CRITICAL EXPONENTS OF SAW AT THE Θ POINT IN TWO DIMENSIONS

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Duplantier and Saleur[1] have proposed the "exact" values of tricritical exponents: γ_t(entropy exponent) = 8/7, ν_t(shape exponent) = 4/7 and ϕ(crossover exponent) = 3/7 for the self-avoiding walk(SAW) at the θ point in two dimensions. These values are derived, however, for a special model of SAW on the hexagonal lattice with randomly forbidden hexagons at the percolation threshold using Coulomb-gas techniques and conformal invariance. Here we examine these results from exact enumeration of SAW on the square lattice on the basis of the Orr model of a polymer chain in dilute solutions.

The partition function of the chain with nearest-neighbor interaction energy ε is written as

$$C_n(\omega) = \sum_{m=0} C_{n,m}\, e^{m\omega} \tag{1}$$

where $\omega = -\varepsilon/kT$ and $C_{n,m}$ is the number of n-step SAW with m non-bonded nearest-neighbors. The reduced free energy is given by

$$F^*(\omega) = \lim_{n\to\infty} n^{-1}\log C_n(\omega) \tag{2}$$

The reduced internal energy and the specific heat are obtained from $E^*(\omega) = \partial F^*(\omega)/\partial\omega$ and $C^*(\omega) = \partial^2F^*(\omega)/\partial\omega^2$, respectively. The mean-square distance is also written as

$$R_n^2(\omega) = \left(\sum_{m=0} r^2_{n,m}\, e^{m\omega}\right)/C_n(\omega) \tag{3}$$

where $r^2_{n,m}$ is the unnormalized square end-to-end distance.

Exact values of $C_{n,m}$ and $r^2_{n,m}$ were enumerated for $n \leq 24$. The locations $\omega_n^{(0)}$ of maximum points on $C_n^*(\omega)$ curves (see Fig.1) for different n are determined by exploiting the series $\{C_{n,m}\}$ obtained. The Neville tables for linear and quadratic extrapolants

$$\omega_n^{(r)} = \left[n\omega_n^{(r-1)} - (n-2r)\omega_{n-2}^{(r-1)}\right]/2r \tag{4}$$

are formed for r = 1 and 2. The tricritical point $\omega_t (= \lim_{n\to\infty}\omega_n^{(0)})$ is determined by plotting these extrapolants against n^{-1}(Fig.2) after taking the average $(\omega_n^{(r)} + \omega_{n+1}^{(r)})/2$ in order to lessen the odd-even effect.

Following scaling assumptions are introduced:

$$C_n(\omega_t) \sim \mu_t^n n^{\gamma_t - 1};\ R_n^2(\omega_t) \sim n^{2\nu_t} \qquad (5)$$

The growth parameter μ_t and the exponents γ_t and ν_t are determined from a conventional technique using Neville tables combined with the ratio method. The crossover exponent ϕ is also estimated from three kinds of approaches based on

$$|\Delta\omega_n| = |\omega_n^{(0)} - \omega_t| \sim n^{-\phi};\ R_n^2(\omega) \sim n^{2\nu_t} f(|\Delta\omega_n| n^{\phi}) \qquad (6a)$$

$$C_n(\omega) \sim \mu_t^n n^{\gamma_t - 1} F(|\Delta\omega_n| n^{\phi}) \qquad (6b)$$

The results thus estimated are listed in Table I together with those of Chang and Meirovich[2] from the scanning Monte Carlo method.

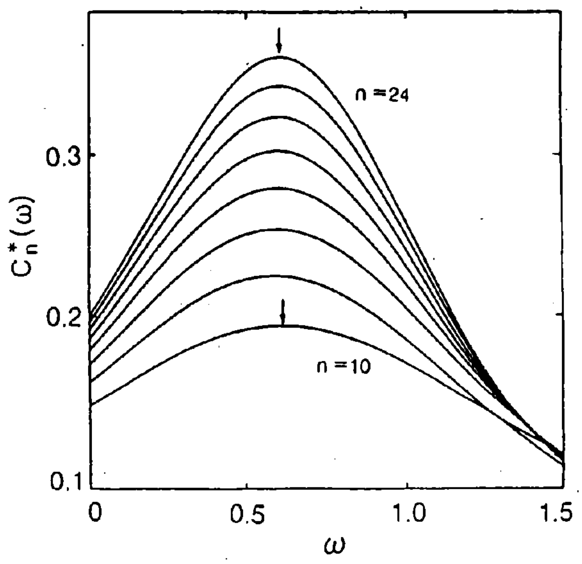

Fig.1 $C_n^*(\omega)$ curves for different n.

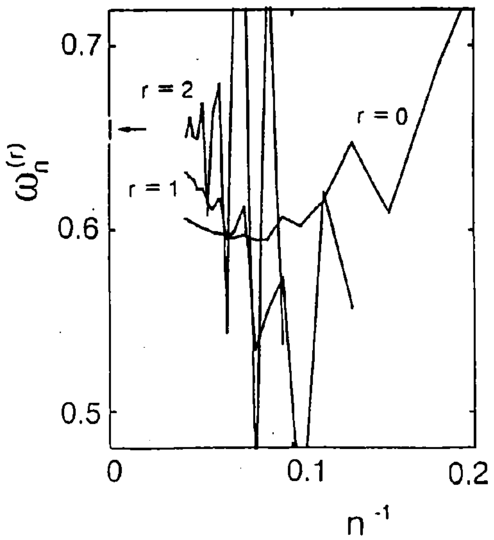

Fig.2 Determination of ω_t.

Table I. Exponents for SAW at ω_t in two dimensions.

Method	ω_t	μ_t	ν_t	γ_t	ϕ
Series analysis	0.655	3.209	0.575	1.095	1.09
Monte Carlo [1]	0.658	3.213	0.574	1.11	0.594
Exact values[2]			0.571...	1.142...	0.428...

1. D.Duplantier and H.Saleur, Phys.Rev.Lett. 59, 539(1987).
2. I.Chang and H.Meirovich, Phys.Rev.E 48, 3656(1993).