Absence of Reentrance in the Two-Dimensional XY-Model with Positional Disorder

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ABSTRACT

We show, that the 2D XY-model with random phase shifts exhibits for low temperature and small disorder a phase with quasi-long-range order, and that the transition to the disordered phase is *not* reentrant. These results are obtained by heuristic arguments, an analytical renormalization group (RG) calculation, and a numerical Migdal-Kadanoff renormalization group treatment.

We reconsider in this paper the 2-dimensional XY-model

$$\mathcal{H} = -J \sum_{<i,j>} \cos(\phi_i - \phi_j - A_{ij}) \tag{1}$$

with quenched random phase shifts A_{ij} on the bonds, where i, j run over the sites of a square lattice. For simplicity we assume, that the A_{ij} on different bonds are uncorrelated and gaussian distributed with mean zero and variance σ.

For vanishing A_{ij} model (1) undergoes a Kosterlitz-Thouless (KT) transition, at which the spin-spin correlation exponent η jumps from 1/4 to zero [1]. Weak disorder, $\sigma \ll 1$, the features of the KT-transition are essentially preserved, but the transition is shifted to lower temperatures and the jump of η at the transition is diminished [2] (strong disorder will suppress the quasi-long-range order of the KT phase).

Rubinstein, Shraiman and Nelson (RSN) [2] extended the Coulomb gas description of the KT-transition [1] to the presence of randomly frozen dipoles arising from the random phase shifts. Surprisingly, they found a second (reentrant) transition at $T_{re}(\sigma)$ ($\leq T_{-}(\sigma)$) to a disordered phase at low temperatures (see Fig. 1). $T_{re}(\sigma)$ bends towards higher temperatures for increasing disorder. Experiments [3] as well as Monte Carlo studies [4] indicate no reentrance. Also, Ozeki and Nishimori [5] have shown for a general class of random spin systems, which include (1), that the phase

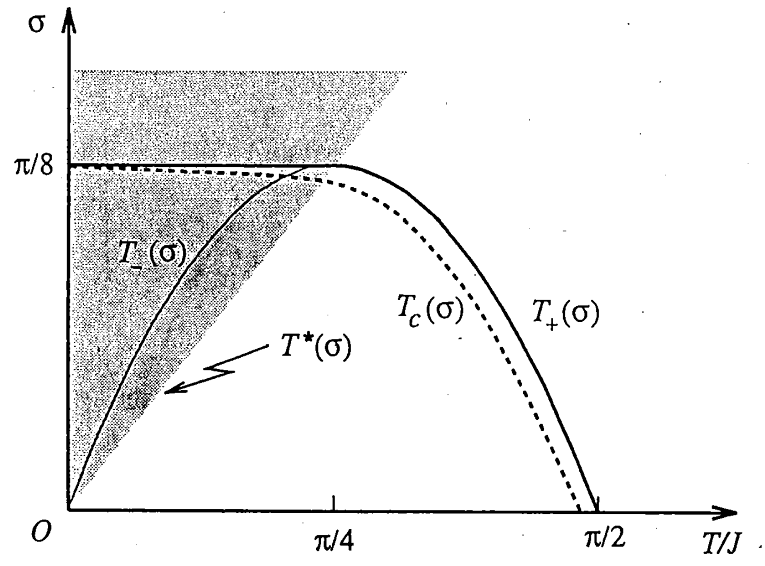

Figure 1: $\sigma - T$ phase diagram of model (1). $T_\pm(\sigma)$ are the upper bounds for the transition temperature $T_c(\sigma)$ between the disordered and the KT phase in the RSN-theory [2]. The T_--line lies completely in the freezing region (hatched area). The true phase transition line $T_c(\sigma)$ is denoted by the dashed line which is bounded by $T_+(\sigma)$ and $\sigma = \pi/8$. The line T_{re} is not shown here.

boundary between the KT and paramagnetic phases is parallel to the T axis for low T. Thus they exclude a reentrant transition, provided the intermediate KT phase exists. However, they cannot rule out the possibility, that the KT phase *disappears* completely.

We will argue below, that the reentrant transition is indeed an artefact of the calculation scheme used in [2] and that the KT-phase is stable at low temperatures with $T_c(\sigma) \to 0$ for $\sigma \to \pi/8$ (see Fig.1).

