研究会報告

26. *Random XY-Spin Chain*

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One-dimensional XY spin model with random exchange interaction is studied and our new approach and its result are reported. The distinguishing feature of this model is that the susceptibility diverges at $T_c(=0)$ due to the off-diagonal disorder, while the susceptibility of the uniform system is constant at T_c. The generality of this divergency is discussed. By our method, a perturbation from the uniform system, the susceptibility has no singularity in the first order of the perturbation.

ランダムな交換相互作用を持つ1次元XY spin モデルについて報告し、我々の新しい方法についてのべる。この系の特徴は一様系において現れなかった感受率の発散($T_c=0$)が、非対角的な乱れの為現れることである。その発散の一般性について議論する。我々の方法(ランダムさの分布の幅についての摂動展開)では、摂動一次まででではその発散は一般に現れない。

§1 Introduction

Many random spin systems have been investigating for the classical Ising models, but no study on the quantum spin systems has been reported except a few cases. We study here the one-dimensional XY-spin model which was solved exactly for the uniform case by Lieb, Schultz and Mattis and by Katsura.[1)] The Hamiltonian we consider here is defined by

$$\mathcal{H} = -\sum_i J_i \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y \right) - \sum_i H_i \sigma_i^z , \tag{1.1}$$

where the exchange interaction $\{J_i\}$ has the distribution function $P(J)$ and the external field $\{H_i\}$ is also randomly distributable.

The disoder of $\{Ji\}$ is an off-diagonal disorder and of $\{H_i\}$ a diagonal one. In almost of the present paper, $H_i=H$. Our main interest is a following problem.

PROBLEM. Does the susceptibility χ^{zz} diverge affected by the randomness of $\{J_i\}$ as

$$\chi^{zz} \sim \frac{1}{T(\ln T)^2} \quad \text{at } H=0 . \tag{1.2}$$

Note here, the susceptibility χ^{zz} is a constant for the uniform case at $T=0$.

Morgenstern and Wültz[2)] and Schüttler, Scalapino and Grant[3)] studied this problem by the Monte Carlo method and concluded that the answer is yes,when $P(J)$ is a gaussian or a rectangular distribution, respectively.

An exact solution was found by Dyson[4] and Smith[5] for the general Poisson distribution given by $P(J)=2m^mJ^{2m-1}e^{-mJ^2}/(m-1)!$ with an integer m. A short review is given in §2. The corrected result is

$$\chi^{zz} \simeq \frac{1}{16}\left(\frac{\pi^2}{6}-t_{m-1}\right)\frac{1}{T\,(\ln T)^2}\,, \tag{1.3}$$

where $t_m=\sum_{r=1}^{m} r^{-2}$.

Another exact solution was also found by Matsubara and Katsura[6] for the distribution: $P(J)=p\delta(J-J_0)+(1-p)\delta(J)$, i.e., this model is a diluted XY chain. From their solution eq.(5) of Ref.6, the susceptibility behaves as

$$\chi^{zz} \simeq \frac{1}{T}\,\frac{1-p}{1+p}\,p\,. \tag{1.4}$$

This has also singularity but without logarithmic correction.

We approach to this problem by the ST-transformation method which was developed by Suzuki and the present author[7,8] and by Koma.[9]

§2 A Short Review of Dyson[4] and Smith[5]

As is well-known, the XY model can be mapped to a harmonic oscillator or a free fermion(electron) problems. By the Jordan-Wigner transformation and diagonalization, our Hamiltonian is represented as the fermion system.

$$\mathcal{H} = \sum_a \varepsilon_a\, C_a^{+} C_a\,, \tag{2.1}$$

where Ca is a spinless fermion. Consequently the free energy is written with the density of state by

$$-\beta\mathcal{F} = \int_{-\infty}^{\infty} \ln\left(1+e^{-\beta\varepsilon}\right) \mathcal{D}(\varepsilon)\, d\varepsilon \tag{2.2}$$

and the susceptibility is given by

$$\chi^{zz} = \frac{\beta}{4}\int_{-\infty}^{\infty}\frac{1}{\cosh^2(\beta\varepsilon/2)}\,\mathcal{D}(\varepsilon)\, d\varepsilon\,. \tag{2.3}$$

In generally, if the density of state has singularity at $\varepsilon=0$, the susceptibility diverges. Numerious studies on the density of state were reportred.[10]

As mensioned in §1, the density of state $D(\varepsilon)$ is obtained by Dyson* for the generalized Poisson distribution. In the vicinity of $\varepsilon=0$, the density of state behaves as

$$\mathcal{D}(\varepsilon) \simeq 2\left(\frac{\pi^2}{6}-t_{m-1}\right)\Big/\left|\varepsilon\,(\ln m\varepsilon^2)^3\right| \tag{2.4}$$

and thus the susceptibility is given by eq.(1.4) in the critical region $T \approx 0$. This is shown in fig.1. The result has not been obtained by Smith. Compare (2.4) with (5.6) of ref.5 and fig.1 with fig.2 of ref.5, respectively.

