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On Ordered States in 4-Fermi and 4-Spin Models

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1 Introduction

I had planed to talk about three topics in my recent studies of low-dimensional 4-Fermi and 4-spin models, including the joint work with I. Ichinose (ref. [2] of sect.2). The two works (sects.2 and 3) on 4-fermi model are motivated by the possibility of low-dimensional superconductivity, while the work treated in sect.4 is inspired by the recent Monte Carlo simulation for the two-dimensional $\mathbb{R}P^{N-1}$ model. Since the time actually given for the talk is too limited, I focused on the latter in the Workshop. In the followings I just present the motivations and the brief summary of the results. Those who are interested in the details are advised to make reference to the papers cited.

2 Phase Structure of quasi (2+1)-dimensional 4-Fermi theory with global chiral U(1) symmetry

Quantum field theories in $(2+1)$-dimensions have recently attracted much attention, due partly to the fact they may have relevance to the long-wavelength properties of possible microscopic models describing high-T_c superconductivity. Since the real system is a quantum many-body system of electrons or holes which are strongly correlated, it is instructive to consider interacting fermion theories with U(1) symmetry. However, real superconducting materials have very weak but finite interlayer coupling, and the long-distance properties of quasi $(2+1)$-dimensional systems have subtleties and require careful investigation [1].

In order to get insight into the long-distance properties of quasi (2+1)-dimensional superconductors with no external magnetic field threatened, we studied in ref.[2] the finite-temperature phase structure of quasi $(2+1)$-dimensional 4-Fermi theory with global chiral U(1) symmetry and large but finite N flavors. The quasi $(2+1)$-dimensionality is taken into account by introducing a small anisotropy parameter in the Dirac-type kinetic term. Using the $1/N$ expansion and the mapping to the XY model, we argued that the model with strong coupling has three phases separated by two different kinds of transitions. The two types of ordered phase appear at low temperatures. The lowest-temperature phase is the Nambu-Goldstone phase with a symmetry-breaking order parameter. The intermediate-temperature regime is the Kosterlitz-Thouless phase where the low-energy excitation is a zero-mode of U(1)-phase wave and its dynamics is described by a classical XY model with quasi two-dimensionality.

References

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3 Long-Distance Properties of Frozen U(1) Higgs and Axially U(1)-gauged 4-Fermi Models in $1+1$ Dimensions

As mentioned in sect.2 we found that the purely (2+1)-dimensional 4-Fermi theory possesses the Kosterlitz-Thouless (KT) phase at low-temperatures. It is then an interesting question to ask whether the Higgs mechanism operates or not in this KT phase if the model is coupled to an axial gauge field. This question may be well parallel to the analogous issue in superconductivity, i.e. if a Meissner effect takes place or not when an external magnetic field is applied to purely two-dimensional superconductors. It will be expected that the qualitative long-distance properties of a (2+1)-dimensional model at finite temperature is essentially similar to those of a (1+1)-dimensional model at zero temperature, in which a local gauge symmetry is easier to treat.

In the KT regime the dynamics of vortex excitations is important, since they, if relevant at long distances, work toward disordering the system. In ref. [1] I then studied the long-distance relevance of vortices (instantons) in an N-component axially U(1)-gauged 4-Fermi theory in $1+1$ dimensions, in which a naive use of $1/N$ expansion predicts the dynamical Higgs phenomenon. Its general effective lagrangian is found to be a frozen U(1) Higgs model with the gauge-field mass term proportional to an anomaly parameter (b). The dual-transformed versions of the effective theory are represented by sine-Gordon systems and recursion-relation analyses are performed. The results suggest that in the gauge-invariant scheme ($b=0$) vortices are always relevant at long distances, while in non-invariant schemes ($b>0$) there exists a critical N above which the long-distance behavior is dominated by a free massless scalar field. Therefore, in any case, the naive Higgs picture seems not to hold, and the similar conclusion is expected for the (2+1)-dimensional system at finite temperatures.

References

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4 Effective Lagrangian for the Two-Dimensional Model at Low Temperatures

In the past decades non-linear sigma models (NLσM) have been one of the important subjects in various contexts of theoretical physics. Among them are various spin systems in statistical physics, string models with backgrounds, Anderson localization in disordered systems, etc.. In spite of such importance and popularity, the classification of their universality classes has not yet been completed, even for the classical spin systems in two dimensions.

