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<th>Title</th>
<th>STATISTICAL MECHANICS OF A GENERALIZED CALOGERO-SUTHERLAND SYSTEM (Session II: Chaos, The 1st Tohwa University International Meeting on Statistical Physics Theories, Experiments and Computer Simulations)</th>
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We report our recent progress in the study of a generalized Calogero-Sutherland system (g-CS), which was first derived by Nakamura and Mikeska \(^1\) for the parametric motion of the eigenvalues of the Floquet operator \( F = \exp(-i\lambda V)\exp(-iH_0) \) of periodically kicked quantum systems \( H(t) = H_0 + \lambda V \sum_{n=-\infty}^{\infty} \delta(t-n) \). The statistical properties of the eigenphases of \( F \) can be studied by applying equilibrium statistical mechanics to the g-CS system. We adopt a specific canonical distribution \( \exp(-\beta E) \) whose validity was verified by Hasegawa and Robnik \(^2\) with a use of the maximum likelihood property of the Gaussian distribution: Here, \( E \) and \( Q \) are the unique two constants of motion of the g-CS system quadratic with respect to the matrix elements of \( V \). The resulting joint distribution function of the eigenphases is

\[
P(\varphi_1, \varphi_2, \ldots, \varphi_N) = C_N \prod_{j<k} \left| \frac{\epsilon_j - \epsilon_k}{\epsilon_j - Z\epsilon_k} \right|^2, \quad \epsilon_j = e^{i2\varphi_j}, \quad 0 \leq Z \leq 1
\]

with \( Z = e^{-2\alpha} \), \( 4\gamma/\beta = \sinh^{-2} \alpha \). Equation (1) was first proposed by Gaudin \(^3\) as an ad hoc interpolation between the Poisson and circular unitary ensembles. Using Gaudin's result for the two-level correlation function

\[
R_2(\varphi_1, \varphi_2) = N(N-1) - 2\mathcal{R} \sum_{\lambda < \mu} \frac{1 - Z^k}{e^{2\pi i(\varphi_1-\varphi_2)} - Z^k},
\]

we obtain the number variance in an interval of length \( S \) (in terms of the mean level spacing \( D = 1/N \))

\[
\Sigma^2(S; a) = S - 2 \int_0^S (S-r)Y(r)dr = S - \int_0^1 dx \int_0^1 dy \frac{\sin^2 SP}{P^2}, \quad P = \frac{1}{2a} \log \frac{e^{2\pi as}}{e^{2\pi at} - 1} - 1
\]

with \( a = \frac{Na}{2\pi} \), which interpolates smoothly between the Poisson and the CUE results (see Fig.1). Its asymptotic behavior is expressed as

\[
\Sigma^2(S) \sim \frac{1 - e^{-2\pi a}}{2\pi a} S + \frac{(1 - e^{-2\pi a})^2}{2\pi^2} (\log S + \text{const.}).
\]

The linearity of the leading term in the large-\( S \) expansion of \( \Sigma^2 \) above means that the level gas is compressible, according to the Ornstein-Zernike relation\(^4\):

\[
\frac{\langle (N_v - \langle N_v \rangle)^2 \rangle}{\langle N_v \rangle^2} = \frac{kT}{v} \kappa_T, \quad \kappa_T = -\frac{1}{v} \left( \frac{\partial v}{\partial P} \right)_{T,(N_v)} = \frac{1}{\langle N_v \rangle} \left( \frac{\partial P}{\partial y} \right)_{T,(N_v)}
\]

Therefore,

\[
\Sigma^2(S)_{\text{linear}} \propto S = \frac{1}{2} \kappa_T S \quad (\langle N_v \rangle = S, \text{ and } \kappa_T = 1/2 \text{ for the unitary ensemble}).
\]
This physical interpretation of the compressible level gas is possible, however, at a cost of violating the sum-rule \( \int_{-\infty}^{\infty} Y(r) \, dr = 1 \) due to the standard thermodynamic limit implied in eq.(3), and contradicts with the recent prediction by Kravtsov et al.\(^5\) for the fractional power law \( \Sigma^2(S) \sim S^\gamma \) (0 < \( \gamma \) < 1) at the mobility edge of the Anderson transition (a detailed discussion given elsewhere\(^7\)).

We show briefly an application of the above model to the intermediate statistics of the survival probability for \( N \)-dimensional \((N \gg 1)\) quantum systems\(^6\), where the correlation effect manifests itself in the "correlation hole" of the time-function

\[
P(t) \equiv \langle |\langle \psi, \psi(t) \rangle|^2 \rangle \sim \frac{2}{N+1} + \frac{1}{N+1}(b(\tau) - b(\tau))
\]

(double average over the initial states and the Hamiltonian ensemble) where \( \tau = t/2\pi \rho \) is the scaled time and \( b(\tau) \) is the two-level form factor (Fourier transform of the cluster function \( Y(r) \), see Fig.2).

The triangle below the horizontal line of the curve \( P(\tau) \) in Fig.2 illustrates (ignoring the rapid oscillation due to the finiteness of \( N \)) the shape of the form factor, \(-b(\tau)\), whose height and total area correspond to the integral \( \int_{-\infty}^{\infty} Y(r) \, dr \) and the value \( Y(0) \), respectively. The height changes depending on the parameter \( a \), while the total area is unchanged; the feature reflecting the violation of the sum rule.

References