SELF-ORGANIZATION IN A TWO-DIMENSIONAL CELLULAR AUTOMATON MODEL OF TRAFFIC FLOW

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1 Introduction

The traffic flow problems have been studied mainly through three modelings. The first is the fluid dynamical model. This method has been studied, for example, with the Burgers equation in one-dimensional cases. The second is the so-called car-following model. Each car controls its velocity by the velocity or distance relative to the car preceding it. The third is the cellular automaton (CA) model. The advantages of the CA models are their computational simplicity. The simplicity of the models enables us to change the models to take many realistic features of traffic problems into account. In spite of the simplicity of the models, they show interesting phenomena as phase transitions and self-organizations.

One of the simplest CA models of traffic flow in one-way expressways is the rule-184 elementary CA, which is a simple asymmetric exclusion rule. There have been great developments in one-dimensional CA models (for example, see [1]). Two-dimensional models, however, have fewer direct connections to real traffic flow problems. They seem to be abstract models with regard to a traffic systems in a whole city or an expressway network.

2 The Model

I study here one of the simplest two-dimensional traffic flow model introduced by Biham, Middleton, Levine (BML)[2]. It is a simple extension of the rule-184 CA model and models simple two-dimensional exclusion processes. Cars are distributed on a square lattice of $N \times N$ sites with periodic boundary conditions in both horizontal and vertical directions. Each car is represented as an arrow directed up or right. A traffic light controls the dynamics, such that the right arrow move only at odd time steps and the up arrows move at even time steps. At odd time steps, each right arrow moves one site to its right neighborhood if and only if the destination is empty. The corresponding rule is applied for up arrows. The density of right (up) cars is given by

$$p_{\rightarrow} = \frac{n_{\rightarrow}}{N^2}, \quad p_{\uparrow} = \frac{n_{\uparrow}}{N^2}$$

where $n_{\rightarrow}(n_{\uparrow})$ denotes the number of right (up) arrows. I examine the isotropic case where $p_{\rightarrow} = p_{\uparrow} = p/2$ following Biham *et al.* They reported the existence of the transition point $p_c \sim 0.35$. Below p_c all cars move freely and the average velocity is $\bar{v} = 1$. All cars are blocked and the average velocity vanishes above p_c.

3 Two Types of Traffic Jam

Two types of traffic jam configurations were found in the BML model (Fig. 1)[3]. In the low-density region ($p_c < p < p_t$), there is a single global cluster of jam. The backbone of the jam lies

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diagonally and branches spread horizontally and vertically. The diagonal correlation function obeys a power law. So the jam state at the low density is far from random configuration and well self-organized. In the high-density region ($p_t < p$), on the other hand, patches of small local clusters of jam cover the whole system. There is no apparent global structure. The diagonal correlation decays exponentially. So the jam state at the high density is random.

The fractal dimension of jam states is studied by the box counting method (Fig. 2)[4]. In the high density region, jam configurations do not show fractal structure reflecting the randomness installed at the initial configuration. Fractal structures appear in the low density jam. Fractal dimension D seems to obey

$$2 - D \sim |p - p_t|^{-\alpha}.$$

The transition point p_t is close to that of point percolation.

The BML model has three phases. The first is the freely moving phase. The second is the self-organized jam phase. In this phase, the spatially diagonal correlation function obeys a power law. The spatial structure of jams is fractal. The third is the random jam phase. The spatially diagonal correlation function decays exponentially.

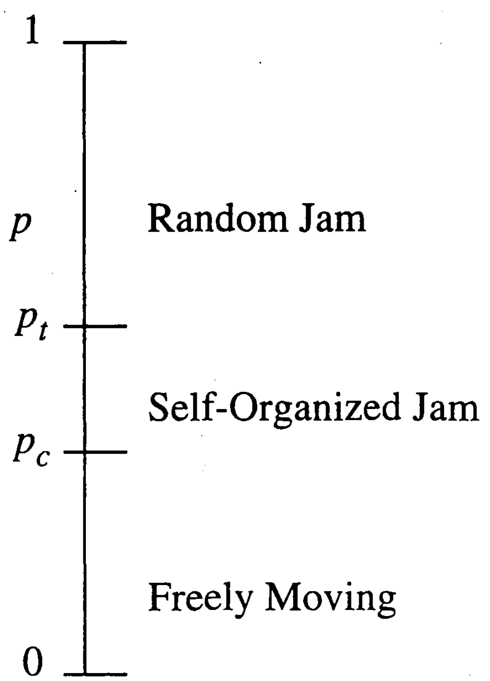

Figure 1: Schematic phase diagram

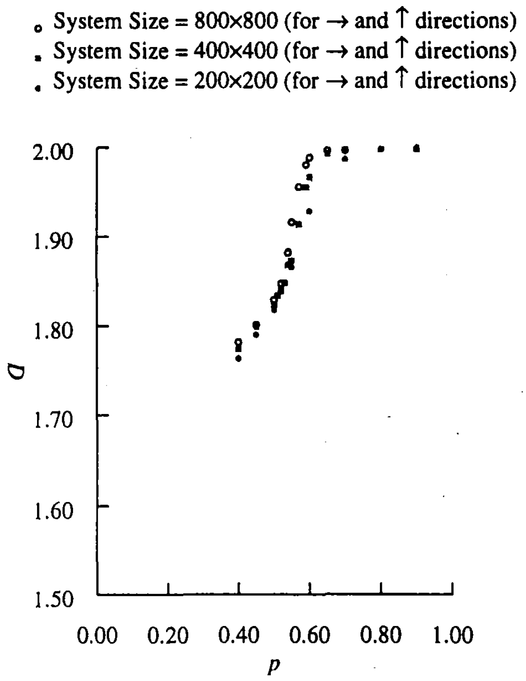

Figure 2: Fractal dimension by box counting

References

[1] S. Yukawa, M. Kikuchi and S. Tadaki, J. Phys. Soc. Japan **63**, 3609 (1994).

[2] O. Biham, A. A. Middleton, and D. Levine, Phys. Rev. A**46**, 6124 (1992).

[3] S. Tadaki and M. Kikuchi, Phys. Rev. E**50**, 4564 (1994).

[4] S. Tadaki and M. Kikuchi, J. Phys. Soc. Japan **64** (1995) in press.