多原子系と電磁界共鳴相互作用における量子カオス

Quantum Chaos in Atoms and Resonant Electromagnetic Field Systems

Toshihiko OHNO (Kurume I.T.)

For the first case of analysis, collective two-level atomic systems driven by a external resonant periodic field are considered. Then there are no other systems coupled to it. The time evolution of density matrix is described by the following master equation.

$$\partial \rho / \partial t = -i\omega_a [S^z, \rho] - i\Omega \cos \omega t [S^+ + S^-, \rho] + 2\gamma \left(S^- \rho S^+ - \frac{1}{2}\rho S^+ S^- - \frac{1}{2} S^+ S^- \rho\right)$$

Density matrix is expanded by the coherent state of spins. Its expansion coefficients are defined as a distribution function of coherent variables. As follows;

$$\rho = \int d\theta \, d\phi \sin\theta \, |\theta, \phi\rangle P(\theta, \phi, t) \langle\theta, \phi|$$

where variables are ones of polar coordinates of Bloch sphere. Function $P(\theta, \phi, t)$ are expanded in terms of spherical harmonics and fourier series.

$$P(\theta, \phi, t) = \sum_{l<N} \sum_{|m|\le l} \sum_n i^m p_{lmn} Y_l^m(\theta, \phi) e^{imwt}$$

Expansion coefficients obey a linear set of coupled equations.

$$\begin{aligned}\frac{d}{dt} p_{lmn} = & -\gamma \left[\frac{(l-m+1)(l+m+1)}{(2l+1)(2l+3)}\right]^{1/2} l(N-l) p_{l+1,m,n} \\ & +\gamma \left[\frac{(l-m)(l+m)}{(2l-1)(2l+1)}\right]^{1/2} (l+1)(N+l+1) p_{l-1,m,n} \\ & +\gamma [l(l+1)-m^2] p_{lmn} + \Omega [(l-m)(l+m+1)]^{1/2} p_{l,m+1,n-1} \\ & - \Omega [(l+m)(l-m+1)]^{1/2} p_{l,m-1,n+1} \\ & (i\omega_a m + i\omega_g n) p_{lmn}\end{aligned}$$

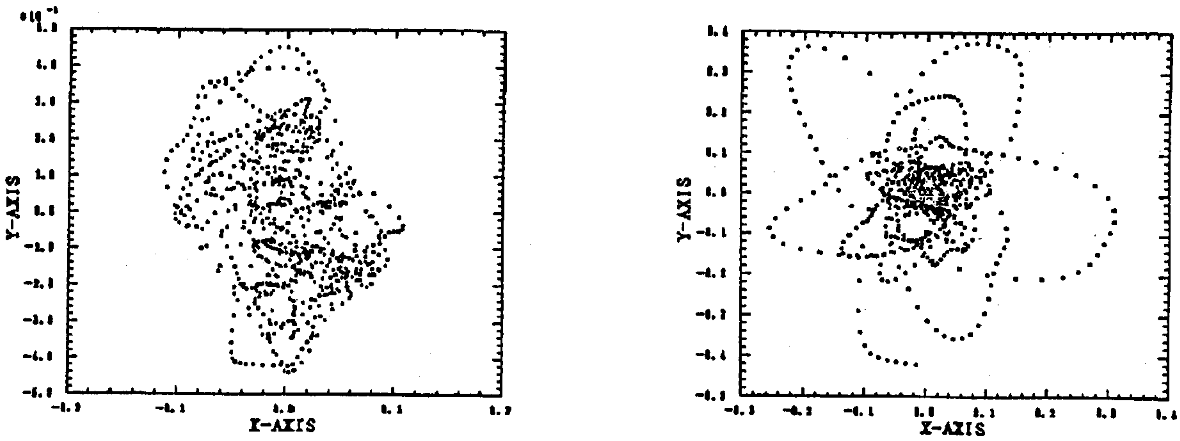
If low order terms of n, which satisfy the resonant condition with atomic frequency, are chosen, rotating wave approximation formula is obtained. In the case of strong interaction of atoms and field these approximation are not applicable.

The most fundamental question for the systems is that the linear systems described by infinite coupled equations have the possibility to chaotic motion. The answer is no, for the meaning of chaos of classical dynamical behavior. However, quantum system may have another property corresponding to the chaos of classical systems. Eigenvalues and eigenvectors are very important for identifying the property of quantum systems which may be in chaotic or non-integrable states. Eigenvalue is not same as the ones of schroedinger representation. Former include the quasi-energy levels owing to the periodic external field. Using the correlation functions, fractal dimension as a correlational dimension will be obtained. These procedure is the one for the classical dynamical systems described by few dimensional nonlinear differential equations. Instead of nonlinear equations, we are now treating linear simultaneous differential equations with periodic varying coefficients. Latter linear systems are solved by diagonalizition using the linear transformations. Limitation for the preciseness of numerical calculation is caused by the termination of infinite series for trial solutions, while finite step width of numerical procedure for non-linear equations for the case of nonlinear dynamical systems.

