Asymptotic Behavior of Spin-Pair Correlation Function of Ising Model on Checkerboard Lattice

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Vdovichenko[1)] introduced the diagrammatical method for the two-dimensional Ising model with non-crossing interaction. Morita[2)] extended the method by introducing techniques of division and integration in the lattice sites for the Ising model.

We consider the Ising model on the checkerboard lattice with additional interaction J_y, as shown in Fig.1(a). The Hamiltonian is given by

$$H = \sum_{k=1}^{K} \sum_{l=1}^{L} \left[-J_y s_{k,l,1} s_{k,l+1,1} + h(s_{k,l+1,2}, s_{k,l,2}, s_{k+1,l,1}, s_{k,l,1}) \right],$$

where $h(s_1, s_2, s_3, s_4)$ is the Hamiltonian of a shaded square cluster. For four spins s_j $(j=1,2,3,4)$ in the shaded-square as shown in Fig.2, we assume that

$$\langle s_i \rangle_2 = 0, \qquad \langle s_i s_j s_k \rangle_2 = 0 \qquad (i,j,k=1,2,3,4),$$

$$\langle s_1 s_2 s_3 s_4 \rangle_2 + \langle s_1 s_2 \rangle_2 \langle s_3 s_4 \rangle_2 = \langle s_2 s_3 \rangle_2 \langle s_1 s_4 \rangle_2 + \langle s_1 s_3 \rangle_2 \langle s_2 s_4 \rangle_2,$$

