Quantum Chaos in Atomic two-level and Harmonic Oscillater Systems

2準位原子ー調和振動子系における量子カオスと蛍光スペクトル

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Laser is an optical oscillater which originate in collective motin of atoms, interacting with resonant field. Atomic motion is, then, described by quantum mechanics, usually based on the model of two level systems,that is, spin systems. Electromagnetic field is, however, described usually by classical ones, that lead to Maxwell Bloch systems, that is, semiclassical models. Quantum theory of field for laser is appropriate only for the stable models of laser oscillation. For the case of unstable or chaotic oscilating laser, quantum theory of field is too difficult to solve the model equations, because the number of vaiables is too many to solve them numerically or analytically.

Neglecting pumping processes, loss and relaxation atomic and field sytems are thougt as isolated dynamical ones with a difinite hamiltonian. That makes it possible to compare the quantum systems and corresponding classical ones. Chaos in laser oscillation is usually described theoretically by semiclassical equations. Almost same condition is made by using two level and resonant field systems without pumping. Master equation is then reduced to Bloch equations of spins which are coupled to the classical field in the form of semiclassical approximations. For fully quantum theoretical treatment, c-number partial differential equaton for expectation values of variables is reduced. These correspondence beteen classical case and quantum ones is characterisic to the spin field systems.

Coupled Bloch equations are shown by using spin vectors and linear polarized electric fiels E, as follows;

$$\frac{d}{dt}\langle S^{+}\rangle = i\omega_a\langle S^{+}\rangle - 2g(a-a^{+})\langle S^{z}\rangle + i\Omega/2\langle S^{z}\rangle\cos\omega t$$

$$\frac{d}{dt}\langle S^{z}\rangle = -g(a-a^{+})(\langle S^{+}\rangle - \langle S^{-}\rangle) + i\Omega(\langle S+\rangle - \langle S-\rangle)\cos\omega t$$

$$A = -i\ (a - a^{+})$$

$$\ddot{A} + \omega_c{}^2 A = -2\omega_c g\ (S^{+} + S^{-})$$

In the case of strong coupling between S and E, that is, $\mu > 0.2$, non-periodic oscillation is observed for spin and field variables. These oscillation might be seen to be chaotic, from the view point of their fourier spectrum, although their fractal parameter and lyapunov number and so on have not been obtained definitly.

The chaotic or nonperiodic oscillation may be seen to be caused by the difference of frequency between spin oscillation and field one, that is induced by the strong coupling of them, or by the detuning given beforehand artifically. The latter case of frequency difference suggests that, if we artifically oscillate the spin system with frequency, whose detuning is irrational, the linear spin system also oscillate nonperiodically or in nearly chaotic motion, the property of which is not neccesarily identified to be chaotic by the usual criterion of chaos such as lyapunov numbers. These are exactly or nearly zero because the system is linear stable ones.

C-number distribution function for the quantum system is described by the following equations obtained from master equations for density matrix.

$$
\begin{aligned}
\frac{\delta}{\delta t}Pc =& -\omega c\left(\frac{\delta}{\delta Ey}Ex-\frac{\delta}{\delta Ex}Ey\right)Pc - 2\Omega\frac{\cos\omega t}{\sin\theta}\left(\frac{\delta}{\delta\theta}(\sin\phi\sin\theta Pc)+\frac{\delta}{\delta\phi}(\cos\psi\cos\theta Pc)\right)\\
&+2\mu Ex\left(\frac{\cos\phi}{\sin\theta}\frac{\delta}{\delta\theta}(\sin\theta Pc)-\frac{\cos\theta}{\sin\theta}\frac{\delta}{\delta\phi}(\sin\phi Pc)\right)\\
&-2\mu Ey\left(-\frac{\sin\phi}{\sin\theta}\frac{\delta}{\delta\theta}(\sin\theta Pc)-\frac{\cos\theta}{\sin\theta}\frac{\delta}{\delta\phi}(\cos\phi Pc)\right)\\
&-\mu\frac{\delta}{\delta Ex}\left((S+1)\sin\theta e^{-i\phi}+\frac{1-\cos\theta}{2}e^{-i\phi}\frac{\delta}{\delta\theta}+i\frac{1-\cos\theta}{2}e^{-i\phi}\frac{\delta}{\delta\phi}\right)Pc\\
&+\mu\frac{\delta}{\delta Ey}(\text{ c. c.})Pc + D\left(\frac{\delta^2}{\delta Ex^2}+\frac{\delta^2}{\delta Ey^2}\right)Pc
\end{aligned}
$$

Variables are angular polar coordinate for spin vector and E and dE/dt are represented by Ex and Ey respectively.

