A Study of Chaos Synergetic Neural Network

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Abstract In this work we shall put forward a novel synergetic neural network involving a chaos dynamics and investigate the dynamic properties in memory retrieval process. The present artificial neuron model is characterized by a sinusoidal activation function as well as competitive nonlinear connections between synergetic neurons. It is elucidated that the present neural network has an advantage in the dynamic memory retrievals beyond the conventional chaotic model with such a monotonous mapping as a sigmoid function. This finding is considered to result from both the nonmonotonous property of the analogue periodic mapping which may be accompanied with a chaotic behaviour of the neurons and the synergetic connections related to the competition between the overlaps. It is also shown that the present analogue neural network may be reduced to the previously proposed synergetic neural network if one assumes an identity mapping in the feedback loop instead of the periodic one.

Key words : chaos, synergetics, periodic mapping, chaotic retrieval, associative memory

§1. Theory

In this section let us formulate a chaos dynamical system to be applied to the memory retrieval under consideration based on the previous works[1,2].

First of all we shall define a dynamic rule for the present neural network with N neurons and L embedded patterns. For this purpose let us define first the internal state and the corresponding output of the ith neuron as σ_i and s_i, respectively, which have to be related to each other in terms of the following sinusoidal mapping[1,3]

$$s_i(n)=f(\sigma_i(n))=\sin\left(\frac{\sigma_i(n)}{\tau}\right) \quad . \tag{1}$$

Here the internal state $\sigma_i(n)$ may be assumed to be related to the order parameter $\eta_r(n)$ introduced in synergetic neural network as follows.

$$\sigma_i(n)=\sum_{r=1}^{L} \sin^{-1}\left(e_i^{(r)}\right)\eta_r(n) \quad , \tag{2}$$

where $e_i^{(r)}$ is assumed to be the rth embedded pattern with $|e_i^{(r)}|\le 1$. The nonlinear mapping concerned with arcsin function is required to achieve a complete association at a fixed point as will be shown below. Then, in analogous to the synergetic neural network[2], the dynamics of the order parameters $\eta_r(n)$ $(1\le r\le L)$ may be defined in terms of

$$\eta_r(n+1)=\eta_r(n)+\kappa_\eta\left(1-2\sum_{p=1}^{L}\eta_p(n)^2+\eta_r(n)^2\right)\eta_r(n) \qquad (1\le r\le L)\ , \tag{3}$$

where $\kappa_\eta(<1)$ is a control parameter. The order parameters $\eta_r(n)$ $(1\le r\le L)$ can be related to the output vector s_i in the following manner,

$$\eta_r(n)=\sum_{i=1}^{N} e_i^{\dagger(r)} s_i(n) \quad , \tag{4}$$

where $e_i^{\dagger(r)}$ $(1\le i\le N,\ 1\le r\le L)$ are the conjugate vector, or the generalized inverse matrix,

corresponding to the embedded vectors, or $e_i^{(r)}$ ($1 \le i \le N$, $1 \le r \le L$).
In similarly with the previous work[1], the internal state has to be updated in terms of

$$\sigma_i(t+1)=\left(1-2/\pi\right)\sigma_i(t)+\sum_{j=1}^{N} w_{ij} s_j(t) \tag{5}$$

Then the periodicity $\tau(t)$ may be controlled as in the following manner.

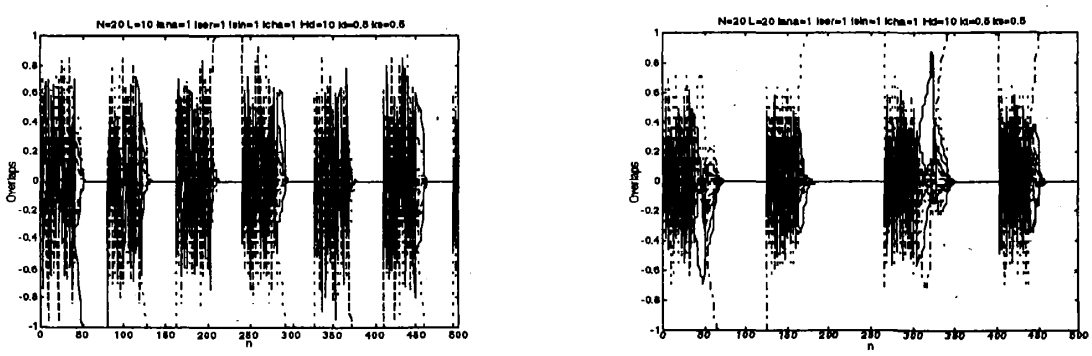
$$\tau(n+1)=\tau(n)+\kappa_\tau \bullet \tau(n)(1-\tau(n)) \quad . \tag{6}$$

In the orthogonal autocorrelation learning model, w_{ij} may be simply defined by

$$w_{ij}=\sum_{r=1}^{L} e_i^{(r)} e_j^{\dagger(r)} \quad . \tag{7}$$

§2. Results

Let us examine below the chaotic memory retrievals derived through the present model.
First of all let us present the association dynamics derived from the present model. In Figs.1 (Overlaps) one may see the transition process from chaos ($\tau \sim \varepsilon$) to non-chaotic state ($\tau \sim 1$), which leads to a fixed-point realising a complete association. Here the embedded patterns $e^{(r)}_i$ and the initial state $s_i(0)$ are set to random vectors in N-dimension space, in which N was set to 20 for the moment. Both the relaxation constant, κ_η and κ_τ, were set to 0.5. From these results, in similarly with the synergetic neural network, one may see that one of the embedded patterns can be certainly associated up to $L=N$. This pattern selection capability is considered to result from the competitive synergetic dynamics expressed by eq.(3) in similarly with the recently proposed synergetic neural network[2]. From the computer simulation results, we confirmed that this ability remains to be valid for any other large L (≥ 1). Therefore the present model involves the pattern search ability as was realised in the synergetic neural network as well as the chaotic wandering as seen in the periodic chaos neural network.[3]



(a) L/N =0.5 (b) L/N=1

Fig.1 Dynamics of overlaps in an association process. Here N =20, $\kappa_\eta=\kappa_\tau$=0.5.

【References】

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