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FLUCTUATION SPECTRUM AND MULTICORRELATION
— APPLICATION TO LOW DIMENSIONAL DYNAMICAL SYSTEMS —

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The statistical mechanics deals with the statistical description of uncontrollable fluctuations whose origin is traditionally believed to be complicated motion of a huge number of elements. Recently however it becomes evident that this is not the sufficient reason but the unpredictability is due to the *trajectory instability* in the phase space, which is present even in a system with a few degrees of freedom.

The global description of characteristics of many degrees of freedom is carried out with the thermodynamics. I talked about the statistical-thermodynamics formalism to temporal fluctuations in a way similar to thermodynamics, and obtained new results by applying it to simple dynamical systems.

Fluctuation spectrum and multicorrelation

Let $\{u_s\}$ be a time series experimentally obtained. Consider the coarse-grained observable

$$\bar{u}_t = \frac{1}{t}\int_0^t u_s ds,$$

which is a fluctuating quantity unless the limit $t \to \infty$ is taken. The probability distribution $p_t(u)$ for $\bar{u}_t$ usually takes the form

$$p_t(u) \sim e^{-s(u)t}, \quad (s(u) \geqq 0)$$

for a large t. The extensivity of $\ln p_t(u)$ is easily derived by assuming the statistical independence of the probability distributions as in the equilibrium statistical mechanics. The existence of the *fluctuations spectrum* $s(u)$ is one of main assumptions in our approach.

On the other hand by introducing the characteristic function $\phi(q)$ by

$$M_q(t) =< \exp(qt\bar{u}_t) >\sim e^{\phi(q)t},$$

(q:real), these functions are related to each other via

$$\phi(q) = \min_{u'}\{qu' - s(u')\}, \quad (s''(u) > 0).$$

The quantity $u(q) = d\phi(q)/dq$ is identical to the weighted average, q having the meaning of the degree of weight. The ensemble average of $\{u_s\}$ is equal to $u(q=0)$. The weighted average $u(q)$ for $q \neq 0$ describes the fluctuation from the ensemble average. It should be noted that the above description has a structure similar to the thermodynamics by making a correspondence $q \leftrightarrow \beta$, $u(q) \leftrightarrow$ internal energy, $s(u)$ $\leftrightarrow$entropy and $\phi(q) \leftrightarrow \beta g$, g being the Gibbs free energy. S_o the present approach is called the *statistical-thermodynamics formalism.*[1].

Although the fluctuation spectrum $s(u)$ and the characteristic function $\phi(q)$ give global statistics of the temporal fluctuation $\{u_s\}$, they contain no explicit correlation. To see it we define the order-q power spectrum [2,3]

$$I_q(\omega) = \lim_{t\to\infty} < F_t(\omega)\delta(\bar{u}_t - u(q)) > /p_t(u(q))$$
$$= \lim_{t\to\infty} < F_t(\omega)e^{qt\bar{u}_t} > /M_q(t)$$

where

$$F_t(\omega) = \frac{1}{t}|\int_0^t (u_s - u(q))e^{-i\omega t}dt|^2$$

is the spectral density. $I_0(\omega)$ is identical to the conventional power spectrum. Generally there exist infinitely many correlations. $I_q(\omega)$ describes the correlation singled out by the parameter value q. In this sense $I_q(\omega)$ is called the *multicorrelation* function.

When the dynamical law on the generation of $\{u_s\}$ in known, the above statistical functions are calculated with a generalized time evolution operator H_q, $H_{q=0}$ being the ordinary evolution operator such as the Frobenius-Perron operator and the Fokker-Planck operator. Especially $\phi(q)$ is determined by the largest eigenvalue of H_q and $I_q(\omega)$ by all eigenvalues of H_q.[2,3].

Periodic-orbit determination

The hallmark of the low dimensional chaotic systems is the strong trajectory instability. This is the reason why the prediction of future behavior is impossible in chaotic systems. However it is interesting to note that in spite of unpredictability statistical quantities can be determined in terms of unstable periodic orbits.

As a typical dynamical system, take a one dimensional chaotic map

$$x_{n+1} = f(x_n).$$

Let $u_s = u\{x_s\}$ be a unique function of x_s, e.g. orbit itself x_s, local expansion rate

$\ln|f'(x_s)|$, coarse-grained position etc, obeying the above dynamical law. An infinitely many unstable periodic orbits is embedded in a chaotic system.

