# SYMMETRY PROPERTIES OF MATSUBARA GREEN'S FUNCTIONS

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In condensed matter theory, a proper method to deal with non-relativistic quantum many-body systems is essential. Since the celebrated work of Matsubara in 1955, the method of the finite temperature Green's functions, i.e., the Matsubara Green's functions, has been proved one of the most powerful tools in various areas of condensed matter physics[1]. The following discovery of the finite temperature version of the Wick's theorem for the Matsubara Green's functions by Ezawa, Tomozawa and Umezawa in 1958, has made the use of so-called diagrammatic perturbative expansion technique extremely popular among condensed matter physicists[2]. It has been used in almost every application of the quantum field theoretical method in condensed matter physics.

In 1987 it was shown that the non-perturbative canonical formulation of quantum field theory can be applied to the Matsubara Green's functions[3]. On the basis of the canonical commutation relations of the Heisenberg operators, without introducing any perturbative approximations, a method to derive exact relations between Matsubara Green's functions has been established. The relations, i.e., the finite temperature generalized Ward-Takahashi relations (FTGWTR) [4-7], rigorously manifest the symmetry properties of model hamiltonians.

The FTGWTR can be used to keep the consistency of approximation schemes such as perturbative expansion methods or variational methods with respect to conservation laws [5,6]. For instance it has been shown that the FTGWTR give a natural generalization of Baym-Kadanoff's conserving approximation. The FTGWTR can also yield various exact relations between physical quantities, which are often obtained by perturbative approximation calculations without proper justification of the results.

The significance of the FTGWTR in the quantum field theoretical formulation of quantum many-body problem can be seen in the diagram in Fig.1. One of the most promising areas for the application of the FTGWTR is the problem of spontaneous symmetry breaking (SSB) such as superconductivity and superfluidity. If the system has SSB, the conventional perturbative approach is in general not valid. On the other hand, the FTGWTR can relate the order-parameter to other relevant Green's functions rigorously. Actually it gives a generalization of the Nambu-Goldstone theorem [3]. The FTGWTR for the scale and the conformal transformation are expected to give a microscopic rigorous extension of the theory of the renormalization group, which may be used together with the Nambu-Goldstone-Ward-Takahashi relation to investigate the microscopic properties of SSB [8].

In this work we present one of the latest results in the FTGWTR theory [7] to illustrate the usefulness of the formalism. In condensed matter physics it is often necessary to check if the one-particle Green's function

$$G_{\alpha\beta}(\tau_1,\tau_2) = -\left\langle T_\tau\left\{c_\alpha(\tau_1)c^\dagger{}_\beta(\tau_2)\right\}\right\rangle \tag{1}$$

is diagonal. A particularly interesting example is a two-dimensional electron gas (2DEG) under a magnetic field, where the hamiltonian is given as

$$H = H_0 + H_{int}, \tag{2}$$

where the first term is the free hamiltonian

$$H_0 = \sum_{N,k}\left\{\left(n+\frac{1}{2}\right)\hbar\omega_c - \mu\right\}c^\dagger{}_{Nk}c_{Nk}, \tag{3}$$

and the second term is the interaction hamiltonian. Some authors hvae assumed the one-particle Green's function for the hamiltonian given by (1) is diagaonal. However there has not been a comprehensive theoretical formulation to verify if a given Green's fucntion is diagonal at least to the knowledge of the present author. Using the general formulation of the FTGWTR we obtained the following formula for the off-diagonal parts of the Landau level Green's function:

$$\hbar\omega_c(N-N')G_{NkN'k'}(\tau,\tau') = \int_o^{\beta\hbar} d\tau''\left\langle T_\tau\left\{\frac{\partial H_{int}(\tau)}{\partial\tau}c_{Nk}(\tau'')c^\dagger{}_{N'k'}(\tau')\right\}\right\rangle. \tag{4}$$

This formula clearly shows that in general the Green's function has non-vanishing off-diagonal parts if the interaction hamiltonian is included in the total hamiltonian of the system. It is possible to estimate the magnitude of such off-diagonal parts at least approximately by applying the Wick's theorem to the time-ordered product in the right-hand side of eq.(4). The formula can be readily generalized to the N-body Green's function [7].