Recent Monte Carlo simulations [1] for the two-dimensional $\mathbb{R}\mathrm{P}^{N-1}$ (4-spin) models, defined by N-components unit spins $\mathbf{n}_x$, $|\mathbf{n}_x|^2 = 1$ with the hamiltonian

$$\begin{aligned} H &= \frac{J}{4}\sum_x\sum_\mu \mathrm{Tr}\left(\mathbf{n}_{x+\mu}\otimes\mathbf{n}^\dagger_{x+\mu} - \mathbf{n}_x\otimes\mathbf{n}^\dagger_x\right)^2 \\ &= \mathrm{const.} - \frac{J}{2}\sum_x\sum_\mu (\mathbf{n}_x\cdot\mathbf{n}_{x+\mu})^2, \end{aligned} \tag{4.1}$$

have shown that the matrix (lattice) NLσM have a long-distance behavior which is different from that for the corresponding vector models (S^{N-1}) defined by

$$H = \frac{J}{4}\sum_x\sum_\mu (\mathbf{n}_{x+\mu} - \mathbf{n}_x)^2 = \mathrm{const.} - \frac{J}{2}\sum_x\sum_\mu \mathbf{n}_x\cdot\mathbf{n}_{x+\mu}. \tag{4.2}$$

Remarkably, in the model (4.1) the strong evidences have been obtained for the low-temperature ordered phase with the diverging spin-spin correlation length. This discrepancy can not be explained by the renormalization-group (RG) applied to the naive continuum lagrangians, which are commonly given by the O(N)- (vector NLσ) model,

$$S = \frac{1}{2t}\int d^2x|\partial\mathbf{n}|^2. \tag{4.3}$$

The results of the simulation thus present an interesting example in the classification theory of universality classes for the two-dimensional classical spin systems. A better analytical understanding is highly desired.

Motivated by this fact, I studied the low-temperature long-distance effective lagrangian for the two-dimensional $\mathbb{R}\mathrm{P}^2$ model by means of the saddle point (SP) plus renormalization methods [2]. Introducing a matrix order parameter Q via the Hubbard-Stratonovich transformation I analyzed the system of a macroscopic effective continuum theory for the Q field. After taking the $\mathrm{O}(1)\times\mathrm{O}(2)$-invariant SP for the potential I renormalized the massive fluctuations suppressed by the cut-off. This procedure allows us to find, in addition to the O(3)-model, a new term for the massless modes. Explicitly,

$$\mathcal{L}_{\mathrm{eff}} = \frac{2}{t}\left[\frac{(\partial b)^2}{(1+b^2)^2} + 4v\,\frac{b^2(\partial b)^2 - (b^\dagger\cdot\partial b)^2}{(1+b^2)^2}\right], \tag{4.4}$$

where $b = (b_1, b_2)^\dagger$ is a stereographic coordinate of S^2 and v is a new coupling constant generated by the massive fluctuations.

Thanks to the new term, it is expected that the infrared RG behavior for the $\mathbb{R}P^2$ model could be distinguished from the S^2 model. The effective lagrangian itself turns out to be perturbatively non-renormalizable. The RG analysis is formulated in the space of general O(2)-invariant non-linear lagrangians with an infinite number of coupling constants

$$\mathbf{u} \equiv (u_1, u_2, \cdots), \quad \mathbf{v} \equiv (v_0, v_1, \cdots), \quad v_0 = 4v. \tag{4.5}$$

We performed the RG analysis in the first-order in t and v, by use of the same field-renormalization constant as of the O(3)-model. The results show that at long distances the effective lagrangian in the vicinity of zero temperature deviates from the O(3)-model as seen in Fig.1.

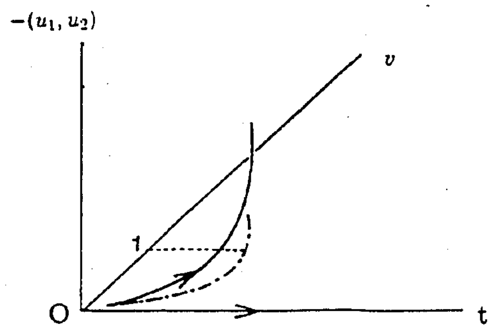

Fig.1. The first order RG flow for $\mathcal{L}_{\text{eff}}$ in the space of O(2) non-linear lagrangians. The dotted line is its projection on to the (t, v) plane. The t axis corresponds to the O(3) model.

The consequence throws a big doubt upon the validity of using the naive continuum model as a correct long-distance effective lagrangian for the low temperature $\mathbb{R}P^{N-1}$ model, and the possibility is left open for an ordered phase. Namely, If the RG flow will come close in the infrared limit to a zero-temperature hyper surface $(t = 0, \mathbf{u}, \mathbf{v})$ so that all the components of $(t\mathbf{u}, t\mathbf{v})$ could go to zero, then there all the interactions effectively vanish and the theory in the infrared limit is described by free massless bosons b_1, b_2. In this way the low-temperature phase could be realized with the diverging correlation length, in accordance with the Monte Carlo results [1]. It would be very interesting to confirm this *non-trivial infrared freedom.*

References

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