A FAST COMPUTATION IN SYMMETRIC TRAVELING SALESMAN PROBLEMS

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This report deals with a computational method for deriving an approximately optimal solution of symmetric traveling salesman problems (TSP), which are currently and widely studied by numerous researchers in scientific and engineering fields. The increase of cities traveled by a salesman, however, brings serious difficulty in solving symmetric TSP, owing to NP characteristics in general. In this respect, a new approach for obtaining an approximate optimum has developed as follows.

By using a concept of critical states in statistical physics, first we divide an area where each city involved is denoted by a point into cells until the number of cells rises over that of the cities, and then number the cells along a fractal line including no intersections of its own; next we connect cities one by one in this numerical order of the cells, skipping over cells having no cities. For a cell having cities, we would divide once more or directly connect them in accordance with their input sequence because of the cell small. Thus a path through the cities, or a solution of symmetric TSP is derived.

With identical square cells connected by a new closed Hilbert line obeying 2 fractal dimension (Fig.1), an NEC-EWS4800/320VX on Fortran77 code ran, for instance (a) 0.00 sec for 4 through 11 cities with 4 different locations, i.e. 32 examples, situated randomly on 16 cells at average 8.5 % over-optimum and (b) 0.18 sec for a USA532-set placed on 1024 cells at 46.9 % over-optimum (Fig.2); for cities on $4^n$ cells with integer $n$ large, the time of computer running takes proportionally a value deviating to some extent to lower side of $4^n$, e.g. 0.20 and only 0.68 sec respectively for 1024($=4^5$) and 4096($=4^6$) cities presented randomly on 1024 and 4096 cells. The worst case where one of $4^n$ cells has almost the cities takes max.2x$4^n$ operations to obtain a solution. Further, the method always derives an optimal path for $4^n$ cities located uniformly on $4^n$ cells, whose distribution is like that in ground state of ferromagnetic spin systems (Fig.1), and also for $4^n/2$ cities that of antiferromagnetic spin systems, while physical meanings of these city distributions are now being sought in symmetric TSP.

The running times observed should become shorter by a fractal function generating the Hilbert line explained, because we devised the code with many IF statements instead of this function. Since we linked geometrical fractal similarity to statistical scale universality in symmetric TSP, an emphasis is placed on a path through the cells rather than that through the cities; in the present study, we treat symmetric TSP by an intensive variable, i.e. a city per cell with the fractal connection above. Note that the method is independent of the integer property of the city places. In addition the method never uses global-local step searches—corrective techniques—such as n-opt orders, simulated annealings, taboo-list searches and genetic algorithm, though the over-optima observed would appear to be small enough.

The method reported is fairly simple and derives a solution considerably fast, and is applicable to multi-dimensional symmetric TSP involving even a barrier between cities.
Fig. 1 Hilbert line through 1024 cities located uniformly on 1024 cells. Each city is denoted by a cross.

Fig. 2 USA532-set placed on 1024 cells.

References