The usefulness of the FTGWTR formalism is due to the fact that it is entirely based on the commutation relations of the Heisenberg fields, i.e., the field operators in the Heisenberg picture or in the $\tau$-Heisenberg picture, and not on the state vectors in the Hilbert space. Therefore, the theory can be applied both to the pure states as well as to the mixtures such as the grand canonical ensemble mixture used in the Matsubara-Green's function theory. In general one can rigorously take into account the symmetry property of a density operator to obtain the FTGWTR, even if the density operator is time-dependent. This indicates the possible extension of the FTGWTR to non-equilibrium problems [9].

## References

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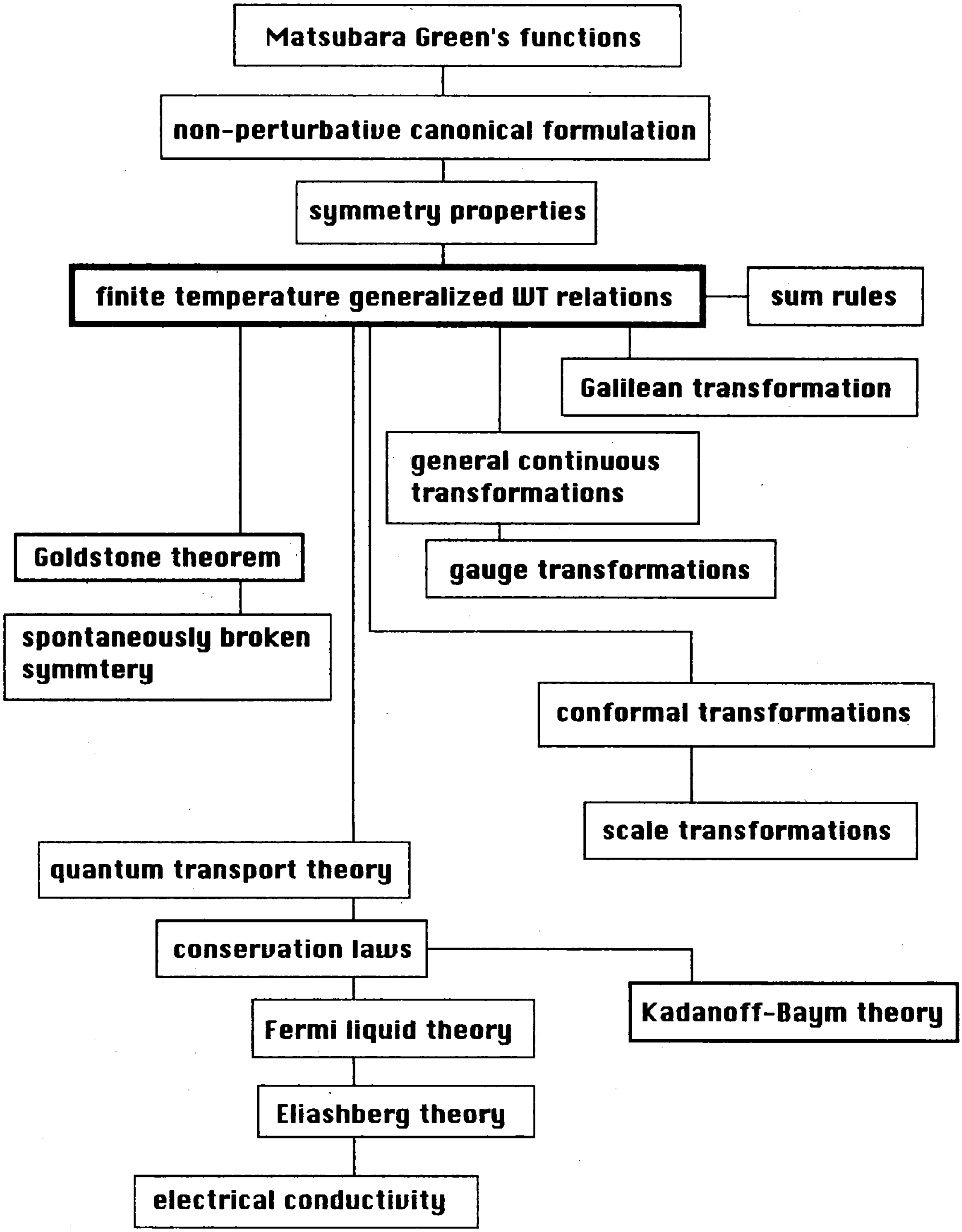
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Fig.1