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* The density of state is obtained as follows. Define $M_S(x) = \int_{-\infty}^{x} D(\varepsilon)d\varepsilon$ which relates to $M_D(x^2)$ by $M_D(x^2) = -1 + 2M_S(x)$. $M_D(x^2)$ is given by eq.(63) of Dyson's paper. $M_S(x)$ is defined as $M_p(\mu)$ in eq.(3.2) of Smith's paper.

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§3 The ST-transformation Method and The Perturbational Expansion

3-1 The ST-transformation method

The one-dimensional quantum system can be mapped to a two-dimensional classical one and the virtual-space transfer-matrix (VTM) can be defined for the present model as was given by Suzuki and the present author for the uniform XY model in ref. 8.

The VTM is explicitly given by

$$\mathcal{T}_\alpha(r) = \sinh^n(\beta J_r/n)\, e^{\tilde{\mathcal{H}}_\alpha(r)}$$

$$\tilde{\mathcal{H}}_\alpha(r) = \sum_{i=1}^{2n} \left[J_1(r)\sigma_i^+ \sigma_{i+1}^- + J_2(r)\sigma_i^- \sigma_{i+1}^+ \right] \tag{3.1}$$

with $i = odd$ for $\alpha = 1$, $i = even$ for $\alpha = 2$ and

$$J_1(r) = e^{-\frac{h(r)}{n}} \ln\left(\coth\frac{\beta J_r}{n}\right), \quad J_2(r) = e^{\frac{h(r)}{n}} \ln\left(\coth\frac{\beta J_r}{n}\right), \tag{3.2}$$

where r denotes a position in the real space and n is the Trotter number. We can reduce this transfer matrix to the 4 by 4 matrix with use of the Jordan-Wigner and the Fourier transformation, independently on r.

The partition function Z of our XY chain with the length m is then written as a product of the 4 by 4 matrices:

$$Z = \prod_{\varepsilon} \mathrm{Tr} \begin{bmatrix} Y_m & 0 & 0 & 0 \\ 0 & A_m & B_m & 0 \\ 0 & C_m & D_m & 0 \\ 0 & 0 & 0 & Y_m \end{bmatrix}, \tag{3.3}$$

where $Y_m = \prod_{r=1}^{m} \sinh(2K_r/n)$,($K_r = \beta J(r)$) and A_r, B_r,..are defined suscessively as follows.

$$\begin{bmatrix} A_r & B_r \\ C_r & D_r \end{bmatrix} = \begin{bmatrix} a_r & b_r \\ b_r^{-1} & a_r \end{bmatrix} \begin{bmatrix} A_{r-1} & B_{r-1} \\ C_{r-1} & D_{r-1} \end{bmatrix},$$

$$a_r = \cosh \frac{K_r}{n} \quad , \quad b_r = \begin{cases} e^{\frac{i\delta}{2} + \frac{h(r)}{2n}} & ; \ r = \text{even} \\ e^{-\frac{i\delta}{2} - \frac{h(r)}{2n}} & ; \ r = \text{odd} \end{cases} \tag{3.4}$$

and q takes the values $q = \pm\frac{\pi}{n}, \pm 3\frac{\pi}{n}, \cdots, \pm\frac{n-1}{n}\pi$.

In the mathematical aspect, the above product of matrices relates to the theorem of noncommuting random products proved by Furstenberg[11] which says that there exists a unique stationary probavility measure ν determined from the convolution equation $\mu * \nu = \nu$ for $m \to \infty$, where a_r or b_r are random variables with a distribution μ.