The function $\phi(q)$ and the poles of $I_q(\omega)$ are determined by these periodic orbits as follows. Let us introduce the generalized Frobenius-Perron operator H_q with the xy element $(H_q)_{xy} = \delta(x - f(y))e^{qu\{y\}}$. The partition function

$$Z_q(n) = \mathrm{Tr} H_q^n = \sum_l (\nu_q^{(l)})^n,$$

where $\nu_q^{(l)}$ is the l-th eigenvalue of H_q, turns out to be expanded as

$$Z_q(n) = \int \delta(x - f^n(x)) \exp[q \sum_{j=0}^{n-1} u\{f^j(x)\}] dx.$$

The contribution to the integration thus comes form periodic orbits satisfying $x = f^n(x)$.[4]

By combining the above two equations, eigenvalues are solved by calculating $Z_q(1)$, $Z_q(2)$, $Z_q(3)$, $\cdots$. To this end we proposed a continued-fraction expansion of the Laplace transform of $Z_q(n)$, whose poles yield the eigenvalues. Figure 1 shows the results with finite-pole approximations for a simple piecewise linear map. As the number of poles is increased, the results tend to agree with the exact results.[4]

Anomalous correlations associated with intermittency

Intermittency is a prominent phenomenon observed in nonlinear dynamical systems. Several years ago we reported that a new intermittency is observed when the synchronization breaks down under the change of control parameter. This is different from the well-known intermittency classified by Pomeau and Manneville.

Yamada *et al* carried out an experiment on the coupled electronic circuit whose unit is composed of a LCR circuit with a voltage-dependent capacitance.[5] They found, changing the coupling constant, a prominent intermittency characteristic slightly below the critical value where the synchronization looses its stability.

Figure 2 shows the voltage difference between two sub-circuits, which vanishes if the synchronization is realized.[5] They found that the order-q power spectrum $I_q(\omega)$ for the voltage difference has the power law

$$I_q(\omega) \sim \omega^{-\nu_q}$$

for certain regions of ω. They have reported the indication of the fact that ν_q depends on q, (Fig.3).[6]

Very recently Just *et al* [3,7] have rigorously obtained the order-q power spectra for the type-I intermittency maps with the order of tangency z, and found that the local maxima of peak trains obey the asymptotic law

$$I_q(\omega) \sim \begin{cases} \omega^{-\nu} & (q < q_c) \\ \text{Lorentzian type} & (q < q_c) \end{cases}$$

for low frequency region, where $q_c(> 0)$ is a characteristic value of q, and the exponent ν is a function of z but is independent of q. This is different from the result observed in the coupled electronic circuit.

After the talk, Yamada and myself[8] succeeded in finding rigorous results of $I_q(\omega)$ for the multiplicative stochastic process $\dot{r}_t = (\Delta - r_t^2 + f_t)r_t$, $(\Delta \gtrsim 0)$, where r_t is a physical quantity, f_t is the Gaussian-white random force. It is known that the statistical property of intermittency observed in the desynchronization of chaotic oscillations is well modeled by the above stochastic process. We found that $I_q(\omega)$ has a structure similar to that in the above type-I intermittency case. Namely the q-dependent exponent ν_q of $I_q(\omega)$ reported in the coupled electronic circuit seems to be not conclusive. A more careful analysis is needed.

References

[1] H. Fujisaka and M. Inoue, Prog. Theor. Phys. **77** (1987) 1334, (Statistical-Thermodynamics Formalism of Self-Similarity).

[2] H. Fujisaka and H. Shibata, Prog. Theor. Phys. **85** (1990) 187, (New Asymptotic Law Governing Overall Temporal Correlations in Chaotic Systems — Ensemble Homogenization and Order-q Power Spectrum —).

[3] W. Just and H. Fujisaka, Physica D, in press, (Gibbs Measures and Power Spectra for Type I Intermittency Maps).

[4] H. Fujisaka, H. Shigematsu and B. Eckhardt, Z. Phys. B, in press, (Continued-Fraction Expansion of Eigenvalues of Generalized Evolution Operators in Terms of Periodic Orbits).

