QUANTUM CHAOS: SOME RECENT PROBLEMS

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1. Introduction

Recent progress in fabrication of nanoscale or mesoscopic structures has been making a nice bridge between high technology and fundamental researches of nonlinear dynamics. Both concave and convex billiards, around which studies of chaos are being accumulated, can be fabricated at the interface layer of semiconductor heterojunctions, e.g., GaAs/AlGaAs.

Concave and convex billiards are prototypes of conservative chaotic systems. In the concave case, a point particle repeats alternately a free motion inside the cavity and an elasic collision (via specular reflection) at the hard wall, resulting in a complicated trajectory extremely sensitive to initial conditions. In the convex case, on the other hand, a particle moves between plural number of convex billiards, e.g., as in case of Sinai billiard. Quantum-mechanical study of these billiards is one of the important subjects of quantum chaos.[1] We shall present quantum and semiclassical theories on both concave and convex billiards **with open channels**. Argument will also be made on level dynamics for parameter-dependent energy spectra of **bounded systems**.

2. Magneto-conductance in open concave billiards

Among concave billiards, Bunimovich's stadium (Sd) billiard has received a wide attention as a paradigm of nonlinear dynamics. It belongs to the K system, showing fully-chaotic orbits in marked contrast to regular orbits in a simple circle (Cl) or rectangle. Its quantum-mechanical study showed the GOE level statistics and periodic-orbit scars in wavefunctions, thereby heralding a new era of quantum chaos. In the presence of perpendicular magnetic field B, the stadium billiard becomes a generic system. Meplan et al's treatment elucidated its characteristic classical features: The erratic and ergodic phase space in a low field region is replaced by KAM tori via a transitional unstable region around $B=B_c$ with increasing the field strength, in contrast to the circle billiard where the phase space is always occupied by periodic (quasi-periodic) and nonergodic orbits. As for the experiment, using the advanced electron transport devices, Marcus et al[2] made a striking experiment on the magneto-conductance of the nanoscale stadium billiard.

Our study on the magneto-conductance $g(B)$ in open Cl and Sd billiards shows fluctuations dependent largely on the stability of phase space in the underlying classical dynamics of closed billiards. While in the Cl billiard the regular modulation of periodic orbits in the phase-space structure gives rise to regular oscillations of $g(B)$, the global chaos and genesis of successive tori in the Sd billiard are responsible to slow and rapid variations of the quantum conductance, respectively. The gradient of the cusp-like central peaks in the autocorrelation function quantifies rich fluctuations of $g(B)$. The present result is qualitatively consistent with the Marcus et al's experiment on a different wire geometry. Further, using the bouncing Larmor-orbit picture, we have derived the classical conductance, which turns out to reproduce most of the locations of peaks in the coarse-grained version of $g(B)$.

Real nanoscale structures are accompanied by extrinsic randomness, e.g., corrugation of walls, impurities and thermal noises. The rapid progress of advanced technology will smear out these extrinsic obstacles preventing us from a simple comparison between theory and experiment. The quantum theory of chaos is thus facing with an era to see its experimental test in the stage of quantum transport in mesoscopic devices. Currently, theoretical interests concentrate on: (1)showing a universality of conductance fluctuations on the basis of random matrix theory; (2) deriving the S-matrices directly by extending the Gutzwiller's semiclassical trace formula to open systems.

Nevertheless, the observed magneto-conductance would demand much deeper insight: Quantitative discrepancy, e.g., of the transition point B_c between the theory and experiment is serious and one should make a challenge into solving a puzzle of spectral structures which cannot be explained by either one of the semiclassical or quantum theory.

3. Chaotic scattering on convex billiards: Success of semiclassical theory

On the other hand, measurements of quantum transport properties have accumulated on the so-called crossroad in the GaAs/AlGaAs interfaces. The four corners of the crossroad consist of electron depletion regions, which correspond to C_{4v} four hard convex disks in the limit where the potential at the border of the circuit is very steep. While some theoretical studies aim at applying Gutzwiller's semiclassical trace formula to the crossroad problem, their issue (e.g., on conductance) is not in good agreement with experimental results. This discrepancy would be due to the serious diffraction effect at leaky regions connected with straight lead wires.

To capture the fluctuation properties truly attributable to chaotic scattering, we shall develop both semiclassical and exact quantum theories for a point particle scattered by a model system consisting of four identical hard disks with C_{4v} symmetry with all its attachments discarded so as to suppress the effect of diffraction. In this simplified system, the semiclassical theory-----Gutzwiller's trace formula-----is extremely powerfull. The reason is: (1) The system is fully chaotic without bearing any bifurcation and therefore symbolic dynamics predicts all the periodic orbits systematically; (2) due to the open-system nature, only short-periodic orbits contribute substantially to the trace formula, suppressing a serious problem of exponential proliferation.

