jeans length in stellar systems corresponds to the Debye length in electrostatic plasmas in some sense. In most stellar systems the radius of the system is of the order of the Jeans length. Therefore stellar systems should be considered intrinsically inhomogeneous.

Most of the discussion here is devoted to the evolution of globular clusters. Section 2 explains gravothermal instability. Section 3 gives the evolution after gravothermal instability. Section 4 explains the roles of binaries. Section 5 explains a self-similar evolution after core-collapse. Finally section 6 discusses gravothermal oscillations.

2. Gravothermal instability.

Gravothermal instability is the main motive force of the evolution of globular clusters. In self-gravitating systems some phenomena which do not occur in terrestrial laboratories are commonly occur. Gravothermal instability is one of these good examples. In laboratories is we get in touch hot material and cold material, heat flows from hot material to cold material and finally the temperature of both materials becomes the same.

In self-gravitating systems the virial theorem

$$2T + W = E + T = 0 \qquad (2.1)$$

holds, which means

$$\Delta T = -\Delta E, \tag{2.2}$$

that is if heat is removed from the system, the temperature of the system increases. Therefore if the central part of the system is hotter than the outer part of the system as shown in Figure 1, heat flows from the inner part to the outer part and the inner part gets hotter and the outer part gets colder. Thus more heat flows from the inner part to the outer part and the temperature difference between the inner part and the outer part becomes larger and larger.

The reason the temperature increases in spite of losing heat is that the system shrinks if it loses heat. Thus if the phenomena occurs once, the temperature and the density of the central part increase indenfinitely. Lynden-Bell and Wood (1968) called this gravothermal catastrophe.

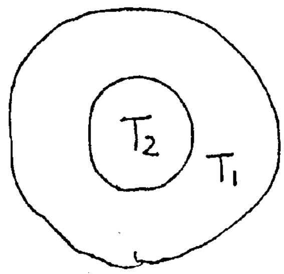

Fig. 1. $T_2 > T_1$.

Lynden-Bell and Wood considered the thermodynamics of isothermal gas spheres enclosed by adiabatic walls. They plotted the total energy against the density contrast, $D = \rho_c / \rho_e$, between the center and the boundary as Figure 2. The total energy oscillates as shown in Figure 2 and concluded that the system is unstable if the density contrast is larger than that at the first minimum of the total energy.

This can be interpreted intuitively as follows. If we consider an adiabatic perturbation to increase the central density, the perturbation is horizontal. If the density contrast of the unperturbed state is lower than the critical value, the pressure of the perturbed state is higher than the corresponding equilibrium state. Thus the system expands and recovers to the original unperturbed state. On the other hand, if the density contrast of the unperturbed state is higher than the critical value, the pressure of the perturbed state is lower than that of the corresponding equilibrium state so that contraction continues. The curve in Figure 2 is called linear series and often used in stability analysis. The validity of linear series to snaulyze stability is found in Inagaki and Hachisu (1978). As we saw that gravothermal instability is a consequence of negative specific heat. Hachisu and Sugimoto (1978) showed this explicitly for isothermal gas spheres. Inagaki (1980) confirmed that gravothermal instability in stellar systems in which the mean free path is much longer than the scale of the systems is also caused by negative specific heat.

3. Evolution after the onset of gravothermal instability.

Cohn (1980) solved orbit-averaged Fokker-Planck equation and obtained the evolution of the density profiles as shown in Figure 3. It is clear from the figure that the evolution takes place homologously.

Lynden-Bell and Eggleton (1980) considered why the evolution is self-similar. If the evolution is self-similar the density should be written

$$\rho(r, t) = \rho_c(t)\rho_*(r_*) \; ; \; r_* = r/r_c(t). \qquad (3.1)$$

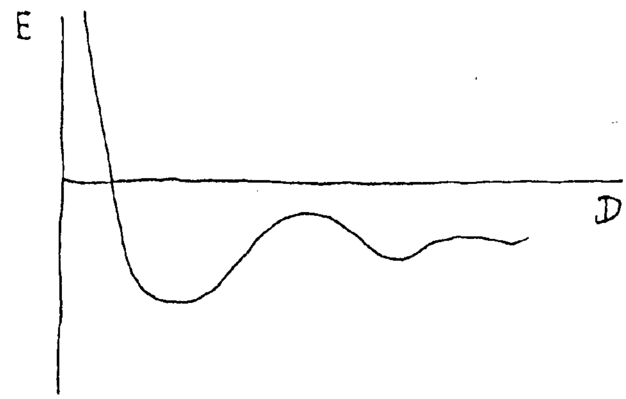

Fig. 2. Linear Series.