3-2 Integral equations

Define that $X_r \equiv A_r + D_r$ and $x_r \equiv C_r/A_r$ and assume that $wA_r = B_r$ and $wC_r = D_r$ for sufficiently large r with a constant w, and thus we get

$$\frac{X_r}{X_{r-1}} = a_r + \frac{w b_r^{-1} + b_r x_{r-1}}{1 + w x_{r-1}}$$

and

$$x_r = \frac{b_r^{-1} + a_r x_{r-1}}{a_r + b_r x_{r-1}} \tag{3.5}$$

for r even. We should exchange $b_r^{-1} \leftrightarrow b_r$ for r odd. With use of the above variables, we have the free energy by

$$\begin{aligned} -\beta \mathcal{F} &= \frac{1}{m} \sum_{\delta} \ln (X_m + 2Y_m) \simeq \frac{1}{m} \sum_{\delta} \ln (X_m) \\ &= \frac{1}{m} \sum_{\delta} \sum_{r=1}^{m/2} \left\{ \ln \frac{X_{2r+1}}{X_{2r}} + \ln \frac{X_{2r}}{X_{2r-1}} \right\} \end{aligned} \tag{3.6}$$

for $m \sim \infty$.

Following Fan and McCoy,[12] the quenched free energy is given by

$$\begin{aligned} -\beta \mathcal{F} &= \sum_{\delta} \int dx_1 \, d\lambda \, \nu_1(x_1) \, \mu(\lambda) \ln \left[a(\lambda) + \frac{w b^{-1}(\lambda) + b(\lambda) x_1}{1 + w x_1} \right] \\ &+ \sum_{\delta} \int dx_2 \, d\lambda \, \nu_2(x_2) \, \mu(\lambda) \ln \left[a(\lambda) + \frac{w b(\lambda) + b^{-1}(\lambda) x_2}{1 + w x_2} \right], \end{aligned} \tag{3.7}$$

where $v_1(x_1)$ and $v_2(x_2)$ are limiting stationary distribution functions of x_1 and x_2, respectivery and these satisfy the following simultaneous integral equations

$$\nu_1(x_1)=\int dx_2 d\lambda\, \nu_2(x_2)\mu(\lambda)\,\delta\left(x_1-\frac{b(\lambda)+a(\lambda)x_2}{a(\lambda)+b^{-1}(\lambda)x_2}\right)$$

and

$$\nu_2(x_2)=\int dx_1 d\lambda\, \nu_1(x_1)\mu(\lambda)\,\delta\left(x_2-\frac{b^{-1}(\lambda)+a(\lambda)x_1}{a(\lambda)+b(\lambda)x_1}\right) \tag{3.8}$$

with a probavility distribution function $\mu(\lambda)$ of $a(\lambda)$ or $b(\lambda)$. When we consider the random exchange model, $\lambda=J$, $\mu(\lambda)=P(J)$ and $b(\lambda)=b$. For the random field model, $\lambda=H, a(\lambda)=a$ and $\mu(\lambda)=P(H)$.

Now we define a new "free energy" as follows

$$-\beta \mathcal{F}_1 \equiv \frac{1}{m}\sum_{g}\sum_{r}\ln\frac{A_r}{A_{r-1}} \quad . \tag{3.9}$$

By the same treatments as above, the "free energy" becomes

$$\begin{aligned}-\beta \mathcal{F}_1 &= \sum_{g}\int dx_1 d\lambda\, \nu_1(x_1)\mu(\lambda)\ln\left(a(\lambda)+b(\lambda)x_1\right)\\ &\quad+\sum_{g}\int dx_2 d\lambda\, \nu_2(x_2)\mu(\lambda)\ln\left(a(\lambda)+b^{-1}(\lambda)x_2\right)\\ &\equiv \sum_{g}\left(\mathcal{F}_g^{1}+\mathcal{F}_g^{2}\right) .\end{aligned} \tag{3.10}$$

We can easily show that $\mathcal{F}=\mathcal{F}_1$ with use of (3.8).

When $\mu(\lambda)$ is a δ-function and H=uniform, we can get the free energy of the uniform XY model from (3.8) and (3.10). The distribution functions become $v_1(x)=\delta(x-x_{10})$ and $v_2(x)=\delta(x-x_{20})$, where x_{10} and x_{20} are given by

$$\begin{aligned}x_{10}&=x_{20}^{-1}=\left[b-b^{-1}+\{4a_0^2+(b-b^{-1})^2\}^{1/2}\right]/2a_0 ,\\ a_0&=\cosh(2\beta J_0/n) .\end{aligned} \tag{3.11}$$

J_0 is defined by $P(J)=\delta(J-J_0)$. Thus the free energy is given by

$$\begin{aligned}-\beta\mathcal{F}&=\lim_{n\to\infty}\sum_{g}\ln\left[a_0^2+1+a_0(bx_{10}+b^{-1}x_{20})\right]\\ &=\frac{1}{\pi}\int_0^{\pi}\ln\left[2\cosh(2\beta J_0\cos\theta+\beta H)\right]\end{aligned} \tag{3.12}$$

with use of the same mathematical formulae as in ref.8 .

