CLUSTER FORMATION IN GAS-SOLID FLOWS

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1. Introduction

Various flow patterns are observed in gas-solid flows according to the flow conditions. Many studies on such flows have been done in the context of industrial applications such as pneumatic conveyers and fluidized beds, and the change of flow pattern is mentioned in many papers (for example Zenz and Othmer(1)). When the gas velocity is large and the solid loading is small, the flow is stable and particles are distributed homogeneously. By contrast, the flow becomes unstable and inhomogeneous at low gas velocity and high solid loading.

Such an unstable flow pattern is observed in fast fluidized beds (Yerushalmi et al.(2)) and in dense phase pneumatic conveyers. This kind of instability is induced by the existence of particles and has significant effects on transport phenomena between gas and solid phases, which is important in many industrial applications. Therefore it is required to specify the critical conditions, and determine the structure of unstable flows.

Tsuo and Gidaspow(3) calculated flow patterns in circulating fluidized beds using the two fluid model, in which solid phase is modeled as a kind of fluid, and predicted unstable flows with clusters, as observed in practice. However, the assumption of continuum becomes invalid, when the mean free path of particles is not sufficiently small compared with spatial scales of changes. The Lagrangian method, in which trajectories of individual particles are calculated, is more suitable for such flows. Another advantage of this method is that particle-to-wall and particle-to-particle interactions can be taken into account based on the physical properties of materials concerned.

In the present work, flow instability induced by particles in the case of low gas velocities was studied numerically. Furthermore the structure of clusters, which are a distinguishing feature of unstable flows, and the dependence of such structures on flow conditions are investigated. Two-way coupling between both phases was of course accounted for. To model interparticle collisions the direct simulation Monte Carlo (DSMC) method, first proposed by Bird(4) for solving the Boltzmann equation of rarefied gas flow, was applied when calculating particle motion.

2. Basic Equations and Modeling

2.1. Particle motion
Particles are assumed to be rigid spheres whose diameter, mass and other physical properties are uniform. Particle motion is described with the equations of translational and rotational motion:

\[
\dot{v} = F_f/m + g,
\]

\[
\dot{\omega} = M_f/I,
\]

and the equations of impulsive motion:

\[
v^* = v + J/m,
\]

\[
\omega^* = \omega + \frac{d}{2I}n \times J,
\]

where \(v\) is the particle velocity, \(m\): the particle mass, \(F_f\): the fluid force acting on the particle, \(g\): the gravitational acceleration, \(\omega\): the rotational velocity of the particle, \(M_f\): the viscous torque caused by the fluid, \(I\): the moment of particle inertia, \(J\): the impulsive force exerted on the particle, \(d\): the particle diameter, \(n\): normal unit vector directed from the particle to outside on the contact point, and \(\cdot\) denotes a time derivative. Post-collision quantities are indicated by an asterisk.

The fluid force acting on a particle are modeled as follows;

\[
F_f = \frac{1}{2} \rho_f |u_R| A \left( C_D u_R + C_{LR} \frac{u_R \times \omega_R}{|\omega_R|} \right) + f_{LG},
\]

where \(\rho_f\) is gas density, \(A\): the projected area of a particle, \(u_R\): gas velocity relative to the particle, and \(\omega_R\) is the rotational velocity of the particle relative to the fluid; \(\omega_R = \omega - \frac{1}{2} \nabla \times u\). The first term in the right side of Eq. (5) represents the drag force, the second term, the Magnus lift force, and \(f_{LG}\), the lift force due to velocity gradient. The standard drag coefficient for a single sphere in a uniform flow was used as \(C_D\). The coefficient of lift force due to particle rotation, \(C_L\), is same with the model by Tsuji et al.\(^{(5)}\). For the lift force due to velocity gradient, Saffman’s\(^{(6)}\) theoretical result was used. Only the transverse component of \(f_{LG}\) was considered here. The viscous torque against particle rotation \((M_f)\) is given following Takagi\(^{(7)}\) and Dennis et al.\(^{(8)}\).