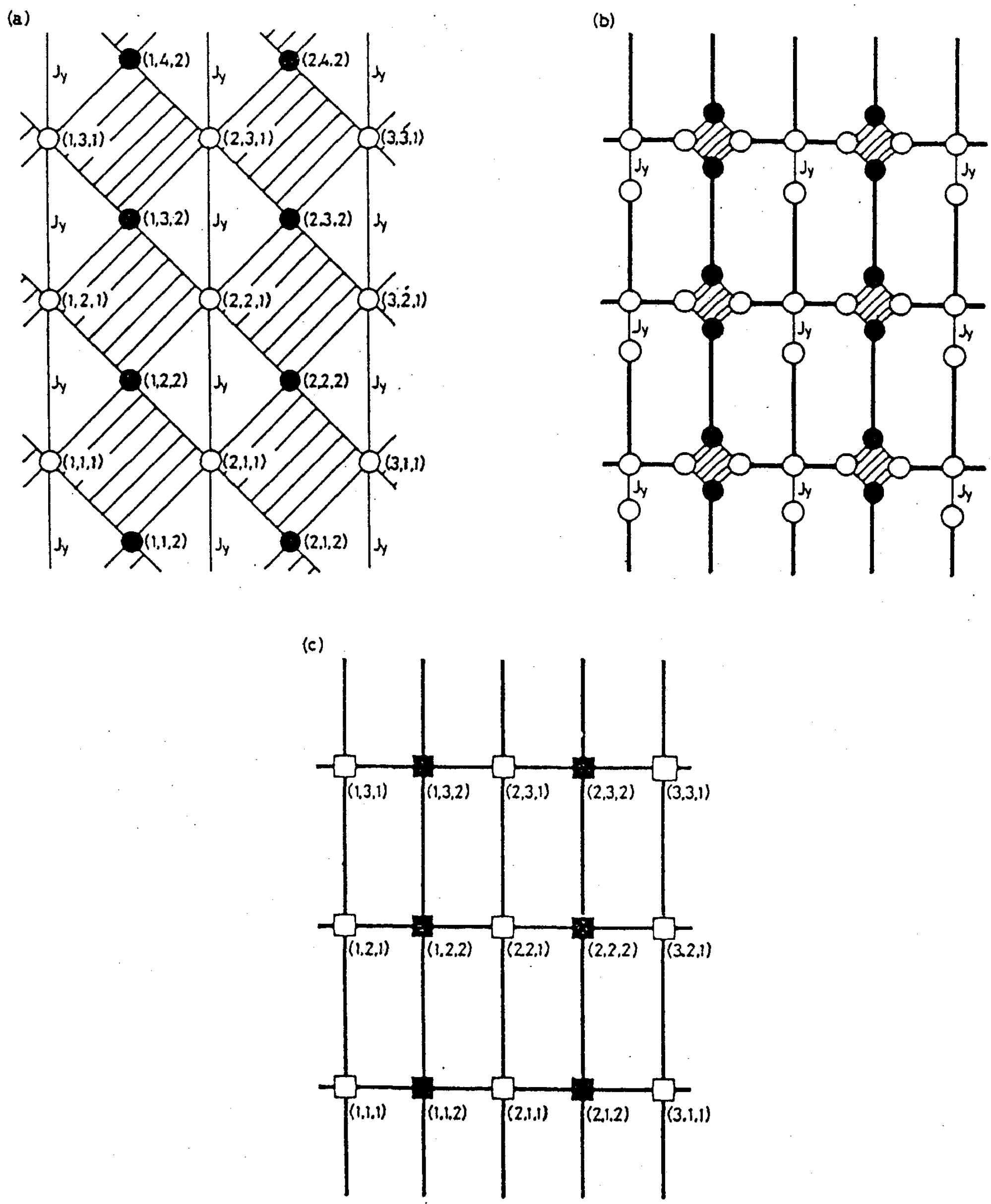

Fig. 1. Ising model on the checkerboard lattice with an additional interaction J_y, and its reduction to the layered free-fermion eight-vertex model on the square lattice. By the technique of division, we obtain (b) from (a). By the technique of integration, we obtain (c) from (b).

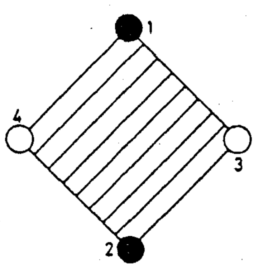

Fig. 2. Cluster consisting of four lattice sites. The spin variable of the spin on a lattice site i $(i=1, 2, 3, 4)$ is denoted by s_i. The hamiltonian of this cluster is denoted by $h(s_1, s_2, s_3, s_4)$.

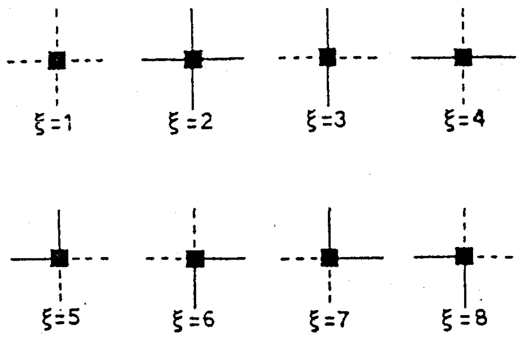

Fig. 3. Vertex configurations $\xi_{k,l,\nu}$ which are permitted in the eight-vertex model.

$$\langle Q\rangle_2 \equiv \frac{1}{Z_2}\sum_{s_1=\pm1}\sum_{s_2=\pm1}\sum_{s_3=\pm1}\sum_{s_4=\pm1}Q\exp\{-\beta_{\mathrm{T}}h(s_1,s_2,s_3,s_4)\},$$

$$Z_2 \equiv \sum_{s_1=\pm1}\sum_{s_2=\pm1}\sum_{s_3=\pm1}\sum_{s_4=\pm1}\exp\{-\beta_{\mathrm{T}}h(s_1,s_2,s_3,s_4)\},$$

where $\beta_{\mathrm{T}}\equiv 1/k_{\mathrm{B}}T$, k_{B} is the Boltzmann constant and T is the absolute temperature. The model is reduced to the free-fermion eight-vertex model consisting of two kinds of sublattice, as shown in Fig.1. The partition function of the equivalent free-fermion eight-vertex model is given as

$$Z = \mathrm{Tr}_{8\mathrm{V}}\prod_{k=1}^{K}\prod_{l=1}^{L}\omega_1(\xi_{k,l,1})\omega_2(\xi_{k,l,2}),$$

where the symbol $\mathrm{T}_{8\mathrm{V}}$ means the summation over all the possible solid-bold complexions. The possible vertex configurations are denoted by $\xi_{k,l,\nu}=1,2,\cdots,8$ as shown in Fig.3. The weights of the eight-vertex model are given by