In the case that atoms interact with electromagnetic field which may be emitted spontaneously, these systems can be able treated as the ones interacting with harmonic oscillators. For simplicity, electromagnetic field is usually assumed to be in single modes. These systems are equivalent to the coupled systems of rotators and oscillators.These are also similar to molecular systems of coupled rotation and vibrations, although the interaction of rotator and vibrators is more complicated. Including the operators of harmonic oscillator and its interaction, density matrix is also described by the master equations. C-number distribution function whose variables are ones represented with coherent representation for both spins and electromagnetic field, is calculated as the series of orthogonal eigenfunction systems. Associated legendre functions are used for variables of spin subspace, and hermite functions for field subspace. Temporal behavior of the c-number distribution function and other behavior of the expectation values are described by the expansion coefficient of the eigen function expansions. These expansion coefficients have the form of the linear systems described by linear simultaneous differential equations with the coefficient of constant numbers or periodic variable coefficients. That is analogous to the case of periodically modulated Bloch systems. This is considered as a generalization to spin systems coupled to harmonic oscillators.

Motion of classical variables of quantum system is expressed in figures in the form of periodic plots in s_x and s_z planes. These variables have the meaning of expectation values of their corresponding operators. They are equal to the corresponding expansion coefficients of series expressing the distribution function. Distribution of the plots are analogous to ones of the case of the classically variables obtained by the non-linear equations of semiclassical theory. Number of plots dada of the figures are smaller than the ones which were actually obtained in the numerical calculation. Log-log plotted correlation function in the 2-dimensional phase space have a constant gradient in a range of distance.

Stationary spectrum of spin variables were calculated by using the eigenvalues and eigenvectors. Absolute value of real part of an eigenvalue are much smaller than one of the imaginary part, which is in the range of numerical errors in the case of no damping or no external fluctuation. Then, time dependent spectrum changes periodically with the frequency equal to the imaginary part of minimum nonzero eigenvalues. In the case that there exists an external resonant periodic field, which is treated in the rotating wave approximations, the change of spectrum owing to the external field is seen to be homogeneous in the whole region of frequency. This fact results in the change of the spectrum to continuously broadened one.



F ig.1 Periodic plots of spin S_x and S_z planes, for atoms coupled to oscillator. $\omega_a=\omega_g=1.0, \omega_c=17711/28657, \mu=1.0$ (coupling constant of oscillators),(a)Ω =0.(b)Ω =0.1

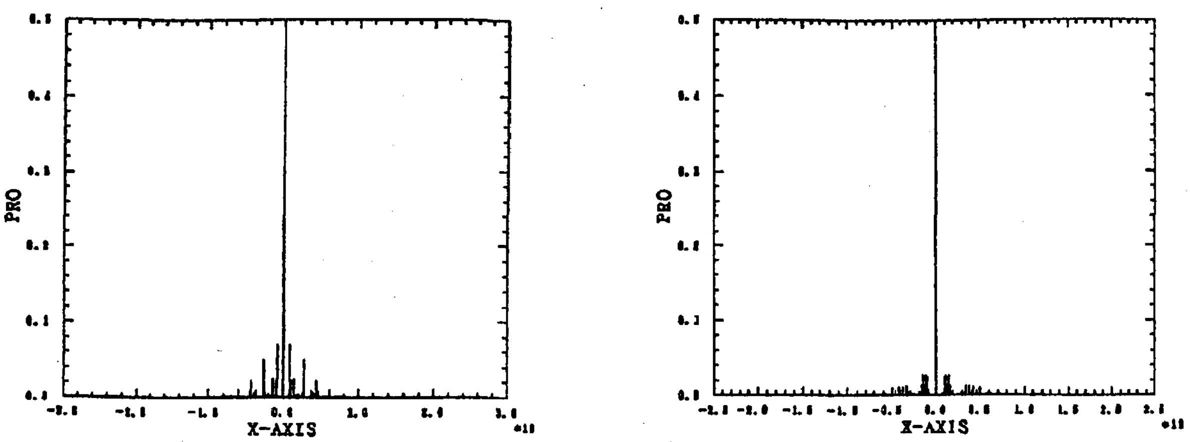


Fig.2 Spectrum of Sx,(a)Ω =0.(b)Ω =0.1