STATISTICAL MECHANICS OF A GENERALIZED CALOGERO-SUTHERLAND SYSTEM

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We report our recent progress in the study of a generalized Calogero-Sutherland system (g-CS), which was first derived by Nakamura and Mikeska[1] for the parametric motion of the eigenvalues of the Floquet operator $F = \exp(-i\lambda V)\exp(-iH_0)$ of periodically kicked quantum systems $H(t) = H_0 + \lambda V \sum_{n=-\infty}^{\infty} \delta(t-n)$. The statistical properties of the eigenphases of F can be studied by applying equilibrium statistical mechanics to the g-CS system. We adopt a specific canonical distribution $\frac{1}{z}\exp(-\gamma Q - \beta E)$ whose validity was verified by Hasegawa and Robnik[2] with a use of the maximum likelihood property of the Gaussian distribution: Here, E and Q are the unique two constants of motion of the g-CS system quadratic with respect to the matrix elements of V. The resulting joint distribution function of the eigenphases is

$$P(\varphi_1, \varphi_2, \cdots, \varphi_N) = C_N \prod_{j<k} \left| \frac{\epsilon_j - \epsilon_k}{\epsilon_j - Z\epsilon_k} \right|^2, \quad \epsilon_j = e^{i2\pi\varphi_j}, \quad 0 \le Z \le 1 \tag{1}$$

with $Z = e^{-2\alpha}$, $4\gamma/\beta = \sinh^{-2}\alpha$. Equation (1) was first proposed by Gaudin[3] as an *ad hoc* interpolation between the Poisson and circular unitary ensembles. Using Gaudin's result for the two-level correlation function

$$R_2(\varphi_1, \varphi_2) = N(N-1) - 2\mathcal{R} \sum_{\lambda<\mu} \prod_{k=\lambda}^{\mu-1} \frac{1 - Z^k}{e^{2\pi i(\varphi_1 - \varphi_2)} - Z^k}, \tag{2}$$

we obtain the number variance in an interval of length S (in terms of the mean level spacing $D = 1/N$)

$$\Sigma^{(2)}(S; a) = S - 2\int_0^S (S-r)Y(r)dr = S - \int_0^1 \int_0^1 dx dx' \frac{\sin^2 SP}{P^2}, \quad P = \frac{1}{2a} \log \frac{e^{2\pi a x} - 1}{e^{2\pi a x'} - 1} \tag{3}$$

with $a = \frac{N\alpha}{2\pi}$, which interpolates smoothly between the Poisson and the CUE results (see Fig.1). Its asymptotic behavior is expressed as

$$\Sigma^2(S) \sim \frac{1 - e^{-2\pi a}}{2\pi a} S + \frac{(1 - e^{-2\pi a})^2}{2\pi^2} (\log S + \text{const.}). \tag{4}$$

The linearity of the leading term in the large-S expansion of Σ^2 above means that the level gas is compressible, according to the Ornstein-Zernike relation[4]:

$$\frac{\langle (N_v - \langle N_v \rangle)^2 \rangle}{\langle N_v \rangle^2} = \frac{kT}{v} \kappa_T, \quad \kappa_T = -\frac{1}{v} \left(\frac{\partial v}{\partial P} \right)_{T, \langle N_v \rangle} = \frac{1}{\langle N_v \rangle} \left(\frac{\partial \rho}{\partial P} \right)_{T, \langle N_v \rangle}$$

Therefore,

$$\Sigma^2(S)_{\text{linear in } S} = \frac{1}{2} \kappa_T S \qquad (\langle N_v \rangle = S, \ \text{and} \ kT = 1/2 \ \text{ for the unitary ensemble}). \tag{5}$$

This physical interpretation of the compressible level gas is possible, however, at a cost of violating the sum-rule $\int_{-\infty}^{\infty} Y(r)dr = 1$ due to the standard thermodynamic limit implied in eq.(3), and contradicts with the recent prediction by Kravtsov et al.[5] for the fractional power law $\Sigma^2(S) \sim S^\gamma$ $(0 < \gamma < 1)$ at the mobility edge of the Anderson transition (a detailed discussion given elsewhere[7]).

We show briefly an application of the above model to the intermediate statistics of the *survival probability* for N-dimensional ($N \gg 1$) quantum systems[6], where the correlation effect manifests itself in the "correlation hole" of the time-function

$$P(t) \equiv \ll |\langle \psi, \psi(t) \rangle|^2 \gg \sim \frac{2}{N+1} + \frac{1}{N+1}(\delta(\tau) - b(\tau)) \tag{6}$$

(double average over the initial states and the Hamiltonian ensemble) where $\tau \equiv t/2\pi\rho$ is the scaled time and $b(\tau)$ is the two-level form factor(Fourier transform of the cluster function $Y(r)$, see Fig.2).

The triangle below the horizontal line of the curve $P(\tau)$ in Fig.2 illustrates (ignoring the rapid oscillation due to the finiteness of N) the shape of the form factor, $-b(\tau)$, whose height and total area correspond to the integral $\int_{-\infty}^{\infty} Y(r)dr$ and the value $Y(0)$, respectively. The height changes depending on the parameter a, while the total area is unchanged; the feature reflecting the violation of the sum rule.

References

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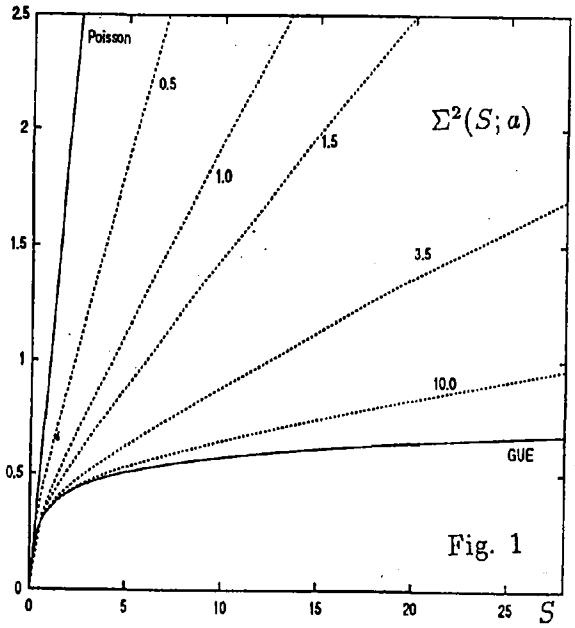

Fig. 1

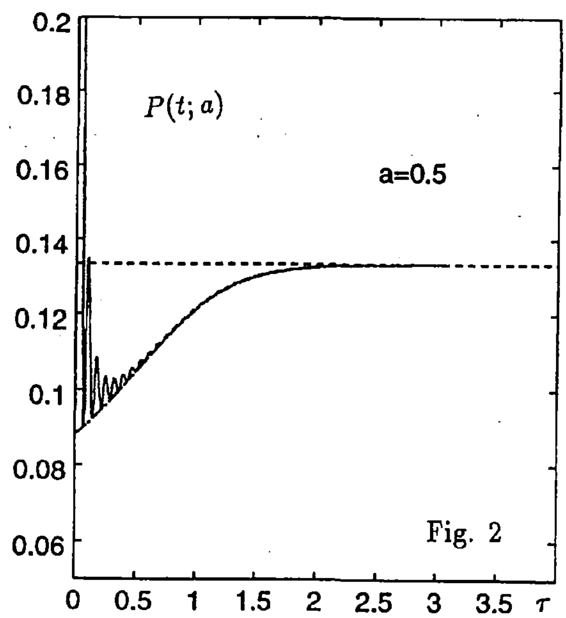

Fig. 2