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ON THE TIME-DEPENDENT DEFORMATION OF A DROPLET UNDER SHEAR FLOW
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Author(s)

Imaeda, Tatsuhiro

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ON THE TIME-DEPENDENT DEFORMATION OF A DROPLET UNDER SHEAR FLOW

Tatsuhiro Imaeda
Faculty of Engineering, Tohwa University, Fukuoka 815, Japan

A perturbation approach is presented to derive a kinetic equation for a droplet under shear flow. A time-dependent deformation of the droplet is calculated to show an overshooting behavior in approach to a steady state. An application of our approach is also discussed.

We start with the interface equation of motion\(^1\) for a droplet under shear flow \(u(r) = Sx \hat{e}_z\),

\[
v(a,t) = Sx(a)n(a) \cdot \hat{e}_z + \int da' \ n(a) \cdot T(r(a)-r(a')) \cdot n(a')oK(a')
\]

\[T(r) = \frac{1}{8\eta} \left( \frac{1}{r} + \frac{xK}{r^3} \right).
\]

where \(S\) is the shear rate, \(\hat{e}_z\) is the unit vector along the \(z\)-axis, \(a\) stands for the position on the interface of the droplet, \(v(a)\) is the normal component of the interface velocity, \(n(a)\) is the normal unit vector at the interface \(K = -\nabla \cdot n\) is the mean curvature, and \(\sigma\) is the surface tension.

We have neglected inertia effects\(^1\) and assumed the common shear viscosity \(\eta\) in the two fluids.

When a dimensionless parameter \(\varepsilon = SR/\sigma\) is sufficiently small, the interfacial profile \(r(\Omega,t) = R(1 + f(\Omega,t))\) would be described by a superposition of the spherical harmonics \(Y_{l}^{m}(\Omega)\).

At the first order in \(\varepsilon\), (1) reduces to the well-known Taylor's solution\(^2\), which constitutes the basis of our theory. The \(O(\varepsilon^2)\) theory was considered in a different point of view from ours.\(^3,4\).

Here we develop a third order theory in a systematic way.

First, we multiply (1) by \(Y_{l}^{m}(\Omega)^*\) and integrate over the interfacial area. Then we expand the resultant equation in terms of \(f(\Omega,t)\) and evaluate the equation up to \(O(\varepsilon^3)\) by assuming the following form for the profile:

\[
f(\Omega,t) = \varepsilon x_{21}(t)f_{21}(\Omega) + \varepsilon^2 \sum_{nm}^{(2)} x_{nm}(t)f_{nm}(\Omega) - r_2(t) + \varepsilon^3 \sum_{nm}^{(3)} x_{nm}(t)f_{nm}(\Omega)
\]

where \(\sum_{nm}^{(2)} = \sum_{n=2,4} \sum_{m=0,2} \sum_{nm}^{(3)} = \sum_{n=4,6} \sum_{m=1,3}\), being \(f_{nm} = 2Re Y_{n}^{m}\) for \(m \neq 0\) and \(f_{n0} = Re Y_{n}^{0}\). Here the term \(r_2(t)\) has been added to
conserve the volume of the droplet at this order.

Thus we have a closed set of equations for $x_{nm}$
with $(n,m) = (2,1), (2,0), (2,2), (4,0), (4,1), (4,2), (4,3), (6,1), (6,3)$. The steady state solution of the equations can readily obtained (Fig.1). There three sets of the solution are obtained for all $\varepsilon$. The linear stability analysis shows that one solution is always stable and two are always unstable. Thus our successive approximation does not reproduce the breakup$^1$, even though the profile for $\varepsilon = 0.5$ is reminiscent of that at the incipient breakup of type $B_2$$^1,5$.

In Fig.2, we have shown a time change of the orientation angle $\alpha$ and $L = r(\theta=\alpha, \phi=0)$. At $t = 0$, a spherical droplet starts to deform under the shear $\varepsilon = 0.1$. After a steady state is attained, the shear is jumped to $\varepsilon = 0.5$. A marked result is an overshooting of the profile in approach to a new steady state.

Finally, as an application of our approach we have derived expressions of the lateral and the slip migration velocity of a droplet in close vicinity of a wall. Our results are in good agreement with a recent computer simulation$^6$.

Fig.1 : The steady state profile at x-z plane for $\varepsilon = 0.1$ and 0.5.

Fig.2 : Time evolution of the orientation angle $\alpha$ and $L$.

References