[5] T. Yamada, K. Fukushima and T. Yazaki, Prog. Theor. Phys. Suppl. No.99, (1989), 120, (A New Type of Intermittency in an Electronic Circuit)

[6] K. Fukushima and T. Yamada, preprint.

[7] T. Kobayashi, H. Fujisaka and W. Just, Phys. Rev. E **47** (1992), No.4, (Thermodynamics and Multicorrelations of Intermittent Dynamics).

[8] H. Fujisaka and T. Yamada, Prog. Theor. Phys. in press, (New Approach to Multiplicative Stochastic Process.II — Infinite Number of Time Correlations —).

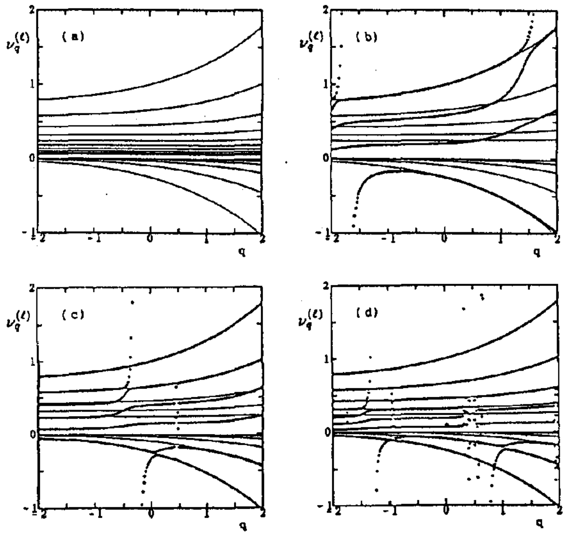

Fig.1 (a) Exact eigenvalues of H_q for a piecewise linear chaotic map.[4] Twenty eigenvalues are shown in the order of increasing values. The results from finite-pole approximations of the continued-fraction expansion are shown in figures (b) using 4 poles, (c) using 6 poles and (d) using 8 poles. Solid and dotted lines respectively indicate results of finite-pole approximation and exact results.

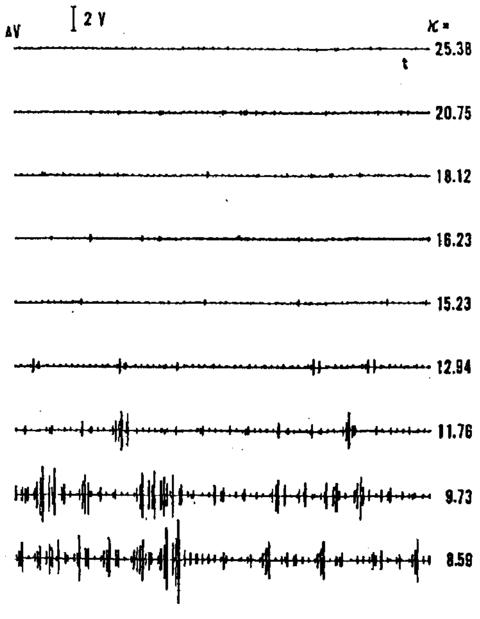

Fig.2 Intermittency observed in a coupled LCR electronic circuit.[5] ΔV is a voltage difference between two partial circuits and κ is the coupling constant.

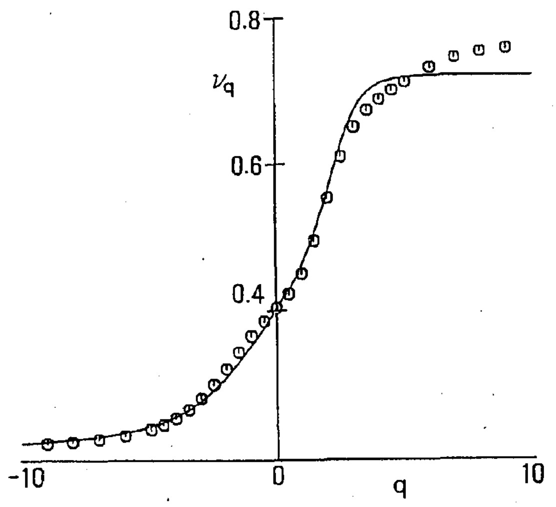

Fig.3 The q-dependence of the exponent ν_q of $I_q(\omega)$ of the voltage difference, observed for the coupled LCR electronic circuit.[6]