Expanding above function by associate legendre functions for spin ones. The series terminate in finite terms. For field variables hermit polynomials are used for the expansion. Expansion coefficients correspond to the classical variables, for example, the 1st order ones of hermite ones to field variable, et al.. Difference between quantum and classical systems are seen in the behavior of the field variables which correspond to the 1st order coefficients and oscillates quasiperiodically in the quantum systems in spite of chaotic behavior of clasicall ones.

The whole image of quantum system may be seen from the distribution function Pc, not the lower order coefficients, whose eigen value may give the image of structure, which are calculated the eigen values of matrix of the differential equations to obtain the expansion coefficients.

Finally quantum correlation functions, for example, of spin variables will show the quality of flourescence from the chaotic quantum systems, such as molecules et. al. That is left for the temporal work, by the following correlaton funtions.

$$\chi(\tau) = \lim_{t\to\infty} \langle S^{+}(t+\tau)S^{-}(t)\rangle$$

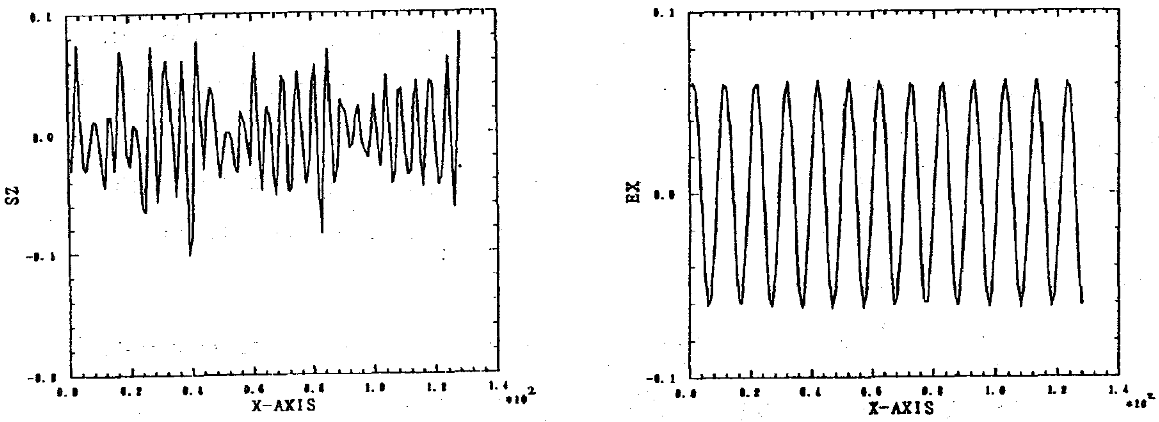

Fig.1 Spins and field motion by quantum theory. η=0.5, ωa=1, ωc=17711/28657, Ω=0, D=0.001, 10×10 terms.

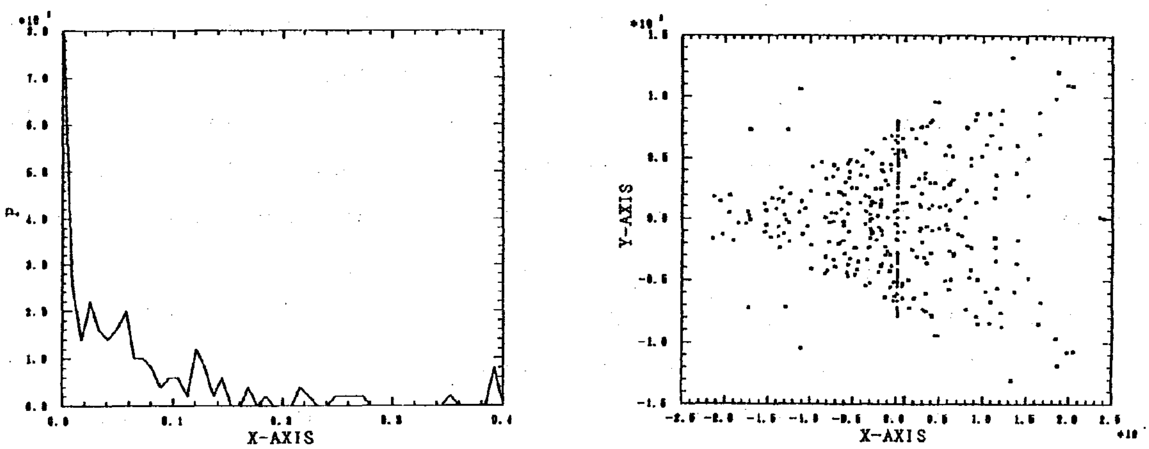

Fig.2 Difference distribution of imaginary part of eigen value, and their distribution on complex plane.