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<th>Title</th>
<th>ON THE TIME-DEPENDENT DEFORMATION OF A DROPLET UNDER SHEAR FLOW (Session IV: Structures &amp; Patterns, The 1st Tohwa University International Meeting on Statistical Physics Theories, Experiments and Computer Simulations)</th>
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<tr>
<td>Author(s)</td>
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<tr>
<td>Citation</td>
<td>物性研究 (1996), 66(3): 598-599</td>
</tr>
<tr>
<td>Issue Date</td>
<td>1996-06-20</td>
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<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/2433/95758">http://hdl.handle.net/2433/95758</a></td>
</tr>
<tr>
<td>Type</td>
<td>Departmental Bulletin Paper</td>
</tr>
<tr>
<td>Textversion</td>
<td>publisher</td>
</tr>
<tr>
<td>Textversion</td>
<td>Kyoto University</td>
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A perturbation approach is presented to derive a kinetic equation for a droplet under shear flow. A time-dependent deformation of the droplet is calculated to show an overshooting behavior in approach to a steady state. An application of our approach is also discussed.

We start with the interface equation of motion for a droplet under shear flow \( u(\mathbf{r}) = S \mathbf{e}_z \),

\[
v(a,t) = S(x) \mathbf{n}(a) \cdot \mathbf{e}_z + \int \mathbf{n}(a) \cdot T(\mathbf{r}(a) - \mathbf{r}(a')) \cdot \mathbf{n}(a') \circ K(a')
\]

where \( S \) is the shear rate, \( \mathbf{e}_z \) is the unit vector along the z-axis, \( a \) stands for the position on the interface of the droplet, \( v(a) \) is the normal component of the interface velocity, \( \mathbf{n}(a) \) is the normal unit vector at the interface. \( K = -\nabla \cdot \mathbf{n} \) is the mean curvature, and \( \sigma \) is the surface tension. We have neglected inertia effects and assumed the common shear viscosity \( \eta \) in the two fluids.

When a dimensionless parameter \( \varepsilon = \eta SR/\sigma \) is sufficiently small, the interfacial profile \( r(\Omega,t) = R(1+f(\Omega,t)) \) would be described by a superposition of the spherical harmonics \( Y_{l}^{m}(\Omega) \).

At the first order in \( \varepsilon \), (1) reduces to the well-known Taylor's solution, which constitutes the basis of our theory. The \( O(\varepsilon^2) \) theory was considered in a different point of view from ours. Here we develop a third order theory in a systematic way.

First, we multiply (1) by \( Y_{l}^{m}(\Omega) \) and integrate over the interfacial area. Then we expand the resultant equation in terms of \( f(\Omega,t) \) and evaluate the equation up to \( O(\varepsilon^3) \) by assuming the following form for the profile:

\[
f(\Omega,t) = \varepsilon x_{21}(t) f_{21}(\Omega) + \varepsilon^2 \sum_{nm}^{(2)} x_{nm}(t) f_{nm}(\Omega) r_2(t) + \varepsilon^3 \sum_{nm}^{(3)} x_{nm}(t) f_{nm}(\Omega)
\]

where \( \sum_{nm}^{(2)} = \sum_{n=2,4} \sum_{m=0,2} \) and \( \sum_{nm}^{(3)} = \sum_{n=4,6} \sum_{m=1,3} \), being \( f_{nm} = 2 \text{Re} Y_{n}^{m} \) for \( m \neq 0 \) and \( f_{n0} = \text{Re} Y_{n}^{0} \). Here the term \( r_2(t) \) has been added to
conserve the volume of the droplet at this order.

Thus we have a closed set of equations for $x_{nm}$
with $(n,m) = (2,1), (2,0), (2,2), (4,0), (4,1), (4,2), (4,3), (6,1), (6,3)$.
The steady state solution of the equations can readily obtained
(Fig.1). There three sets of the solution are obtained for all $\epsilon$.
The linear stability analysis shows that one solution is always
stable and two are always unstable. Thus our successive
approximation does not reproduce the breakup, even though the
profile for $\epsilon = 0.5$ is reminiscent of that at the incipient breakup
of type B$_2$.

In Fig.2, we have shown a time change of the orientation angle $a$
and $L = r(\theta = a, \phi = 0)$. At $t = 0$, a spherical droplet starts to
deform under the shear $\epsilon = 0.1$. After a steady state is attained,
the shear is jumped to $\epsilon = 0.5$. A marked result is an overshooting
of the profile in approach to a new steady state.

Finally, as an application of our approach we have derived
expressions of the lateral and the slip migration velocity of a
droplet in close vicinity of a wall. Our results are in good
agreement with a recent computer simulation.

Fig.1 : The steady state profile at x-z plane for $\epsilon = 0.1$ and 0.5.
Fig.2 : Time evolution of the orientation angle $a$ and $L$.

References