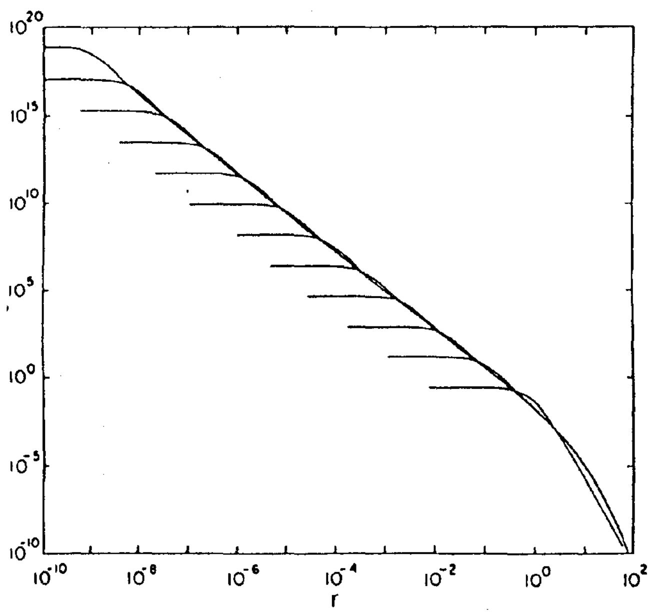

Fig. 3. Density profiles after the onset of gravothermal instability.

Since the time scale of evolution is the relaxation time and it is very long at the halo, the density does not change in the limit of $r \to \infty$.

$$\frac{\partial \rho(r,t)}{\partial t} = \dot{\rho}_c \rho_* - \frac{\dot{r}_c}{r_c} r_* \rho_c \rho_*' = 0 \qquad (3.2)$$

and so

$$\frac{r_* \rho_*'}{\rho_*} = \frac{r_c \dot{\rho}_c}{\rho_c} = -\alpha \qquad (3.3)$$

Since the l.h.s. is a function of r_* alone and the r.h.s. is a function of t alone, α must be a constant. We deduce that

$$\rho_* = A r_*^{-\alpha} \qquad (3.4)$$

in the halo and that

$$\rho_c \propto r_c^{-\alpha}. \qquad (3.5)$$

Since ρ_c and r_c are functions of t alone, this last relationship holds everywhere. Now the core mass is some definite multiple of $\rho_c r_c^3$ and so $M_c \propto r_c^{3-\alpha}$. Similarly the core energy is some multiple of $-GM_c^2/r_c \propto -r^{5-3\alpha}$. Hence the core energy is related to the core mass by

$$E_c \propto -M^{\zeta} \quad \text{where} \quad \zeta = \frac{5-2\alpha}{3-\alpha} \qquad (3.6)$$

Similarly calling v_c^2 the central velocity dispersion, we have

$$v_c^2 \propto \frac{GM_c}{r_c} \propto r_c^{2-\alpha}. \qquad (3.7)$$

Now the standard formula for the relaxation time in a stellar system is

$$T_r = v^3 (8\pi G^2 mp \ln N)^{-1}. \qquad (3.8)$$

For the evolution of our system we must have

$$\frac{1}{\rho_c}\frac{d\rho_c}{dt} \propto \frac{1}{T_{rc}} \propto \frac{\rho_c}{v^3}\,(8\pi G^2 m \ln N)^{-1} \tag{3.9}$$

From (3.9), (3.7) and (3.5)

$$-\alpha\frac{dr_c}{dt} \propto \frac{r_c^{\,1-\alpha}}{r_c^{\frac{3}{2}(2-\alpha)}} = r_c^{\frac{\alpha}{2}-2}\ .$$

Hence

$$r_c \propto (t_0 - t)^{\frac{2}{6-\alpha}}, \tag{3.10}$$

where t_0 is a constant of integration and we take $\alpha \neq 6$. From (3.7) and (3.9) we obtain

$$\begin{aligned} v_c^2 &\propto (t_0 - t)^{\frac{4-2\alpha}{6-\alpha}} \\ \rho_c &\propto (t_0 - t)^{\frac{-2\alpha}{6-\alpha}} \\ M_c &\propto (t_0 - t)^{\frac{6-2\alpha}{6-\alpha}} \\ E_c &\propto (t_0 - t)^{\frac{2(5-2\alpha)}{6-\alpha}} \end{aligned} \tag{3.11}$$

Notice that $\rho_c \to \infty$ and $r_c \to 0$ at the finite time $t = t_0$ provided $\alpha < 6$.

