LARGE DEVIATION STATISTICAL MECHANICS

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Coarse-graining is one of the most important concepts in statistical physics. Recently it has been established that a new coarse-graining procedure gives a universal statistical asymptotic law in strange objects known as fractal sets, for temporal domain and in fluid turbulence, etc. [1,2]. This is known as the thermodynamics formalism and is closely related to the large deviation theory in the probability theory [3,4].

Let $u\{t\}$ be a steady time series experimentally observed. Without loss of generality, let t be discrete,($t = 0, 1, 2, \ldots$). If we define the time coarse-graining quantiry by $\bar{u}_T = T^{-1}\sum_{s=0}^{t+T-1} u\{s\}$,then it approaches the long time average as $T \to \infty$. For large but finite T, depending on a time region where the average is carried out, it takes various values. As known in the large deviation theory, the probability distribution $\rho_T(u)$ for $\bar{u}_T$ takes the asymptotic form

$$\rho_T(u) \sim e^{-S(u)T}$$

for a sufficiently large T. The function $S(u)$ called the fluctuation spectrum is a concave function. $S(u)$ has a relation with the characteristic function $\phi(q)$ defined by

$$M_q(T) \equiv \langle e^{qT\bar{u}_T}\rangle \sim e^{\phi(q)T}$$

for large T,$\langle\ldots\rangle$ being the ensemble average. The shape of $S(u)$ characterizes the "static" statistics of the fluctuation [4].

The explicit time correlation can not be described by the fluctuation spectrum, but can be described with the Fourier spectrum. As mentioned above the time series is not "homogeneous" in the sense that $\bar{u}_T$ takes various values, depending on the time region. It also implies that the Fourier spectrum is not same for different time regions. We postulated that Fourier spectrum can single out different dynamical processes, if it is appropriately defined.

In order to single out the "fluctuations" of temporal correlations we introduce the generalized power spectrum,

$$I_q(\omega) \equiv \lim_{T\to\infty}\langle I\{\omega;T\}\delta(\bar{u}_T - u(q))\rangle/\rho_T(u(q)) = \lim_{T\to\infty}\langle I\{\omega;T\}e^{qT\bar{u}_T}\rangle/M_q(T)$$

, where $I\{\omega;T\}$is the conventional power spectrum calculated over the time span T,and $u(q) = \phi'(q)$. $I_q(\omega)$ is identical to the power spectrum over the region whose $\bar{u}_T$ is equal to $u(q)$.

CHAOTIC SYSTEMS:

Consider the Poincare map,$x_{t+1} = f(x_t)$. Let $u\{t\}$ be a unique function $u[x_t]$ of x_t. The eigenvalues of the operator $H_q = He^{qu[x]}$, H being the Frobenius-Perron operator determines $S(u)$ and $I_q(\omega)$. I reported the application of the present new approach to the intermittent dynamics of the type I intermittency [5].

LANGEVIN DYNAMICS:

When the temporal fluctuation is generated by a Langevin dynamics for $A(t)$ via $u\{t\} = u[A(t)]$, $u[a]$ being a unique function of a,where the Langevin equation is given by,

$$\frac{dA(t)}{dt} = [\Delta - A^2 + f(t)]A,$$

$(\Delta > 0)$, where $f(t)$ is the gaussian-white noise. In this case the eigenvalues of the operator $H_q = H + qu[a]$,H being the Fokker-Planck operator, determines the statistical quantities above mentioned [7]. I reported the application for the multiplicative noise model. Near the critical point the system shows a highly intermittent behavior.

For the above cases, rigorously solving the eigenvalue problems of the generalized evolution operator, I reported the anomalous statistical characteristics of the intermittency dynamics. We found that (1) at the onset of intermittency, the system undergoes the so-called q-phase transition, near which an anomalous behavior for thermodynamics quantities are observed, and then (2) the generalized power spectrum can single out both the laminar and burst dynamics separately. Although for the burst dynamics the power spectrum is approximately white, it shows a typical power law dependence in the laminar phase.

References

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