3-3 The perturbational expansion ----- 1st order -------

We put our attention on the random exchange model hereafter. The exact solotion of the integral equations (3.8) is difficult, we shall only explore some general characterictic of solutions for the class of narrow distributions $\mu(\lambda)$. The method we employ was developted by Fan and McCoy[12] for the one-dimensional Ising model.

First, we change the variables J to $a = \cosh(2\beta J/n)$ and thus $\mu(J)dJ = \mu(a)da$. Assume that

$$\bar{\mu}(a) = \frac{N}{\Delta} f\left(\frac{N}{\Delta}(a-a_0)\right) + \cdots \tag{3.13}$$

where Δ and N have the same meaning as in ref.12, i.e., Δ is a unit for the energy spread and N is a dimensionless scaling factor introduced to indicate the narrowness of the width. The function f should have some mathematical properties which are given in eqs.(3.5)-(3.6) of ref.12 .

When $N \to \infty$, the distribution μ becomes a δ-function and then the corresponding distribution functions $v_1(x)$ and $v_2(x)$ are the functions of $N(x-x_{10})$ and $N(x-x_{20})$, respectively. We expand them as

$$\nu_1(x) = N g_{10}(N(x-x_{10})) + g_{11}(N(x-x_{10})) + \cdots$$

and

$$\nu_2(x) = N g_{20}(N(x-x_{20})) + g_{21}(N(x-x_{20})) + \cdots \tag{3.14}$$

with the condition: $\int g_{ij}(x)dx = \delta_{j,0},\ i=1,2.$

We change the variables to $\xi_i = N(x-x_{i0})$ and $y = N(a-a_0)/\Delta$ and expand the free energy defined by (3.10)

$$\mathcal{H}_q' = \ln(a_0 + b x_{10}) + \frac{1}{N}\frac{\Delta \overline{my_1} + b\,\overline{m_1\xi_{01}}}{a_0 + b x_{10}} + O(N^{-2}) \tag{3.15}$$

where

$$\overline{my_\ell} \equiv \int_{-\infty}^{\infty} dy\, y^\ell f(y),$$

$$\overline{m_j\xi_{k\ell}} \equiv \int d\xi_j\, \xi_j^\ell\, g_{jk}(\xi_j) \tag{3.16}$$

$\mathcal{H}_q^2$ is the same as (3.15) exchanging b to b^{-1}, x_{10} to x_{20} and $\overline{m_1\xi_{01}}$ to $\overline{m_2\xi_{01}}$. From (3.8) with (3.14), we have

$$\overline{g_{20}}(\eta_2) = \int d\eta_1\, \overline{g_{10}}(\eta_1)\, f(\eta_2 - A\eta_1) \tag{3.17}$$

in the first order of N, where we have re-exchanged the variables as

$$\eta_1 \equiv \frac{a_0}{\Delta(b^{-1}-b)}(b^{-1}x_{20} + a_0^{-})\xi_1, \quad \eta_2 \equiv \frac{a_0}{\Delta(b-b^{-1})}(b x_{10} + a_0)\xi_2$$

and

$$g_{10}(\xi_1) = \bar{g}_{10}(\eta_1)\frac{d\eta_1}{d\xi_1}\ , \text{ etc.} \qquad A_1 \equiv -\frac{a_0 - b x_{20}}{a_0 + b^{-1} x_{20}} \ . \tag{3.18}$$

By the Fourier transformation, $f(y) = \int F(k)e^{iky}dk$ and $g_{ij}(\eta) = \int G_{ij}(k)e^{ik\eta}dk$, we get

$$G_{20}(k) = G_{10}(A_1 k)F(k) \text{ and } G_{10}(k) = G_{20}(A_2 k)F(k)\ , \tag{3.19}$$

where A_2 is defined by (3.18) exchanging $b \leftrightarrow b^{-1}$ and $x_{10} \leftrightarrow x_{20}$. From (3.19), we have

$$G'_{10}(0) = \frac{1+A_2}{1-A_1A_2}F'(0) \text{ and } G'_{20}(0) = \frac{1+A_1}{1-A_1A_2}F'(0) \ . \tag{3.20}$$