$$\left.\begin{array}{ll}\omega_1(\xi) \equiv \sinh(\beta_{\mathrm{T}}J_y), & (\xi = 2,3,6,8),\\ \omega_1(\xi) \equiv \cosh(\beta_{\mathrm{T}}J_y), & (\xi = 1,4,5,7),\end{array}\right\}$$

$$\left.\begin{array}{ll}\omega_2(1) \equiv (Z_2/4), & \omega_2(2) \equiv (Z_2/4)\langle s_1s_2s_3s_4\rangle_2,\\ \omega_2(3) \equiv (Z_2/4)\langle s_1s_2\rangle_2, & \omega_2(4) \equiv (Z_2/4)\langle s_3s_4\rangle_2,\\ \omega_2(5) \equiv (Z_2/4)\langle s_2s_3\rangle_2, & \omega_2(6) \equiv (Z_2/4)\langle s_1s_4\rangle_2,\\ \omega_2(7) \equiv (Z_2/4)\langle s_1s_3\rangle_2, & \omega_2(8) \equiv (Z_2/4)\langle s_2s_4\rangle_2,\end{array}\right\}$$

$$\omega_\nu(1)\omega_\nu(2)+\omega_\nu(3)\omega_\nu(4)=\omega_\nu(5)\omega_\nu(6)+\omega_\nu(7)\omega_\nu(8), \quad (\nu = 1,2).$$

Next, we consider the spin-pair correlation function of the Ising model

$$\langle s_{1,1,1}s_{1,1+N,1}\rangle = \frac{1}{Z}\,\mathrm{Tr}_{\mathrm{I}}s_{1,1,1}s_{1,1+N,1}\exp(-\beta_{\mathrm{T}}H).$$

The numerator $\mathrm{Tr}_{\mathrm{I}}s_{1,1,1}s_{1,1+N,1}\exp(-\beta_{\mathrm{T}}H)$ is equal to the partition function of the free-fermion eight-vertex model which is obtained by replacing the weight $\omega_1(\xi_{1,l,1})$ for $2\le l\le N$ by

$$\left.\begin{aligned}\omega_1(\xi) &\equiv \cosh(\beta_{\mathrm{T}}J_y), \quad (\xi = 2,3,6,8),\\ \omega_1(\xi) &\equiv \sinh(\beta_{\mathrm{T}}J_y), \quad (\xi = 1,4,5,7).\end{aligned}\right\}$$

Hence, the spin-pair correlation function can be obtained by applying the Vdovichenko's method to the layered free-fermion eight-vertex model and is expressed in terms of a block Toeplitz determinant whose generating function is a 2×2 matrix function. The detailed calculation was given in Ref.3. And the more generalized version for the diagrammatical techniques in reducing from two-dimensional Ising models to vertex models was given in Ref.4. The asymptotic behavior is obtained by using the theory of the block Toeplitz determinant reformulated by Tanaka, Morita and Hiroike.[5)]

The model studied here includes the Ising model on the generalized Kagomé lattice as shown in Fig.4.[3,6)] The Ising model with $J_5=J_6=J_7=J_8$, $J_1=J_2$ and $J_3=J_4$ corresponds to the model treated by Debauche et al.[7,8)] In Ref.6, We calculated the asymptotic behavior of the spin-pair correlation function of the Ising model and discussed about the nature of the disordered points.

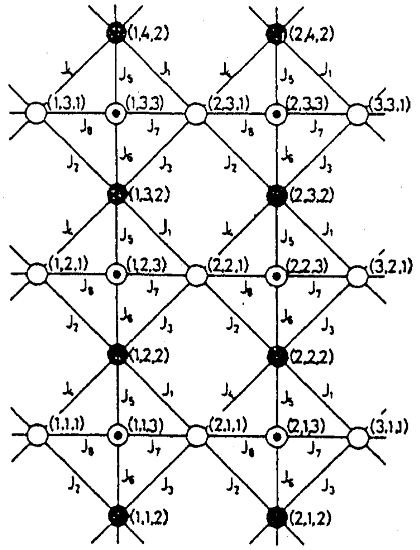

Fig. 4. Ising model on the generalized Kagomé lattice.

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