HISTOGRAM-IMPORTANCE-SAMPLING MONTE CARLO APPROACH TO THE Q-STATE POTTS MODEL

Jau-Ann Chen, Chin-Kun Hu
Institute of Physics, Academia Sinica, Nankang, Taipei, Taiwan, 11529, Republic of China

In order to make efficient usage of data from Monte Carlo simulations, various histogram methods have been developed [1, 2, 3, 4, 5]. In recent papers [3, 4], Hu proposed a histogram Monte Carlo method and applied it to the q-state bond-correlated percolation model (QBCPM) [6, 7] corresponding to the q-state Potts model (QPM) [8]. Instead of calculating the geometrical quantities of the QBCPM at various discrete bond probability weight p, in [4] Hu used a bond random percolation process to generate histograms of important quantities from which various properties of the QBCPM and the QPM at any bond probability weight p may be calculated. Hu has also suggested an algorithm [3, 5] which combines his sum rule [3, 4] and important Monte Carlo sampling method, e.g. the Swendsen-Wang cluster Monte Carlo simulation method [9], to perform histogram importance sampling. Later, Chen and Hu [11] realize Hu's idea of histogram Monte Carlo important sampling [3, 5] and calculate the geometrical factors $N_{tp}(b,n)$ and $N_{tf}(b,n)$ of the QBCPM and the QPM, where $N_{tp}(b,n)$ is the total number of percolating subgraphs with b occupied bonds and n clusters and $N_{tf}(b,n)$ is the total number of nonpercolating subgraphs with b occupied bonds and n clusters. The the specific heat C_h of the Ising model, i.e. the QPM with $q = 2$, calculated from such geometrical factors agree very well with the corresponding exact quantities of Ferdinand and Fisher [10]. The specific heat C_h for the Ising model on a 20×20 square lattice is shown in Fig. 1.

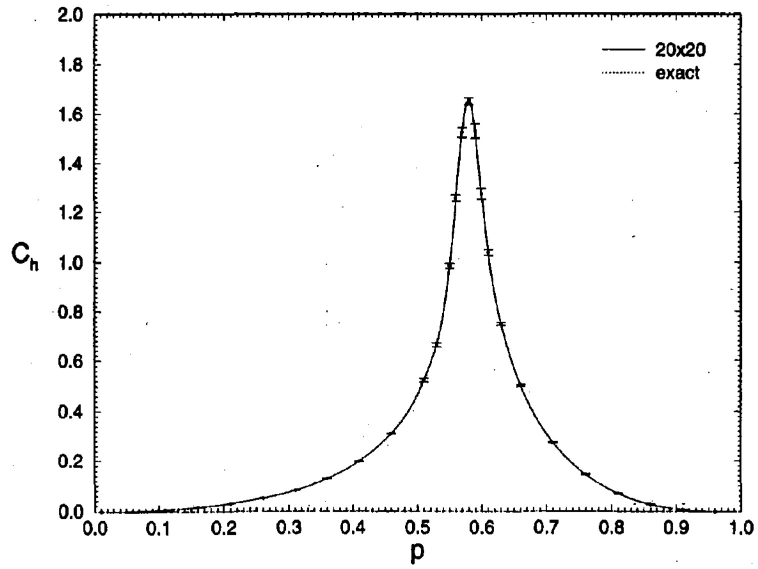

FIG 1. Calculated specific heat C_h for the Ising model on a 20×20 square lattice. Our results and the exact results of Ferdinand and Fisher are shown by soild and dashed lines, respectively. The vertical bars are standard deviation calculated from ten independent sets of simulations.

In this paper, we use this histogram-importance-sampling Monte Carlo method [11] to calculate the geometrical factors $N_{tp}(b,n)$ and $N_{tf}(b,n)$ of QBCPM with $q = 2,3$ and 4 in various two and three dimensional lattices. Thus, the free energy f, the internal energy $-U/J$, the specific heat C_h, the existence probability E_p and percolation probability P of the QBCPM (or QPM) can be calculated from these geometrical factors. The specific heat C_h for the Ising model on a $4 \times 4 \times 4$ and $6 \times 6 \times 6$ simple cubic lattices is shown in Fig. 2. We also study the scaling functions of E_p and P with respect to different lattices.

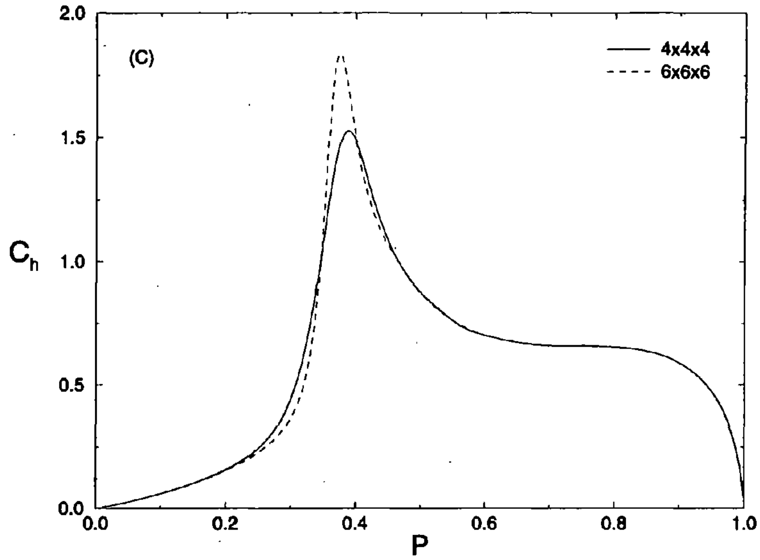

Fig. 2. Calculated specific heat C_h for the Ising model on the simple cubic lattices with linear dimensions L=4 and 6. The specific heat C_h as a function of p.

References

[1] A. M. Ferrenberg and R. H. Swendesn, Phys. Rev. Lett. **61**, 2635 (1988).

[2] A. M. Ferrenberg and R. H. Swendesn, Phys. Rev. Lett. **63**, 1195 (1988).

[3] C.-K. Hu, Phys. Rev. B **46**, 6592 (1992).

[4] C.-K. Hu, Phys. Rev. Lett. **69**, 2737 (1992).

[5] C.-K. Hu, Phys. Rev. Lett. **70**, 2045 (1993).

[6] For a recent review, see C.-K. Hu, in *Computer-Aided Statistical Physics*, edited by C.-K. Hu (AIP, New York, 1992), pp. 79-101.

[7] C.-K. Hu, Phys. Rev. B **29**, 5103 and 5109 (1984).

[8] F. Y. Wu, Rev. Mod. Phys. **54**, 235 (1982).

[9] R. H. Swendsen and J. S. Wang, Phys. Rev. Lett. **58**, 86 (1987).

[10] A. E. Ferdinand and M. E. Fisher, Phys. Rev. **185**, 832 (1969).

[11] J.-A. Chen and C.-K. Hu, Phys. Rev. B **50**, 6260 (1994).