Universal and Individual Dynamics of Pattern Selection in Extended Systems

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The goal of the present paper is to call attention to some new aspects of two traditional questions of the pattern formation problem in extended dissipative systems, namely creation and/or annihilation of defects and the role of an additional continuous group of symmetry, respectively.

Interest to the process of creation and/or annihilation of defects in pattern dynamics is related generally to the following reasons (i) it is one of the most common mechanism to change the wavenumber in unstable patterns, (ii) annihilation of defects is the only way for an imperfect texture to evolve into a perfect defectless pattern, (iii) such a process is extremely important for understanding of the so-called defect turbulence, where a chaotic state is achieved due to spontaneous defect generation. Despite the abovementioned reasons the problem of defect interaction and the dynamics caused by the interaction is still far from completion. In particular, most of theoretical works are limited by the framework of different versions of the perturbation theory, see, e.g., [1]. Such an approach in principle cannot describe the process of creation (annihilation) at the stage when "cores" of defects are close to each other so that their interaction may not be regarded as a small perturbation. We are going to show that the latter case often may be described by certain universal dynamics that weakly depends on particular details of the initial and lateral boundary conditions. The dynamics is associated with self-similar solutions of the underlying equations that is the only possibility to describe the process when the distance between centers of interacting defects becomes much smaller than all characteristic spatial scales of the corresponding pattern-forming system. Note, that similar arguments in case of interaction of two point defects in two-dimensional roll-patterns of electro-convection in nematic liquid crystals (EHC) were also mentioned by Aranson [2], however he did not apply this idea to any concrete calculations. The first application of such a kind was made in Refs. [3, 4, 5] for the case of Eckhaus instability arises in one-dimensional roll-patterns in EHC.

Let us consider here a two-dimensional roll pattern. In case of EHC in a certain approximation the problem is governed by a two-dimensional version of the time-dependent

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Ginzburg-Landau equation [6]. The only locally stable point defect in this case is a dislocation with topological charge ±1. A center of the dislocation may be related to the points where the Ginzburg-Landau order parameter ψ vanishes. In Fig. 1 the results of experimental study of dynamics of annihilation of a pair of dislocations in EHC is presented. The experimental technique and the data treatment were standard, see e.g. Refs. [7, 8]. Detailed discussion of these results will be a subject of a separate publication. Here we note only that the profiles of the lines $\mathrm{Re}\psi = 0$ and $\mathrm{Im}\psi = 0$ shown in Fig. 1 cover all, topologically different possibilities may be realized in this problem. In all cases close to the annihilation moment the spatial form of the curves may be approximated by conic curves (ellipse, parabola or hyperbola). The latter brings about the scaling $\Delta X \propto |\Delta T|^{1/2}$ that connects the distance between centers of two annihilating defects ΔX with time ΔT downcounted from the annihilation moment. The scaling is in good agreement with our experimental results. The important point is that the scaling is exactly the same as that obtained in one-dimensional case [3, 4, 5], despite the different meaning of ΔX in one-dimensional systems.