F' denotes a derivative of F by its argument and so on. The moment defined by (3.16) is then represented by $F'(0)$ as follows

$$\overline{my_1} = iF'(0) \text{ and}$$

$$\overline{m_1 \xi_{01}} = i\frac{\Delta(b^{-1}-b)}{a_0(a_0+b^{-1}x_{20})}\frac{1+A_2}{1-A_1A_2}F'(0)\ . \tag{3.21}$$

Consequently, the free energy of (3.10) becomes

$$\mathcal{F}_q^1 = \ln(a_0+bx_{10}) + \frac{\Delta}{N}\frac{i}{a_0+bx_{10}}\left\{1 - \frac{b(b-b^{-1})x_{10}}{a_0^2(x_{10}+x_{20})}\right\}F'(0)\ . \tag{3.22}$$

$\mathcal{F}_q^2$ has also a similer form.

3-4 Final results

The $O(N^{-1})$ term of the free enrgy is thus obtained by

$$-\beta\mathcal{F}^{(1)} = \frac{\Delta}{N}\sum_{\theta}\frac{2\cosh\left(\frac{2K}{n}\right)}{\sinh^2\left(\frac{2K}{n}\right)}\left\{1 - \frac{\cos\left(\frac{\theta}{2}+\frac{ih}{2n}\right)}{\sqrt{\sinh^2\left(\frac{2K}{n}\right)+\cos^2\left(\frac{\theta}{2}+\frac{ih}{2n}\right)}}\right\}\ . \tag{3.23}$$

Let $\delta = \overline{my_1}\cosh(2K/n)/\sinh^2(2K/n)$, $\gamma = \Delta/N$ and $t = -1$, eq.(3.23) becomes

$$\frac{-\beta\mathcal{F}^{(1)}}{\delta} = \sum_{\theta}\ln\left[1 + \gamma\left(1 + t\frac{\cos\left(\frac{\theta}{2}+\frac{ih}{2n}\right)}{\sqrt{\sinh^2\left(\frac{2K}{n}\right)+\cos^2\left(\frac{\theta}{2}+\frac{ih}{2n}\right)}}\right)\right]\ . \tag{3.24}$$

With the help of the mathematical formulae used to derive (3.12), we have the following result after tedious calculations.

$$-\beta \mathcal{F}^{(1)} = \frac{\delta}{\pi}\int_0^{\pi} d\theta \, \ln\left[\frac{\cosh nv + \cosh h}{\cosh 4K + \cosh h}\right], \tag{3.25}$$

where

$$\cosh v \equiv 1 + 2\sinh^2\left(\frac{2K}{n}\right)\left\{1 + \left(\frac{\gamma t}{1+\gamma}\tan\frac{\theta}{2}\right)^2\right\}^{-1}. \tag{3.26}$$

The result (3.25) is for the finite Trotter number, we should take the limit $n \to \infty$ to return to the original Hamiltonian (1.1)

Keeping in mind that we are now considering the order N^{-1} term, we integrate by θ in (3.25). For large n, v becomes

$$v \simeq \frac{4K}{n}\sqrt{D}\left\{1 + \frac{2K^2}{3n^2}(1-D)\right\} \tag{3.27}$$

with $D = \{1 + (t\gamma\tan(\theta/2)/(1+\gamma))^2\}^{-1}$. The free energy (3.25) becomes finally,

$$-\beta \mathcal{F}^{(1)} = -\frac{4K}{3}\left\{\frac{\sinh 4K}{\cosh 4K + \cosh h} + 1 - \frac{1}{16K^2}\right\}\frac{\Delta}{N}\overline{my_1}. \tag{3.28}$$

This is the order N^{-1} term of the free enregy and thus the susceptibility is given by

$$\chi^{zz(1)} \simeq \frac{4}{3}\beta^2 J_0 \frac{\sinh 4\beta J_0}{\{\cosh 4\beta J_0 + 1\}^2}\cdot\frac{\Delta}{N}\overline{my_1}. \tag{3.29}$$

This has no singularity at $T=0$. This is our final result. The second or higher terms can be in principle calculatable,

§4 Summary and Discussions

We have studied the XY chain with random distributed exchange interactions. The free energy up to the first order is obtained by the perturbation method. The susceptibility has no singularity in this order. However, this result does not mean that χ^{zz} is a smooth function of T (no singularity).

We should remark two points. i) Our perturbational calculation has no meaning in the $\beta\Delta/N > 1$ region, because the free energy does not converge in this region. ii) The free energy (3.10) is defined for the quenched system, but up to the first order (N^{-1}) it is the same as for the annealed system.

