STABILITY AND STOCHASTICITY OF QUANTUM CHAOS

Nobuhiro Suda

Nippon Medical School, Kawasaki-shi, kanagawa 211, Japan

With respect to quantum chaos, particularly in view of quantum-classical correspondence, many studies have been made until now originating from Casatti, Chirikov, Ott, Shepelyansky, Adachi, Yukawa, etc. Casati, etal., indicate a lack of stochasticity in the quantum motion. Ott et al. report addition of small noise to a time-dependent quantum system which exhibits as a result different dynamical bahaviors from its classical counterpart. Shepelyansky, et al. have investigated statistical properties (correlation functions) of quantum movement and corresponding classical movement. Yukawa has shown that a quantum system under the influence of classical chaos behaves as if it is a classical one with respect to the quantum chaos within a thermodynamic limit including infinite degrees of freedom.

In the present study, classical and quantum kicked rotators are taken as a model, to which various kinds of noise such as random noise, colored noise, and chaotic noise as well as a small number to an infinite number of other degrees of freedom are coupled to pursue the stability of the quantum system and the mechanisms of the stability of the quantum system and the diffusion retrieval for the purpose of elucidation of the origin of the quantum chaos.

Procedures are as follows: In accordance with the quasi classical approximation of Zaslavsky there is constructed a Hamiltonian in which random noise and a coupling term with other systems are incorporated, from which an iterative equation of the standard map is formed based upon the Heisenberg equation of motion. The iterative equation is simulated with changes in involved parameters.

Herein, Hamiltonian is

$$
\begin{aligned}
H = {} & H_0(a_i^{+} a_i) + \varepsilon\hbar(a_i^{+} + a_i) \sum_n \delta(t - nT) \\
& + \eta f(a_i^{+}, a_i, t) \sum_n \delta(t - nT) \\
& + \Sigma_{ij} \xi_{ij} g(a_i^{+}, a_i, t) \sum_n \delta(t - nT)
\end{aligned}
$$

where the term η is a noise term, ξ_{ij} is a coupling term with other dimensions.

The wave function Φ is

$$
\Phi(\theta, t+1) = e^{-iF\cos\theta}\, 1/\sqrt{2}\,\pi \times \sum_n A_n(t) \exp(in\theta - iTn^2/2)
$$

$$A_n(t) = 1/\sqrt{2\pi} \int \Phi(\theta, t)\, e^{-in\theta} d\theta$$

Correlation of the quantum system is expressed as follows.

$$R_t(\tau) = \langle 0|\cos\theta_t \cos\theta_{t+\tau} + \cos\theta_{t+\tau}\cos\theta_t|0\rangle$$

In a quantum rotator, FT = 1 defines a boundary of stability. Herein, F ≒ K(kick term) + Y(random noise term) + Z(coupling term with other systems) defines a boundary of stability. As these parameters are increased, the motion gets unstable and an action variable I of the rotator becomes larger. In this case, nearby trajectories diverge exponentially. Namely, assumed a distance between adjacent trajectories to be l, $l = l_0\exp(st)$ holds and $s \fallingdotseq \ln(kT/2)$ is KS entropy.

From numerical simulation, without any noise, energy of the quantum rotator system grows diffusively at an approximately classical rate for a time shorter than a certain time t_1. This corresponds to Shepelyansky. With $t > t_1$, the speed of the diffusion drops off while with $t \gg t_1$ an increase of the energy is actually interrupted.

It is found that with the addition of any random noise and chaotic noise to the quantum system the increases of the energy of the quantum system at $t \gg t_1$ are prevented from being interrupted, respectively. The degree of the prevention in the present case is clearly different from that in the case of the random case. This asserts a difference between the statistical properties of chaoses and random noize in view of the occurrence of the chaoses and the disappearance of quantum correction terms. This further asserts that random noise does not cause any chaotic behavior.

We are now in a study to clarify the unified description on the retrieval of chaos in the quantum system caused by its coupling with various kinds of noise and systems of other degrees of freedom for further primitive investigation of the origin of the quantum chaos combined with statistical properties including the large deviation principle.

(1) G.Casati, etal., : vol.93 of Lecture Notes in Physics, 1979 (2) E.Ott.etal.,: Phys.Rev.Lett, Vol.53, No.23 1984 2187-2190 (3) D.L. Shepelyansky, PHYSICA 8d 1983 208-222 (4) Yukawa: Proceedings of the International Conference on DYNAMICAL SYSTEMS AND CHAOS Vol. 2 Physics 23-27 May 1994 (5) N.Suda: presented to the annual meeting of the physical society of Japan (1995)