The following assumptions are introduced to calculate the impulsive force \(J\); (1) Particle is spherical, and particle deformation negligible. (2) The coefficient of restitution ‘\(e\)’ is constant. (3) The tangential impulsive force during the slip motion is given by the Coulomb’s friction law. (4) Slip between particles does not occur again after the initial slip dies out. According to these assumptions \(J\) is given as follows (Tsuji et al.\(^{(9)}\), Tanaka and Tsuji\(^{(10)}\)).

\[
J = J_n n + J_t t
\]

\[
J_n = (1 + e)M_n \cdot v_R
\]

\[
J_t = \min[ -\mu_f J_n, \frac{2}{7} M |v_S| ]
\]

In the above equation, \(v_R\) is the relative velocity of the collision partner to the particle, \(v_S\): the slip velocity of the collision partner with respect to the particle, \(t\): the tangential unit
vector directed to \( v_S, \mu_F \): the coefficient of friction, and \( M = m/2 \) for collision with another particle, \( M = m \) for collision against a wall.

2.2. Fluid motion

In contrast to the microscopic description of the solid phase flows, the fluid motion is described with macroscopic quantities. The fluid is assumed to be incompressible, and inviscid except for the force on particles. The fluid flow field is described with "local mean variables", defined as locally averaged values in the volume actually occupied by fluid (i.e. not by particles; cf. Anderson and Jackson\(^\text{(11)}\)). The spatial scale of averaging is larger than that of a particle but small compared with the scale of variation of macroscopic flow properties. The equation of continuity and the equation of motion used here are as follows.

\[
\frac{\partial \varepsilon}{\partial t} + \nabla \cdot (\varepsilon \mathbf{u}) = 0, \tag{9}
\]

\[
\frac{D}{Dt}(\varepsilon \mathbf{u}) = -\frac{\varepsilon}{\rho_f} \nabla p + \frac{F_p}{\rho_f}, \tag{10}
\]

where \( \varepsilon \) is the void fraction, \( \mathbf{u} \): (local mean) gas velocity \( p \): (local mean) pressure, and \( F_p \) is the force exerted on the fluid by particles per unit volume of fluid.

3. Numerical method

3.1. DSMC method

In the DSMC method, each simulated particle represents a large number of "physical" particles, so that number of particles treated can be greatly reduced. The flow field is divided into small cells in which the change in flow properties is small, within which particles are allowed to collide by a Monte Carlo procedure.

According to the DSMC method the calculation of particle motion was carried out by repeating the following procedure: (1) First, the motions of all simulated particles in the time interval \( \Delta t \) are calculated by the equations of motion, Eqs.(1) and (2), taking account of boundary conditions. (2) Secondly, inter-particle collision during the time interval \( \Delta t \) is examined by means of the Monte Carlo procedure. The collision partner and geometry of collision are also chosen with the Monte Carlo method. (3) If a particle collides with another particle, the post-collision velocities of the collision pair are calculated by the impulsive equations, Eqs.(3) and (4). The particle velocities are replaced by the post-collision velocities, but without changing their positions.

Several schemes to examine whether a particle collides with another particle in a time step have been proposed. The modified Nanbu method (Illner and Neunzert\(^\text{(12)}\)) was used in the present calculations.

The collision probability of particle \( i \) during a time step \( \Delta t \) is given by,

\[
P_i = \sum_{j=1}^{N} P_{ij}, \tag{11}
\]

where \( N \) is the number of simulated particles in the cell and \( P_{ij} \), the probability of collision between the particle \( i \) and particle \( j \) in the cell during the time step \( \Delta t \). Assuming the particles
are spheres which have a uniform diameter \( d \), \( P_{ij} \) is given by,

\[
P_{ij} = \frac{n}{N} \pi d^2 v_{Rij} \Delta t,
\]

(12)

where \( n \) is the number density of the real flow at the corresponding position, and \( v_{Rij} \) is the magnitude of relative velocity between both particles.

According to the collision probability given by Eqs. (11) and (12), the occurrence of inter-particle collision and the collision partner are decided. If the particle \( i \) collides with another particle, the velocity of particle \( i \) (though not that of collision partner) is replaced by velocity after the collision.