For the further discussion one has to take into account only a vortex part $\mathcal{H}_v$ of the Hamiltonian (1). In the continuum description we have

$$\begin{aligned} \mathcal{H}_v &= -J\pi \sum_i m_i \{ \sum_{j \neq i} m_j \ln |\mathbf{r}_i - \mathbf{r}_j| + \\ &+ 2 \int d^2 r Q(\mathbf{r}) \ln |\mathbf{r} - \mathbf{r}_i| - \frac{E_c}{J\pi} m_i \}. \end{aligned} \tag{2}$$

The integer vortex charges m_i satisfy $\sum_i m_i = 0$. $Q(\mathbf{r})$ is a quenched random charge field, which is related to the phase shift $\mathbf{A}(\mathbf{r})$ by $2\pi Q(r) = -\partial_x A_y + \partial_y A_x$. E_c is a core energy of a single vortex.

To find the correct behaviour at low temperatures, we we have to take into account the screening of the vortex and quenched random charges by other vortex pairs. This

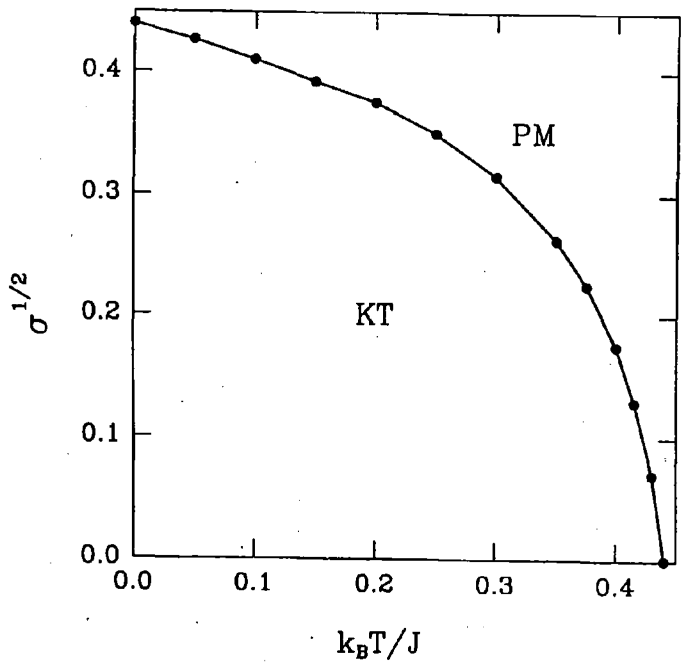

Figure 2: $\sigma - T$ phase diagram obtained by the discretized MKRG scheme. PM and KT denote the paramagnetic and KT phases respectively. The critical value $\sigma_c^{1/2}(T=0) \approx k_B T_c(\sigma=0)/J \approx 0.44$.

can be done most easily by using the dielectric formalism [6]. At $T = 0$ we have the following system of RG equations for the couplings J, the vortex fugacity y ($y = e^{-F_c/T}$, F_c is the core free energy of a single vortex on the scale e^l) and σ

$$\begin{aligned} \frac{dJ}{dl} &= -4\pi^3 c \frac{J}{\sqrt{\pi\sigma l}} y^2 \\ \frac{dy}{dl} &= (2 - \frac{\pi}{4\sigma}) y \\ \frac{d\sigma}{dl} &= 0 \ , \end{aligned} \tag{3}$$

where we neglected terms of the order σ/l. Within this approximation, the system undergoes a phase transition at $\sigma_c = \pi/8$ from a KT to a disordered phase. At σ_c the exponent η shows a universal jump from $1/16$ to zero.

We have extended our analysis to $T \neq 0$. One can show that for $0 \leq T \leq T^* = 2J\sigma$ system of equations (3) is still valid and theory of RSN is correct for $T \geq T^*$. This means that there is no reentrancy transition from KT phase to the paramagnetic phase.

Our conclusions about the absence of reentrance are confirmed also by a discretized Migdal-Kadanoff renormalization group (MKRG) scheme [7]. In the discretized scheme instead of allowing ϕ to be a continuous variable, we constrain it to take one of many discrete values which are uniformly distributed between 0 and 2π. Hamiltonian (1) is now defined for values of ϕ restricted to $2\pi k/q$, where $k = 0, 1, 2, \ldots, (q-1)$ and q is a number of clock states. We define

$$J_{ij}(q,k) \;=\; J\cos(2\pi k/q - A_{ij}). \tag{4}$$

The recursion relations for $J_{ij}(q,k)$ may be found in [7]. One can iterate those recursion relations to obtain the phase diagram. The result is shown in Fig. 2. Thus the MKRG gives us an additional evidence that the reentrance is absent in model (1). The details of this report may be found in [8].

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