Collective mechanism for attraction of charged particulates in plasmas with finite flows

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In a series of recent experiments [1–5], the formation of microscopic Coulomb crystals [6] of solid particles as well as particulate coagulation [7,8] have been demonstrated. In a typical experiment, the dust is embedded in the sheath region [9] where the balance between the gravitational and electrostatic forces is established (see, e.g., [1,3]).

The physics of the sheath region of two–component plasmas has been qulalitatively understood for many years [9]. It was demonstrated that strong electric fields, plasma density gradients, and ion flows are established there. According to the Bohm criterion, the average velocity of the plasma ion flow in the sheath region must exceed that of ion sound [9]. The effects of dust grains in the plasma–wall region have been investigated in Ref. [10–12]. In particular, it was found that, similar to the Bohm criterion for two–component plasmas, the ions entering the sheath region must have a velocity exceeding the critical one (the latter depends on the dust concentration and is not less than the ion sound velocity) [12].

Here, we point out a novel possibility of charged particle attraction in dust plasma systems with finite ion flows. The mechanism is similar to the recently proposed scenario of attractive forces between *moving* charged particulates which involves the collective interactions via low–frequency electrostatic fluctuations [13] of dusty plasmas. The effect is analogous to the Cooper pairing [14] of electrons in superconductors, and has been studied for two–component electron–ion plasmas by Nambu and Akama [15] in which the possibility of electron attraction has been demonstrated. The important feature of the present investigation is that we consider the situation close to laboratory experiments, namely, *static* dust particulates which can interact through the low–frequency oscillations in the ion flow which velocity v_{i0} exceeds the ion–acoustic velocity v_s [16].

The electrostatic potential around the isolated test dust particle can be written as

$$\Phi(\mathbf{x},t) = \int \frac{q_t}{2\pi^2 k^2}\frac{\delta(\omega - \mathbf{k}\cdot\mathbf{v}_t)}{\varepsilon(\mathbf{k},\omega)}\exp(i\mathbf{k}\cdot\mathbf{x} - i\omega t)d\mathbf{k}d\omega, \tag{1}$$

where q_t and $v_t \ll v_{Ti}$ are the charge and velocity of the test dust particle, respectively. The dielectric response function of the plasma in the presence of finite ion flow with the speed v_{i0} ($v_{Ti} \ll v_{i0} \ll v_{Te}$, where $v_{Ti,e} = (T_{i,e}/m_{i,e})^{1/2}$ is the electron (ion) thermal velocity, $T_{e,i}$ is the electron (ion) temperature, $T_i \ll T_e$, and $m_{e,i}$ is the electron (ion) mass) is calculated under condition

$$kv_{Ti} \ll |\omega - k_z v_{i0}| \ll kv_{Te}, \tag{2}$$

where z–axis is directed along the ion flow.

For the plasma dielectric response, we have

$$\varepsilon(\mathbf{k},\omega) = 1 + \frac{1}{k^2\lambda_D^2} - \frac{\omega_{pi}^2}{(\omega - k_z v_{i0})^2}, \tag{3}$$

where $\lambda_D = (T_e/4\pi n_e e^2)^{1/2}$ is the electron Debye radius, $\omega_{pi} = (4\pi e^2 n_i/m_i)^{1/2}$ is the ion plasma frequency, and $n_{e,i}$ are the electron and ion number densities, respectively. The inverse of function (3) can be written as

$$\frac{1}{\varepsilon(\mathbf{k},\omega)} = \frac{k^2\lambda_D^2}{1 + k^2\lambda_D^2}\left[1 + \frac{\omega_s^2}{(\omega - k_z v_{i0})^2 - \omega_s^2}\right], \tag{4}$$

where $\omega_s = kv_s/(1 + k^2\lambda_D^2)^{1/2}$ is the frequency of the oscillations in the ion flow, and $v_s = \lambda_D\omega_{pi}$ is the ion sound velocity.

Substituting Eq. (4) into (1), we obtain that the total electrostatic potential can be represented as the sum of the following two potentials:

$$\Phi(\mathbf{x},t) = \Phi_D(\mathbf{x},t) + \Phi_C(\mathbf{x},t), \tag{5}$$

where

$$\Phi_D = \frac{q_t}{r}\exp(-r/\lambda_D) \tag{6}$$

is the usual static Debye screening potential, and $\Phi_C(\mathbf{x},t)$ is the additional potential involving, in particular, the collective effects caused by the oscillations in the ion flow. We have

$$\Phi_C = \int \frac{q_t}{2\pi^2 k^2} \frac{k^2\lambda_D^2\omega_s^2 \exp(i\mathbf{k}\cdot\mathbf{x} - i\omega t)}{(1+k^2\lambda_D^2)[(\omega - k_z v_{i0})^2 - \omega_s^2]}\delta(\omega - \mathbf{k}\cdot\mathbf{v}_t)d\mathbf{k}d\omega. \tag{7}$$

Integrating Eq. (7) over frequencies and assuming $\mathbf{k}\cdot\mathbf{v}_t \approx 0$, we obtain

$$\Phi_C(\mathbf{x},t) = \mathrm{Const}(t) = \frac{q_t}{2\pi^2\lambda_D M^2}\int \frac{k^2}{1+k^2}\frac{\exp(i\mathbf{k}\cdot\mathbf{x}/\lambda_D)}{(k_z^2+k_0^2)(k_z^2-k_1^2)}dk_z d\mathbf{k}_\perp, \tag{8}$$

where we introduced the dimensionless $k = k\lambda_D$. Furthermore, in Eq. (8), $k^2 = k_z^2 + k_\perp^2$, $k_{0,1}^2 = \pm(1-M^{-2}+k_\perp^2)/2 + [k_\perp^2 M^{-2} + (1-M^{-2}+k_\perp^2)^2/4]^{1/2}$, and $M = v_{i0}/v_s$ is the Mach number. We note that contribution from the poles at $k_z = \pm ik_0$ provides the non-oscillating part which changes the effective Debye shielding scale in plasmas with finite ion flows [11].

We are interested now in the oscillating contribution to the collective potential (7) which arises from the residues at the poles at $k_z = \pm k_1$ in (8). Integration over angles in Eq. (8) can be proceeded using an expansion in spherical harmonics [15]. Furthermore, we assume $k_\perp^2 \ll (M^2-1)$ as well as $k_\perp \ll 1$, and obtain

$$\Phi_C(z,\rho) \approx \frac{2q_t}{\lambda_D\sqrt{M^2-1}}\int_0^1 J_0(k_\perp\rho)\frac{k_\perp^2}{1-M^{-2}}\sin\left(k_\perp z/\lambda_D\sqrt{M^2-1}\right)dk_\perp, \tag{9}$$

where z and ρ are the cylindrical coordinates of the field point, and J_0 is the Bessel function. For small distances in the perpendicular direction, $k_\perp\rho \ll 1$, and for $|z| > \lambda_D\sqrt{M^2-1}$, the main contribution to the stationary wake potential is given by

$$\Phi_C(\rho=0,z) \approx \frac{q_t}{|z|}\frac{2\cos(|z|/L_s)}{1-M^{-2}}, \tag{10}$$

where $L_s = \lambda_D\sqrt{M^2-1}$ is the effective length. From Eq. (10), we can conclude that the wake potential is attractive for $\cos(|z|/L_s) < 0$.

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