WAVES AND PATTERNS IN A MEDIA WHICH HAS BOTH OF THE EXCITABLE AND OSCILLATORY CHARACTERS

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Reaction diffusion system has been succesfully used for modeling pattern formations and self-organization far from equilibrium. We consider a reaction diffusion system which has two different characters, say, excitability and oscillation, at the same time. The model equation for the system is a BvP-type equation whose concrete form is given as follows.

$$\tau\frac{\partial u}{\partial t} = \nabla^2 u + f(u) - v$$

$$\frac{\partial v}{\partial t} = D\nabla^2 v + u - \gamma v$$

where $f(u)$ contains N-shaped nonlinearity defined by $f(u) = \frac{1}{2}[tanh(\frac{u-a}{\delta}) + tanh(\frac{a}{\delta})] - u$, which approximates McKean's piecewise linear nonlinearlity. If the parameters are appropriately chosen, the ODE dynamics admits a coexistence of a stable stationary solution and a stable limit cycle which are separated by the unstable limit cycle (Fig.1). We carried out computer simulations of the model equation in one and two dimensional spaces by changing the parameters a and D.

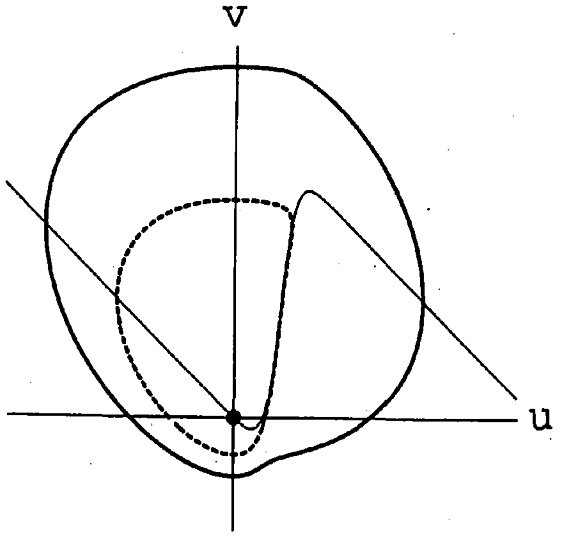

Fig.1 Limit cycle orbit (thick line) and equilibrium solution (black circle). The dotted line means the separatrics.

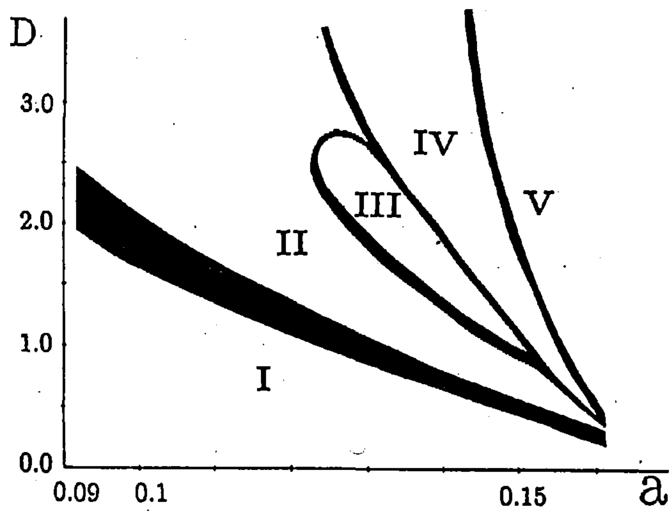

Fig.2 Phase diagram in the a-D plane.

In the one dimensional simulations, the dynamics of pulses and wave trains are examined. We give a phase diagram in $a - D$ plane (Fig.2). There exists a propagating pulse train in the region I. Its front destabilizes as the parameters go into region II, while the front itself propagates at the constant velocity. In the region III, the oscillating region is localized and shows breathing motion. The breathing motion disappear in the region IV. In the region V, no oscillating region can survive, which means that the front moves backward.

In the two dimensional simulations, we obtain a target pattern without heterogeneous pacemakers. Only the initial deviation at the center is needed for the self-organization of the pacemaker (Fig.3). Also it is shown that the two types of spiral patterns appear for the same parameters. The one is made from pulses (Fig.4) and the other from phase waves (Fig.5), which cleary exibits the two-facedness of this system.

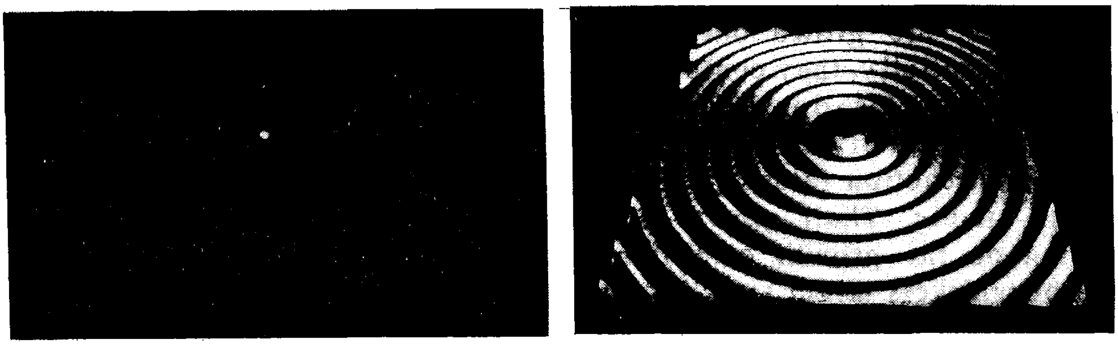

Fig3. Propagating target pattern.

Fig.4 Spiral made from pulse.

Fig. 5 Spiral made from phase wave.

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