Note on Woodford's Conjecture: Constructing Stationary Sunspot Equilibria in a Continuous Time Model

Kyoto Institute of Economic Research, Kyoto University　Tadashi Shigoka

If for given equilibrium dynamics there exist a continuum of non-stationary perfect foresight equilibria all converging asymptotically to a steady state (a deterministic cycle resp.), we say the equilibrium dynamics is indeterminate near the steady state (the deterministic cycle resp.). Suppose that the fundamental characteristics of an economy are deterministic, but that economic agents believe nevertheless that equilibrium dynamics is affected by random factors apparently irrelevant to the fundamental characteristics (sunspots). This prophecy could be self-fulfilling, and one will get a sunspot equilibrium, if the resulting equilibrium dynamics is subject to a nontrivial stochastic process and confirms the agents' belief. See Cass-Shell [2].

Woodford [6] suggested that there exists a close relation between the indeterminacy of equilibrium near a deterministic steady state and the existence of stationary sunspot equilibria in the immediate vicinity of it. We might summarize Woodford's conjecture as what follows: "Let $\bar{x}$ be a steady state of a deterministic model which has a continuum of non stationary perfect foresight equilibria all converging asymptotically to the steady state. Then given any neighborhood $U(\bar{x})$ of it, there exist stationary sunspot equilibria with a support in $U(\bar{x})$.

Azariadis, Farmer, Grandmont, Guesnerie, Woodford, Chiappori, and Geoffard investigate the connection between the local indeterminacy of equilibria and the existence of local stationary sunspot equilibria thoroughly and show the conjecture holds good in extremely general situations. However the existing results supporting Woodford's conjecture are all derived from discrete time models. See Chiappori-Guesnerie [3] and Guesnerie-Woodford [5] for thorough surveys on the existing sunspot literature. The purpose of this note is to show that Woodford's conjecture extends to a continuous time model. We present the method of constructing stationary sunspot equilibria near a steady state (a closed orbit resp.) in a continuous time

model, where equilibrium is indeterminate near the steady state (the closed orbit resp.). One can use our method to show there exist stationary sunspot equilibria in such models as treated by Diamond-Fudenberg [4], and Benhabib-Farmer [1]. The model treated by [4] includes a stable limit cycle, where equilibrium is indeterminate around the stable limit cycle. One can use our method to construct stationary sunspot equilibria around the stable limit cycle in these models.

References

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