ROTON-LIMITED MOBILITY OF POSITIVE IONS IN BULK LIQUID ^{4}HE

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Abstract

It is shown that the roton-limited mobility of the positive ion in bulk liquid ^{4}He has the temperature variation with $\bar{\sigma}_r T^{1/2} e^{-\Delta/k_B T}$, where $\bar{\sigma}_r$ is the roton scattering cross section and is good agreement with the experiments for the roton-limited temperatures ranges from $\sim 0.5K^{-1}$ to $\sim 1.85K^{-1}$.

In the previous paper, using the rate of the phonon momentum transfer via scattering of phonons by positive ion and temperature-dependent anomalous excitation spectrum, we have obtained the mobility of positive ion in phonon-limited temperature region in bulk liquid helium. The mobility of a slowly moving charged ion in relatively high temperature from $\sim 0.6K$ to $\sim 1K$ is closely related to the collisions with the elementary excitations in three-dimensional liquid ^{4}He. Baym et al[1] and Schwarz[2] have shown that the averaged transport cross section and the mobility of the negative and positive ions can be put in the form

$$\bar{\sigma}_+ \simeq 2.8 \times 10^{-14} T^{-1/2} cm^2, \tag{1}$$

$$\mu_\pm \simeq e^{-\Delta/k_B T}, \tag{2}$$

through an generalized equation of ion mobility resulting from microscopic scattering process and hydrodynamic approximation repectively. Here, the subscript symbol ($\pm$) represents the case of the positive and negative ions repectively. The roton energies are arranged over 6.65-8.8K for Δ_+ and 7.7-8.1K for Δ_-.

As, in the previous paper, we have estimated the temperature limit whether or not the collisons between phonons and ions is elastic, we can easily estimate it in the same way. At the density of $2.18 \times 10^{-2} \AA^{-3}$, the typical roton momentum $\hbar k_0 \simeq \hbar(1.93\AA^{-1})$[3] will transfer to an ion through a collision with a roton. In this case the characteristic roton temperature T_r can be obtained as $T_r = \hbar^2 k_0^2 / 2m_4 k_B \simeq 22K$, where m_4 is the ^{4}He atomic mass. The dimensionless ratio $< E_{rec} > /k_B T$ implies that the recoil effect is negligible for $T \gg T_r m_4/m^*$, where $< E_{rec} >$ and m^* are the mean recoil energy and the effective mass respectively. For $m^* \simeq 40m_4$ we may confirm the condition $T \gg 0.55K$ for the positive ion. Therefore below $\sim 0.6K$ the ionic recoil in the roton collison is generally negligible, but the roton collison become significant for the positive ion below about $1K$. Therefore, as a first approximation we treat the roton-ion collision as elastic in the temperature ranges over $\sim 0.6K < T <\sim 1K$[4]. Through the same processes as we did in previous paper we obtain the mobility of the positive ions in the roton-limited region in the bulk liquid ^{4}He :

$$\frac{e}{\mu_+} = \frac{2(2\pi\mu_r k_B)^{1/2}}{3\pi^2} P_0^3 \bar{\sigma}_r T^{1/2} e^{-\Delta/k_B T}, \tag{3}$$

where $\bar{\sigma}_r$ corresponds to the thermal averaged transport cross section for roton energy.

To investigate and analyze the experimental mobility of the positive ion in roton-limited temperature ranges, we have taken same parameters given in the previous paper. In the subtraction of the putative phonon contribution from the measured values in the overlapped temperature region, Schwarz[2] obtained the roton contribution to the mobility e/μ_+ of the positive ion. Schwarz has taken the temperature dependent roton energy gap $\Delta/k_B = (8.65 \pm 0.04)K$, while we have chosen $0.153m_4$ which is taken to fit the phase velocity data.

Comparison of Eq.(3) with Schwarz's Eq.(29)[2], hereafter referred to Eq.(4), for the mobility of positive ion as following

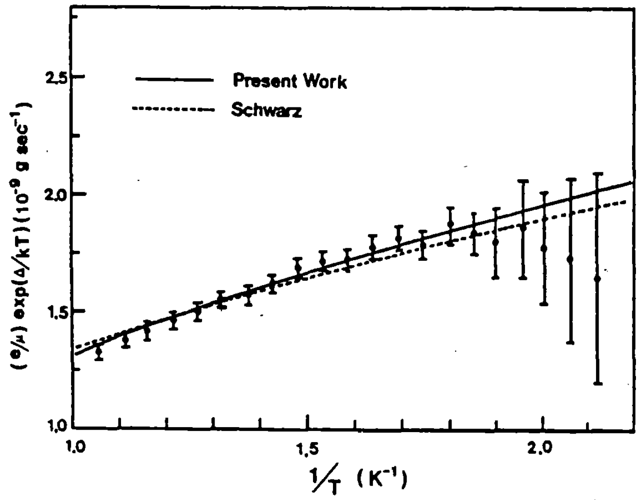

Fig. 1. The mobility versus inverse temperature.

$$\frac{e}{\mu_+} = 1.34 \times 10^9 T^{-1/2} e^{-\Delta/k_B T} \tag{4}$$

gives the discrepancy between temperature prefactor. The simplest kinetic theory from Reif and Meyer[5] gives a T^1 dependence of prefactor. However, Eq.(3) shows the variation with $T^{1/2}$, while Eq.(4) has $T^{-1/2}$. Clearly in Eq.(3) $\bar{\sigma}_r$ is not determined. Comparing both equations we may estimate the roton scattering cross section $\bar{\sigma}_r$:

$$\bar{\sigma}_r = 0.93 \times 10^{-13} T^{-1}. \tag{5}$$

Reif and Meyer[5] has verified that e/μ_+ is proportional to $\exp[-\Delta/k_B T]$ below $2K$. In this case we may educe that the roton scattering cross section $\bar{\sigma}_r$ varies with $T^{-1/2}$ in Eq.(3). Therefore we may confirm that the roton contribution to the mobility of positive ion has the variation of $T^{-1/2}e^{-\Delta/k_B T}$ as far as $\bar{\sigma}_r \propto T^{-1}$. Figure 1 illustrates the mobility due to positive ion from roton scattering multiplied the exponential temperature dependence as a function of inverse temperatures. The dotted line represents the theoretical curve by Schwarz, while the solid line is our theoretical curve based on Eq.(3) together with Eq.(5). The theory agrees very well with experiments in the temperature ranges from $\sim 0.5K^{-1}$ to $\sim 1.85K^{-1}$.

In conclusion, the mobility of positive ion in roton limited region has $\bar{\sigma}_r T^{1/2} e^{-\Delta/k_B T}$ dependence and our theory is in good accordance with experiments.

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