### 3.2. Fluid motion

The fluid motion was solved alternately with the motion of particles, using the SIMPLE scheme (Patankar, 1980). The flow region was divided into rectangular cells of dimensions \( \Delta y_f \times \Delta z_f \). \( \epsilon \) is given explicitly by

\[
\epsilon = 1 - \frac{\alpha N_f V_p}{\Delta y_f \Delta z_f},
\]

(13)

where \( \alpha \) is the ratio of "physical" particle concentration to that of simulated particles, \( N_f \) is the number of simulated particles contained in the fluid calculation cell, and \( V_p \), the volume of a particle. \( F_p \) in Eq. (10) is the reaction of fluid forces acting on particles:

\[
F_p = -\frac{\alpha}{\Delta y_f \Delta z_f} \sum_{i=1}^{N_f} F_{fi}.
\]

(14)

### 4. Results

Flows in two-dimensional vertical channels are calculated. The principal conditions are as follows; channel length: 2 m, \( \rho_f = 1.205 \text{ (kg/m}^3\text{)} \), \( d = 0.5 \text{ mm} \), particle density: 2620 kg/m\(^3\), mean particle velocity at the inlet: 0.4 m/s, fluctuation of particle velocity at the inlet: 0.2 m/s. The terminal velocity of the particles in such a gas is 3.7 m/s. The calculation region is divided into 20 x 100 rectangular cells for both phases.

In the case of high superficial gas velocities, the particle concentration shows a nearly homogeneous distribution. In such conditions the effect of particles on the gas flow field is negligible, so that the distributions of particle velocity are nearly flat across the channel. When the superficial gas velocity decreases, the flow becomes inhomogeneous and unstable, as shown in Fig. 1. It must be noted that the vertical scale of the channel is compressed in this figure. In this figure, \( W \) is the channel width, \( U \) is the superficial gas velocity, \( Q_p \) is the solid mass flux at the inlet, \( e_p \) and \( e_w \) are the coefficients of restitution, \( \mu_{Fp} \) and \( \mu_{Fw} \) are the coefficients of friction, where a subscript \( p \) denotes quantities relevant to inter-particle contact, while \( w \) indicates particle-wall contact.

It is observed that nonuniformities appear at some distance from the inlet and develop into clusters. These particle clusters substantially affect the gas flow. Gas velocity and particle velocity are reduced in particle clusters and in their wakes. In the region in which clusters are formed the flow shows an unstable behavior in space and time. It is observed that especially dense clusters which grow through coalescence can move downward along the wall.
These features of unstable flows with cluster formation qualitatively agree with experimental observations in fast fluidized bed by Yerushalmi et al. (2).

Fig. 2 shows the relation between the superficial gas velocity and the RMS of solid concentration fluctuation in the fully developed region of time and space. Local concentration values which were used to evaluate these RMS values are those in calculation cells. It can be seen that flows rapidly become unstable as $U$ approaches 5 m/s from above. It is known that particle clusters can be formed by inelastic collisions even without fluid effect (Shida and Kawai (13)). This type of cluster formation is caused substantially by energy dissipation of kinetic energy through particle-to-particle collisions. Therefore effects of inelasticity and friction on cluster formation were studied. These effects on the RMS of solid concentration fluctuation are shown in Fig. 2. In the case of $e_p = 1$ and $\mu_{FP} = 0$, kinetic energy does not decay through particle-to-particle collisions. Even in this case, the flow becomes unstable, but the growth of clusters is suppressed in comparison with other cases. From these results it can be seen that the inelasticity and the friction of particle-to-particle collision promote the growth of clusters.

5. Conclusion

Unstable phenomena and cluster formation of dispersed gas-solid flows were investigated by numerical simulation. The results are summarized as follows.

(1) The flow becomes unstable and inhomogeneous when the superficial gas velocity is low and the solid mass flux is large. The process of growth, coalescence and disappearance of particle clusters was observed.
Figure 2  Relation between the superficial gas velocity and the RMS of solid concentration 
\(W=80\text{mm}, U=5\text{m/s}, Q_s=25\text{kg/m}^2\text{s}, e_w=0.94, \mu_{fp}=0.28\)

(2) Flow instability and cluster formation are promoted by inelasticity and friction of 
particle-to-particle collisions.

References