PATTERN FORMATIONS IN REACTION-DIFFUSION SYSTEM

Katsuya Ouchi, Hirokazu Fujisaka [a)]

Department of Design, Kobe Design University, Kobe 651-21, Japan

[a)] Department of Physics, Kyushu University 33, Fukuoka 812, Japan

We first discussed the asymptotic pattern formation dynamics of a reaction-diffusion system. The Brusselator equation, which is a well known model in studies of chemical dissipative structures,

$$\begin{aligned} \partial X/\partial t &= A-(B+1)X+X^2Y+D_x\nabla^2X \\ \partial Y/\partial t &= BX-X^2Y+D_y\nabla^2Y \end{aligned} \tag{1}$$

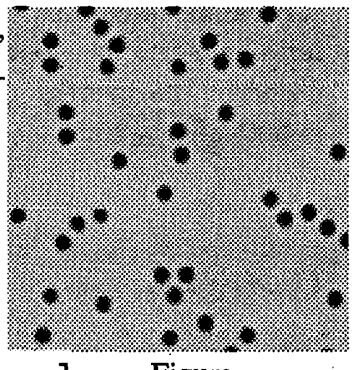
is solved numerically, and we have revealed that the system with a specified control parameter evolves much slower than the Swift-Hohenberg(S-H) equation. The peak width of the structure function depends on time as $t^{-1/8}$ asymptotically, on the other hand, it depend on time as $t^{-1/5}$ for the Swift-Hohenberg equation[1]. The roll and triangular pattern coexist in this parameter region, and the triangular pattern seems to be more robust than defects of the roll pattern. The slow decay of the triangular pattern causes the slow relaxation of the structure function and brings about the value 1/8 of the dynamic scaling exponent different from 1/5 observed in the Swift-Hohenberg equation[2].

Reaction-diffusion equations slightly above the Turing instability point, where the steady uniform state loses its linear stability, reduce qualitatively to the asymmetric Swift-Hohenberg (aS-H) equation

$$\partial w/\partial t = [\varepsilon-(\nabla^2+k_c^2)^2]w + aw^2-w^3 = -\frac{\delta H\{w\}}{\delta w}, \tag{2a}$$

where $H\{w\}$ is the Lyapunov functional and is denoted as follows

$$H\{w\} = -\int d\vec{x}\left[\frac{1}{2}\varepsilon w^2-\frac{1}{2}[(\nabla^2+k_c^2)w]^2+\frac{a}{3}w^3-\frac{1}{4}w^4\right]. \tag{2b}$$



Figure

The quadratic term is included because chemical systems have no symmetry of dynamical variable in general. Indeed we can lead to (2) by making use of amplitude equation of (1) near its onset of Turing instability . The equation is then studied, and spotty patterns are observed for some control parameter, which is a spatially highly localized pattern (Figure). The asymptotic form of the spot solution is obtained analytically. We further find that spot positions obey relaxation equations of motion interacting via the short range pair potential

$$\eta(r) = r^{-1/2}e^{-r/\xi}\cos[Q(r-r_0)] \tag{3}$$

for interspot distance r, where ξ evaluates the spot radius, and Q and r_0 are constants. The pair poten-

tial is explicitly determined by the spot shape. This yields many equilibrium positions, and therefore the system can take infinitely many spatial disorderd states.

We finally investigate the phase ordering kinetics of the S-H equation, which has been mainly studied in terms of the TDGL equation following a temperature quench from a homogeneous phase into a two-phase region. The S-H equation also show the phase ordering for $\varepsilon > 3/2k_0^4$ where two uniform states stably exist. The domain dynamics is discussed analytically, which revealed that the domain wall propagates with the velocity

$$c(R) = -\frac{2}{R}\frac{\varepsilon+8k_0^4}{k_0^4} \tag{4}$$

where 1/R is the local curvature. Then the equation is solved numerically for $\varepsilon > 2k_0^2$ by setting the initial condition with w=0. The roll patterns are observed for $\varepsilon < \varepsilon_{cr}$, and phase ordering is observed for $\varepsilon > \varepsilon_{cr}$, respectively. This transition of asymptotic pattern is found by observing the values of the Lyapunov functional.

The asymptotic dynamics of phase ordering is discussed by introducing the structure function

$$S(k, t) = \langle | \iint X(\vec{r}, t)\, e^{i\vec{k}\cdot\vec{r}}\, d\vec{r} |^2 \rangle , \tag{5}$$

and we reveals that S(k,t) is scaled as $S(k,t) = t^{\beta} g(kt^{\beta})$ with $\beta \cong 1/2$, where g(x) is a scaling function. The structure function S(k,t) for $\varepsilon > \varepsilon_{cr}$ is characteristic that S(k, t) has the maximal value at k=0, which is often observed in TDGL equation. S(k, t) for $\varepsilon < \varepsilon_{cr}$, on the other hand, has the maximal value at $k=k_0$ which is a property of the S-H equation. The asymptotic dynamics of phase ordering for $\varepsilon > \varepsilon_{cr}$ is much faster than that of roll pattern formation for $\varepsilon < \varepsilon_{cr}$.

References

[1] K. R. Elder, J. Vinals and M. Grant, Phys. Rev. A46 (1992) 7618
K. R. Elder, J. Vinals and M. Grant, Phys. Rev. Lett. 68 (1992) 3024
[2] K. Ouchi, T. Horita, K. Egami and H. Fujisaka, Physica D71 (1994), 367