We now turn to the proof that $2 < \alpha < 2.5$. In any similarity solution it is the scales that vary so the dimensionless quantities appearing in the solution are constant. In our problem constants with dimensions are:

$[G] = M^{-1}L^3T^{-2}$,

$[M] = M$,

$[E] = ML^2T^{-2}$.

Since each is constant and $\rho_*(r_*)$ is some definite functional form, we deduce that the core radius is some fixed fraction of the natural length and so

$$r_c \propto \frac{GM^2}{-E} = [\,L\,].$$

Thus if M and E are finite constanst, r_c is fixed, the core does not shrink and no evolution occurs. Therefore we are not interested in this case. Now M finite implies $\alpha > 3$, whereas E finite only requires $\alpha > 2.5$. Thus we now try the supposition that $3 \geq \alpha > 2.5$ so that M is infinite but E is finite. Looking at our asymptotic form for ρ_*, and remembering that the outer halo is unchanging on the time scale of the evolution of the core, we see that A is a constant of dimensions $ML^{\alpha-3}$. Hence A, G and E form three dimensionfull constants and so

$$r_c \propto \left(\frac{GA^2}{-E}\right)^{1/(2\alpha-5)} = [\,L\,].$$

Hence r_c is again fixed and again no evolution is possible.

Next we examine the case $\alpha = 2.5$. Consider for a moment the system with $\rho \propto r^{-5/2}$ everywhere; than $M(r) = A_1 r^{1/2}$ where A_1 is constant. Hence

$$-\int \frac{GMdM}{r} = -\int \frac{GMdM}{M^2} A,$$

The integral is logarithmically divergent not only at large M but also at small M. Thus if the core should ever shrink to zero in finite time, the inner halo would have to lose an infinite energy. However, the transport would be through a cluster that was of finite density and velocity dispersion away from the central core and the transport process could not carry an infinite energy in a finite time. As we have already shown that r_c shrinks to zero in a finite time, both $\alpha = 5/2$ and constant E_c are impossible.

We have thus shown that $\alpha < 5/2$ but we still need to show that $\alpha > 2$. This follows from the remark that $\alpha = 2$ is the case of an isothermal sphere. We require that our solutions lose heat from the core to the halo so the temperature of the core must be greater. There is only a decrease of temperature outward for $\alpha > 2$ and only then does the central temperature, v_c^2, increases towards t_0 — see (3.11).

4. Effects of binaries.

Equation (3.11) implies that the central density diverges when t tends to t_0 and the mass contained in the core tends to zero then. This means that the Fokker-Planck approximation breaks down and some new physical process occurs. Binaries are formed through three-body encounters with the formation rate,

$$N_{B3} = 0.9\, G^5 m^5 n^3 / \sigma^9 . \tag{4.1}$$

Equation (3.11) means that both central density and the central velocity dispersion grow at t approaches t_{cc} but the rise of the central velocity dispersion is much milder than that of the central density. Therefore the binary formation rate increases according to equation (4.1).

Another formation mechanism of binaries is two-body tidal dissipative encounters, which are proposed by Fabian et al. (1975). Press and Teukolsky (1977) calculated the detailed cross section for binary formation and Lee and Ostriker (1986) superceded it.

If the binding energy of a binary exceeds the mean kinetic energy of a field star, the binary is hardened as the results of encounters between the binary and the field star and the excess energy is transferred to the energy of the translational motion of stars. This kind of binaries are called hard binaries and play an important role to heat the core of the cluster (Heggie 1975). On the other hand, if the binding energy of a binary is less than that of a field star, the binding energy of the binary becomes less than the original value. This kind of binaries are called soft binaries and have tendency to be destroyed. Thus soft binaries are not important in cluster dynamics.

5. Post-collapse evolution

If the heat released from binaries is enough to stop the core collapse, the core collapse is reversed then. Inagaki and Lynden-Bell (1983) obtained a self-similar solution which is connected to the solution of Lynden-Bell and Eggleton (1980).

Inagaki and Lynden-Bell considered an idealized problem: According to Lynden-Bell and Eggleton, the gravothermal instability must leat to a similarity solution and in finite time the natural length scales becomes zero. At that moment the system takes a power-law structure with density $\rho = Ar^{\alpha}$. Inagaki and Lynden-Bell considered the subsequent evolution. The only way that the mass m of an individual star comes into the problem is through the relaxation time

$$T_r \propto \frac{v^3}{G^2 m \rho}$$

The physical problem is set by the dimensionful constants $[\,G\,] = M^{-1} L^3 T^{-2}$, $[\,A\,] = ML^{\alpha-3}$ and $[\,m\,] = M$.