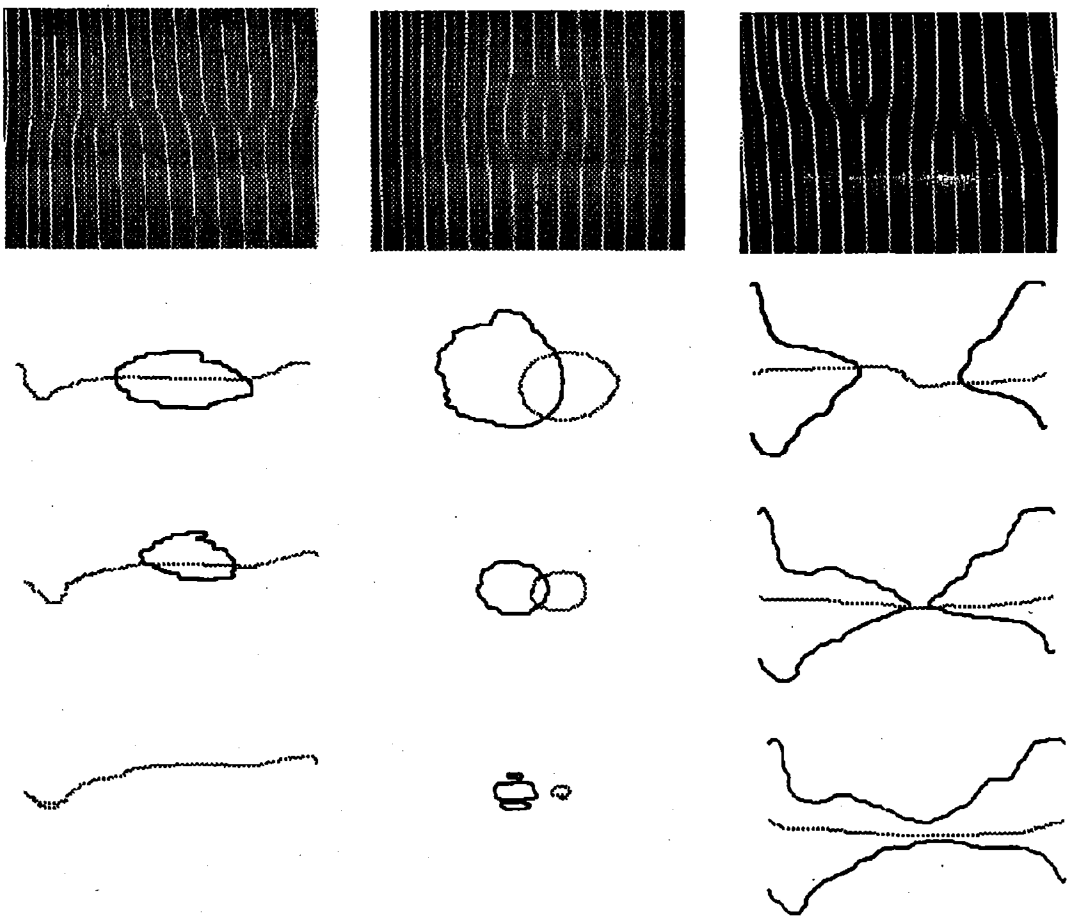

Fig. 1. Annihilation of a pair of dislocations in planar EHC. Nematic liquid crystal, MBBA; $\varepsilon = 0.1$. The real image at a certain moment (upper panel) and the result of treatment of its temporal evolution at three consecutive moments (lower panel). $\mathrm{Re}\psi = 0$ on the black solid line, while the gray one corresponds to $\mathrm{Im}\psi = 0$. Three typical topologically-different annihilations of a pair of defects are shown. Time proceeds from top to bottom. The bottom figures show the treatments just after annihilations for all cases.

Let us discuss now roll patterns in case when the system, undergoing short-wave instability, has an additional one-dimensional continuous group of spatial symmetry. The latter means that beside the conventional invariance under translations and reflections in the plane of pattern formation, the system remains invariant with respect to one extra spatial transformation parametrized by one continuous quantity. Examples of such a kind may exhibit convection in a liquid layer with "stress-free" boundary conditions [9, 10,11], systems with Galilean invariance [12], travelling front in phase transition phenomena or in reaction-diffusion systems [13, 14] and others [1]. In EHC with a homeotropic alignment of the director by top and bottom surfaces of a container an additional symmetry exists beyond the threshold of the *Fredericksz* [15] transition. In this case the equilibrium orientation of the director in the midlane is tilted, so that the director has a non-zero projection into this plane. The system is degenerate with respect to rotation of the director through an arbitrary angle around an axis perpendicular to the midplane, that provides desirable additional symmetry. Thus, the corresponding pattern-formation problem has a new slowly varying field of long-wave modes related to this symmetry. The field describes the director's angular orientation in the midlpane and includes a neutrally-stable (*Goldstone*) mode with zero wavenumber generated by the additional symmetry transformation. Coupling of the long-wave orientational modes with short-wave EHC modes brings about dramatic changes in the pattern formation problem. In particular for roll pattern the relevant pattern stability problem must possess *two* Goldstone modes: one associated with the same orientational degeneracy and another generated by spatial translation of the pattern along the roll's wave vector **k** (translation in direction perpendicular to **k** makes no influence into the roll pattern, and therefore does not generate a Goldstone mode). Recent theoretical results [16] say that in such a case all spatially periodic patterns may be unstable and a threshold of short-wave instability (EHC) corresponds to a bifurcation from a spatially uniform state directly into spatiotemporal chaos.