We have now two exact solutions, the susceptibility for the case of the generalized Poisson distribution behaves as (1.2) , another one for the diluted system behaves as (1.4). We shall ask again how generally the susceptibility diverges and if it diverges whether the logarithmic correction like (1.2) exists or not.

Eggarter and Riedinger[13] studied this problem very plausibly with using a random walk representation and concluded that the answer of the problem (1.2) is yes in general if σ^2 exists. σ^2 is the variance of $\ln J^2$ defined as $\sigma^2 = <(\ln J^2)^2> - <\ln J^2>^2$ and the susceptibility is given by

$$\chi^{zz} \simeq \frac{\sigma^2}{16T(\ln T)^2} . \tag{4.1}$$

If their result is correct for all cases up to the coefficient (for the case of the generalized Poisson distribution, $\sigma^2 = \pi^2/6 - t_{m-1}$ which is the same as the exact one), it is clear why we have obtained the susceptibility with no singularity.

We have studied the effect of the off-diagonal disorder to the diagonal susceptibility. How about the diagonal disorder? The problem of the random field corresponds to this case and which was also solved for the Lorentzian distribution by Nishimori [14] with use of the corresponding Lloyd's solution[15] for the electron system. Nishimori showed that the random field destroy the ordering of the XY model in ground state and that the both of the susceptibilities χ^{zz} and χ^{xx} are suppressed by the randomness.

Acknowledgement

I would like to express my sincere appreciation to Professor Masuo Suzuki for his continual encouragement. I also thank Professor Shigetoshi Katsura for useful discussions.

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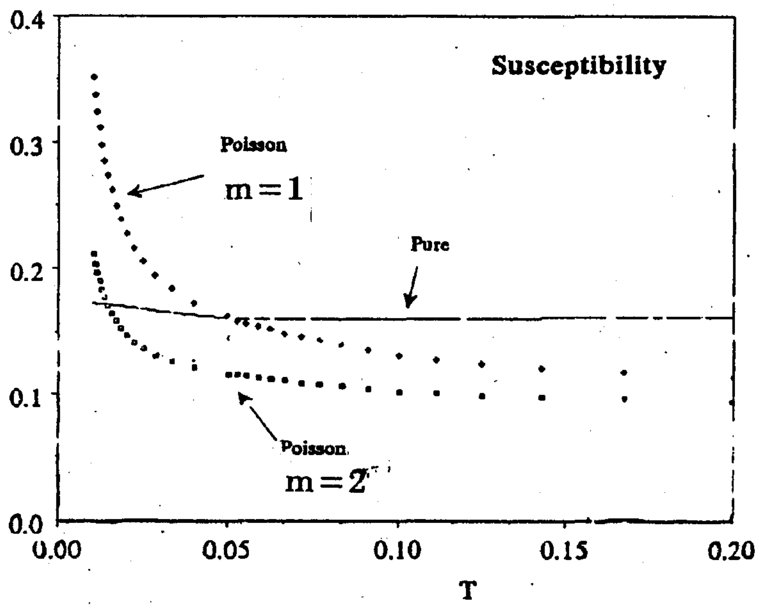

Fig.1.

The susceptibility of Dyson-Smith model. This graph is obtained by numerical integration of eq.(2.3) with the density of state given by Dyson.

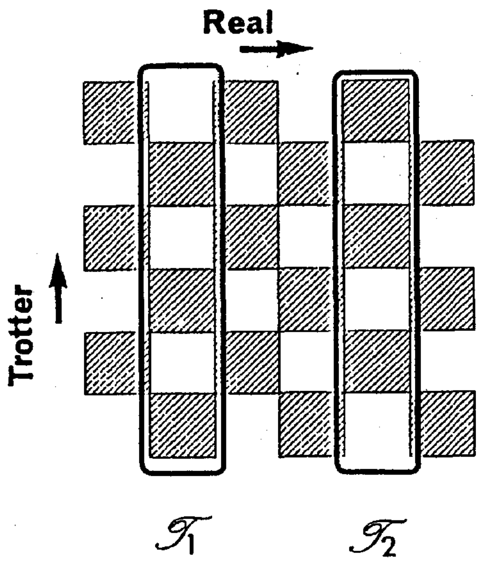

Fig.2.

The checkerboard decomposition. Each shaded square means four-spin interactions. T_1 and T_2 are given by eq. (3.1)