COMMENSURATE-LOCKING OF THE ORBITALS OF THE SATURNIAN RING PARTICLES IN THE COMMENSURABILITY WITH A SATURNIAN SATELLITE

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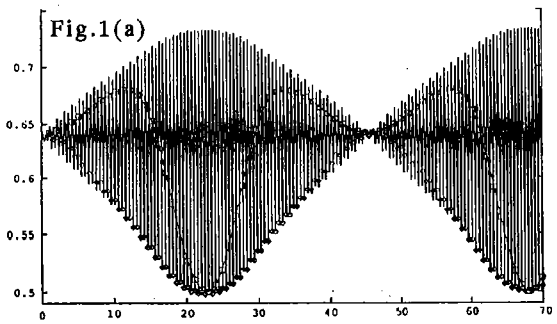
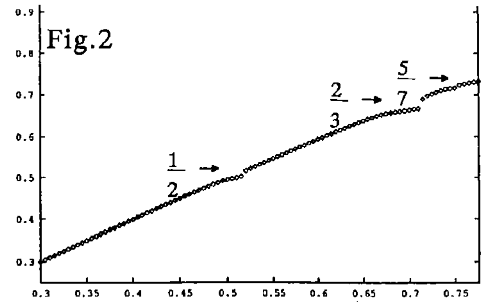
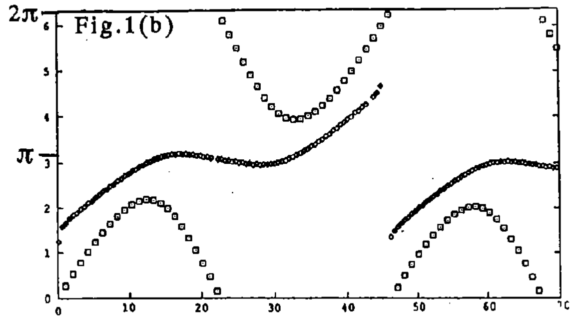
The commensurate (C) locking is one of the important concepts on physics, where the ratio between the periods of two sub-systems tends to a rational number. The famous example is the system of the atomic array connected by springs on a periodic substrate potential ("Frenkel Kontorova (FK) model"). This system has one important pair of quantities: (i)b_{nat}, which is the distance between the neighboring atoms without the substrate potential, (natural distance); and (ii) b_{av}, the averaged value of the realized atomic separation with the potential. Hereafter we rather use the non-dimensional parameters $r_{av}=b_{av}/a$, and $r_{nat}=b_{nat}/a$, where a is the period of the substrate. Considering the function, $r_{av}=f(r_{nat})$, r_{av} is locked at the infinite rational values while r_{nat} is smoothly moved. This function is called "devil's staircase" function.

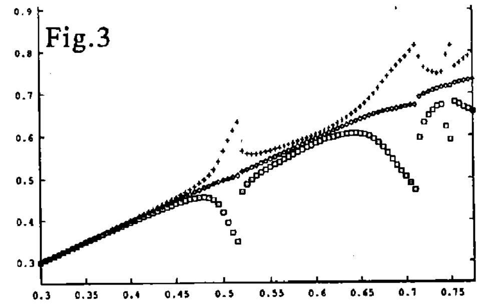
In this paper, we will argue on the C-locking on the system of the Saturnian ring particles perturbed by the gravity of one Saturnian satellite. Here we consider the ratio, t, between the period of the revolution of the ring particle, T, and the period of the revolution of the satellite, T_{satell}. In detail, similar to the FK model, there are the important pair of the quantities, one is this ratio without the perturbation, $t_{nat}=T_{nat}/T_{satell}$, the other is the ratio with the perturbation, $t_{av}=T_{av}/T_{satell}$, where T_{nat} is the period of the revolution of the ring particle without the perturbation, and T_{av} is the averaged value of the one with the perturbation. The important function which shows the C-locking is $t_{av}=f(t_{nat})$, and the C-condition is t_{av} = (a rational number). We can also consider the similar relation between these radii: $r_{av}=f(r_{nat})$, where $r_{nat}=R_{nat}/R_{satell}$, and $r_{av}=R_{av}/R_{satell}$. (R_{nat} is the radius of the particle without the perturbation, R_{av} is the averaged value of the one with the perturbation, and R_{satell} is the one of the satellite).

We study these motions by the simulation using Verlet (leap frog) method. We neglect the masses the satellite, compared with the ones of Saturn ($M_S=1$) and the satellite (M_{satell}). Besides, Saturn is fixed at the center. We can see the eccentricity of the particle changes periodically where t_{av} is near rational numbers. Hereafter we call this period as one resonant period. (See Fig.1(a).) In the figure, the radius of the particle where $t_{av}=1/2$ (where t_{nat} must be slightly larger than 0.5), $M_{satell}=10^{-3}$, and the initial condition is choosed that the motion is circular in case of non-perturbation. Then the rotational motion repeats as: circle → ellipse → circle, etc. We also show the longitude of the synod of the particle and the satellite(LOS) (square symbol) and the longitude of the aphelion (rhombic symbol). (Fig.1(b)) In Fig.2, where the horizontal and vertical axes represent t_{nat} and t_{av}, with same condition as in Fig.1, we see the locking at $t_{av}=1/2$, 2/3, 5/7. From the other results using various masses of









the satellite, we can see the locking at 1/2, 3/5, 2/3, 5/7, 3/4. These graphs corresponds to the devil's staircase, however, different from devil's staircase, the graph gradually approaches to the commensurate value of t_{av} from lower region. After t_{av} reaches the strict rational number, it jumps to the un-locking state. In Fig.3, the upper/ lower symbols represent the max./min. of the radius in one resonant period; besides, the central symbols represent the averaged value of the radius, r_{av}, where the horizontal axis represent r_{nat}. Here we do not use r , but $r^{2/3}$ to compare with Fig.2. We can see the averaged radius is also locked at the same C-points as t_{av}. It means that the stabilization of the C-orbit. If the ring particles have inelastic collisions, there may be the accumulations of the particles towards the C-orbits. On the other hand, we can see the large eccentricity at the same C-orbits. If there are the other ring particles or moonlets exist, the larger eccentricity causes the collisions or gravitational interactions with these objects, thus, the particle will apart from the original orbit, which means the de-stabilization of the orbit near the C-condition.

We can interpret the C-locking and un-locking, in terms of the LOS. (See Fig.1(b)) Hereafter we use the C- locking at t_{av}=1/2 for the explanation. The angular motion of the synod is originated by two mechanism. One is the non-perturbed(NP) angular velocity part of LOS, ω(NP), which is linear on the NP angular velocity of the particle. The other is the angular velocity part by the perturbation, ω(P). The latter one is negative (clock-wise) motion and the speed changes periodically with one resonant period. These amount is : ω_{tot} = ω(NP) + ω(P). The shift of the LOS in one resonance period, in other words, the integrals of these angular velocity on time is : $\Delta\theta_{tot}$ = Δθ(NP)+Δθ(P). Near the C-point, broadly speaking, Δθ(NP) ≈ (t_{nat}-1/2), Δθ(P) <0, and $\Delta\theta_{tot}$=0 at t_{av}=1/2. Then, t_{nat} which gives the C-condition t_{av}=1/2, is slightly larger than 1/2, (t_{nat}=0.5142, in Fig.1), and the derivative of the graph in Fig.1(a) become lower at t_{av}=1/2. More detail, Δθ(P) depends on $\Delta\theta_{tot}$. It brings a self consistent mechanism, which causes the locking below the C-point, and un-locking above the point.

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