An example of a weakly nonlinear pattern obtained experimentally in EHC under the abovementioned conditions is displayed in Fig. 2a. Being quite irregular, the pattern, nevertheless, has a certain characteristic spatial scale equal to that for the short-wave instability. The pattern slowly evolves in time but the evolution does not yield any steady state. All these

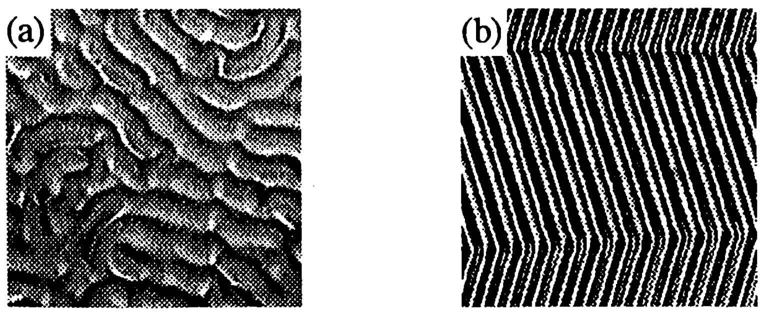

Fig. 2. Patterns in homeotropic EHC. (a) without a magnetic field; (b) with a magnetic field directed along the plane of pattern formation (H = 1600 G).

features are in good agreements with theoretical predictions of Ref. [16].

The orientational degeneracy may be lifted by an external force directed along the plane of pattern formation, that has to reduce the problem to the conventional case and to stabilize the patterns. Indeed, action of a constant magnetic field (1600 G) transforms the irregular pattern into a perfect steady zig-zag one, typical for EHC, see Fig. 2b. As soon as the magnetic field is removed the regular pattern becomes unstable and the system returns to the state shown in Fig. 2a. A limited length of the present paper does not allow us to pay more attention to these results. In the nearest future their detailed discussion will be presented elsewhere.

References

[1] M. C. Cross, and P. C. Hohenberg, Rev. Mod. Phys. **60**, Part II, 851 (1993).

[2] I. Aranson, private communication.

[3] M. I. Tribelsky, S. Kai, and H. Yamazaki, Phys. Rev. A **45**, 4175--4177 (1992).

[4] H. Yamazaki, M. I. Tribelsky, S. Nasuno, and S. Kai, in *Pattern Formation in Complex Dissipative Systems*, Ed. by S. Kai (World Scientific, Singapore, 1992), pp. 346--357.

[5] M. I. Tribelsky, S. Kai, H. Yamazaki, and M. G. Velarde, Phys. Rev. E **51**, 5132--5135 (1995).

[6] E. Bodenschatz, W. Pesh, and L. Kramer, Phys. D. **32**, 135, (1988).

[7] G. Goren *et al*, Phys. Rev. Lett. **63**, 1237, 1989.

[8] S. Nasuno, O. Sasaki, S. Kai and W. Zimmermann, Phys. Rev. A **46**, 4954 (1992).

[9] E. D. Siggia, and A. Zippelius, Phys. Rev. Lett. **47**, 835 (1981).

[10] F. H. Busse, and E. W. Bolton, J. Fluid Mech. **146**, 15 (1984).

[11] A. J. Bernoff, Euro. J. Appl. Math. **5**, 267 (1994).

[12] P. Coullet, and S. Fauve, Phys. Rev. Lett. **26**, 2857 (1985).

[13] S. I. Anisimov, M. I. Tribelsky, and Ya. G. Epelbaum, Zh. Eksp. Teor. Fiz. **78**, 1597 (1980) [Sov. Phys. ---- JETP **51**, 802 (1980)].

[14] A. J. Bernoff *et al*, SIAM J. Appl. Math. **55**, No 2, 485 (1995).

[15] P. G. de Gennes, *The Physics of Liquid Crystals* (Clarendon, Oxford, 1982).

[16] M. I. Tribelsky, submitted to Int. J. Bif. Chaos (special issue devoted to The International Workshop on Nonlinear Dynamics and Chaos, Pohang, Korea 1995).