The issue of classical treatment is as follows:[3] Suppose a system of convex hard disks (with radii a and inter-disk distance R) lying within the area B with size $\mathcal{R} \times \mathcal{R}$. The escape rate γ of a point particle moving outside of B is expressed in terms of KS entropy h_{KS} and the sum of positive Lyapunov exponents λ as

$$\gamma = \lambda - h_{KS} \tag{1}$$

(Eckmann and Ruelle, 1985). Since $\lambda > h_{KS}$, $\gamma > 0$ is guaranteed. This means the existence of an upper limit for the confining time. On the other hand, let's consider a gas of particles confined initially between disks inside B and eventually escaping outside B. The particle density function f satisfies inside B a diffusion equation (with diffusion coefficient D), $\partial_t f = D\nabla^2 f$, with the boundary condition $f=0$ outside B. By assuming the long-time behavior of the solution, $f \sim \exp(-\gamma t)$, one obtains the relationship

$$\gamma \sim D/\mathcal{R}^2 . \tag{2}$$

Combining (1) and (2), one reaches[3]

$$D \sim \mathcal{R}^2(\lambda - h_{KS}) . \tag{3}$$

D proves to be nonvanishing owing to the presence of the lower limit for γ.

The Lyapunov exponent strongly depends on the degree of opening of the system, $\sigma = R/a$. We find λ deceases with increase of σ. In case $\sigma >> 1$, $\lambda >> h_{KS}$ and hence $\gamma(\sim\lambda)$ decreases with increase of σ.

In our study on the semiclassical and quantum-mechanical counterparts of these properties, we shall evaluate wavenumbers k_{res} associated with scattering resonances. The scattering resonance means a state in which an incident electron is transiently trapped within the quasi-closed region surrounded by disks. The imaginary part of k_{res} is related to the inverse life time of the electron. We have elucidated the presence of a gap in the distribution of k_{res}, corresponding to the classical issue, e.g., the presence of the lower limit of γ and its σ dependence.[4,5]

Resonances can provide information on the dynamical behavior of a semiconductor mesoscopic device like the crossroad. The reaction time of the device may be estimated from the size of the aforementioned gap in the distribution of the resonances. Indeed, according to Schrödinger's equation applied to scattering systems, the time evolution of an electronic wavepacket is given by a linear superposition of damped exponentials, $\exp(\mathrm{Im}E_n t/\hbar)$, controlled by the imaginary parts of the complex energies of the resonances. In the present system, we showed that the imaginary parts of the wavenumbers are bounded by the value of the gap according to $\mathrm{Im}k_n < -x_{gap}$. According to the relation between energy and wavenumber, we infer that the probability for the electron to remain in the scatterer decays like $\exp(-t/\tau_{gap})$. The upper bound on the lifetime is given by $\tau_{gap}=1/(2v_F\, x_{gap})$ in terms of the semiclassical gap and the Fermi velocity v_F of the electron gas. Since the gap is related to the Lyapunov exponents of the periodic orbits, our analysis shows how the reaction time of the device depends on the geometry of the system:

$$\tau_{gap} \sim \frac{R}{v_F \ln(R/a)}, \tag{4}$$

which is valid in the regime kR, ka >>1 where diffraction effects can be neglected. In the case of crossroad we see that the shortest reaction times are obtained for the smallest values of the corner radius a, assuming a fixed value for the width R of the lead wires. For GaAs/AlGaAs heterojunctions, the effective mass of the electrons is m=0.067m_e and an electron density of $n_s=3\times10^{11}$ cm^{-2} can be obtained so that the Fermi velocity would then take the value $v_F=(2\pi\hbar^2 n_s)^{1/2}/m=2.4\times10^5$ m/s. For a nanometric circuit of size R=100nm, the time unit is therefore of $R/v_F=0.4\times10^{-12}$ s so that the lifetimes of the resonances are in the subpicosecond domain. We suggest that the dynamical behavior of such devices could be probed by femto-second laser experiments.

4. Level dynamics and random matrix theory

Finally, some argument will be devoted to the parameter-dependent energy spectra for bounded systems. Statistical aspects of classically-chaotic quantum systems are described by random matrix theory and its constrained variants. These apparent irregularity can be explained more profoundly by statistical mechanics of the completely-integrable Calogero-Moser and Calogero-Sutherland systems derived from quantum-mechanical eigenvalue problems **by regarding a nonintegrability parameter as a pseudo- time**.[4,5] The idea is traced back to Dyson's level dynamics, but the modern framework described in this talk is based on conservative Newtonian dynamics rather than an overdamped limit of Langevin equation.

References

1) K. Nakamura: *Quantum Chaos: A New Paradigm of Nonlinear Dynamics* (Cambridge University Press, Cambridge, 1993).
2) C.M. Marcus et al: *Phys. Rev. Lett.* **69**, 506 (1992).
3) P. Gaspard and G. Nicolis: *Phys. Rev. Lett.* **65**, 1693 (1990).
4) For a variety of recent topics in this field, see articles in *Quantum Chaos--- Present and Future*, edited by K. Nakamura: Special issue of *Chaos*, *Solitons & Fractals* (Pergamon) Vol.**5**, #**7** (1995).
5) K. Nakamura: *Quantum versus Chaos: Questions Emerging from Mesoscopic Cosmos* (Kluwer Academic, Dordrecht, 1996) , to be published.