From these we can construct a length $(A/m)^{1/(\alpha-3)}$. This is the very small radius which contains the mass of one star at the center of the cluster's density profile. The density at that radius is $\rho_m = A(A/m)^{-\alpha/(\alpha-3)}$ and $(G\rho_m)^{-1/2}$ gives the only time-scale we can make. However, that is the crossing time at a tiny radius — we are neither interested in structures of this scale nor in times so short. Our interest lies in the evolution of the cluster over several relaxation times. Thus no time of interest to us can be made from the constants of the problem. However, if we give ourselves the time since the core collapse $\tau = t - t_0$ and remember that we are interested in the behavior on the relaxation time-scale appropriate to the radius at which we look, then it must be the ratio τ/T_r that arises. Notice that this ratio always involves m and τ in the combination $m\tau$. Thus we look again at dimensional arguments using G, A and the combination of $m\tau$ of dimensions MT as our basis. With these assumptions we find a natural unit of length

$$r_c = [Gm^2\tau^2/A]^{\frac{1}{6-\alpha}}, \tag{5.1}$$

and similarly a natural unit of mass

$$M_c = [(Gm^2\tau^2)^{3-\alpha}A^3]^{\frac{1}{6-\alpha}} .$$

If we now ask again what is the form of the density evolution of the globular cluster, it can only involve the constants that specify the problem and so

$$\frac{\rho}{\rho_c} = \rho_*(r/r_c) \tag{5.2}$$

Notice that ρ_c and r_c are both τ dependent and that this density is of similarity form. Using the method of dimensions, we find that the characteristic radius r_c, mass M_c, density ρ_c, velocity v_c, energy E_c and relaxation time T_r have the following time dependence.

$$r_c \propto |\tau|^{\frac{2}{6-\alpha}}, \quad M_c \propto |\tau|^{\frac{2(3-\alpha)}{(6-\alpha)}}, \quad \rho_c \propto |\tau|^{\frac{-2\alpha}{6-\alpha}}, \quad v_c^2 \propto |\tau|^{\frac{-2(\alpha-2)}{6-\alpha}},$$
$$E_c \propto GM_c^2 r_c^{-1} \propto |\tau|^{\frac{5-2\alpha}{6-\alpha}}, \quad T_r \propto \frac{v_c^3}{\rho_c} \propto |\tau|. \tag{5.3}$$

These are identical to those found by Lynden-Bell and Eggleton, but we are now discussing τ positive instead of negative.

Inagaki and Lynden-Bell further found that in post-collapse phase the inner region of the core is isothermal and outer region has the power law profile with $\rho \propto r^{-2.2}$, which is the relic of the pre-collapse phase. The transition radius, r_*, between the isothermal region and the power law region moves out according to the law,

$$r_* \propto (t - t_0)^{0.53}. \qquad (5.4)$$

When the transition radius becomes larger than the half-mass radius, their solution becomes invalid and the solution found by Goodman (1984) takes the place. Goodman's solution is essentially the same as Henon's (1965) solution, both are valid when there is escape of stars. Both Inagaki and Lynden-Bell's solution and Goodman's solution have infinite central density. Goodman (1987) found another self-similar solution with finite central density. Since the central density of real globular clusters should be finite, neither Inagaki and Lynden-Bell's solution nor Goodman's solution is realized.

6. Gravothermal oscillations

Realistic solutions were found by numerical integration of equations for a gas model or Fokker-Planck model of globular clusters. Bettwieser and Sugimoto (1984) found oscillations of the central density and Heggie (1984) and Cohn (1985) found monotonic decrease of the central density. The oscillations found by Bettwieser and Sugimoto are called gravothermal oscillations and their existence was contraversial at first. Later both Heggie (see Heggie and Ramamani 1987) and Cohn (see Cohn et al. 1986) found gravothermal oscillations and the existence is confirmed at least in continuum models (i.e., a gas model of Fokker-Planck model).

Inagaki (1986), however, pointed out the importance of fluctuations because the number of stars in the core at the time of the reversal of core collapse is about 50. In order that gravothermal oscillations occur, inverse temperature gradient is necessary (Bettwieser and Sugimoto 1984). Inagaki (1986) pointed out that the fluctuations due to small number of stars in the core may erase the inverse temperature gradient. In fact he did N-body simulations and found that the oscillations of the core are directly related with binary activity; if the binaries release energy, the core expands and if the binaries do not supply energy to the core, the core contracts. McMillan (1986) found similar evolution, using a hybrid-code.

Inagaki (1984) pointed out that binaries formed by two-body tidal dissipational encounters are more important than binaries formed by three-body encounters in the evolution of real globular clusters. Statler et al. (1987) did detailed calculation of the effects of two-body binaries.

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