# RESPONSE PROPERTY OF COMPLEX SYSTEM INTERPRETED BY INFORMATION GEOMETRY

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We tried to express a particular model of a complex system, which responds to external forces, as a statistical manifold constructed by probarbility distribution functions $p(x,\theta)$. This model is characterized by a distribution of receptors and the response property of the receptor. In this model if we regard the distribution of receptor as the probarbility function $p(x,\theta)$, and the response to the external force as the function of $p(x,\theta)$ defined on the statistical manifold, we can describe this model as a geometrical structure of statistical manifold.Then an index which characterizes the response property of the model is related with the $\alpha$-connection of the statistical manifold[1]. This result suggests an analysis of a system which has invariant behavior for a kind of changes of distribution, although this model proposed here is not always the best example.

In this paper, at first a model is explained and then the model is related with a statistical manifold. Finally the property of the manifold which determines the system is described.

## 1, Explanation of the model

Here we make three assumptions. To explein this model,it is sufficient only to describe these assumptions.These assumptions are described as follow.

1)The distribution of receptors is determined by paramater $\theta$.
For example, the distribution is determined by the average and deviation in Gaussian distribution cases.

2)When an external force $\psi$ is applied to the receptor,the domain where the distribution of receptors is $f(x,\theta)$ responds as a set of the next expressions.

$$R_i(t) = a_i(1-\exp(-b_i t)) \tag{1}$$

where $i$ expresses a path explained at next and $t = \psi\ f(x,\theta)$.

3)This responses are transmitted through many paths, and these paths are related by following formulae.

$$R_i = aR_{i-1}(bt) \tag{2}$$

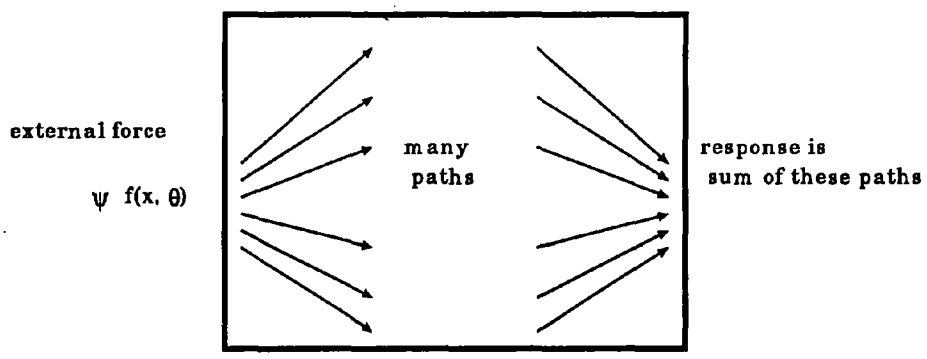
Then the factors are $a_i = a^i, b_i = b^i$ from (1) and (2), if $a_1 = a$ and $b_1 = b$.

The third assumption means that these response paths are related by a kind of scaling law each other. Then the total response $F(t)$ at $x$ is

$$F(t) = \sum_i a_i(1-\exp(-b_i(t)) \tag{3}$$

$$= \sum_i a^i + \sum_i \exp(g(t)) \tag{4}$$

where $g(t) = (i)\log a - b^i t$.




The response function $F(t)$ is then evaluated by the saddle point method[2] as

$$F(t) = \sum_i a^i - const * t^{-\eta} \tag{5}$$

where $\eta = (\log a / \log b)$.

2, Response function and $\alpha$-connection on statistical manifold

If we claim that the geometrical structure of statistical manifolds is invarient for the transformation of probarbility variables, the function of $p(x, \theta)$ defined on the statistical manifold must be

$$\frac{1}{(1-\alpha)} p^{(1-\alpha)/2}(x, \theta) \tag{6}$$

where $\alpha$ is the index of $\alpha$-connection[3].

Note that this formura is the same as the response function evaluated above aside from constant factors. We assert that the response function is defined on statistical manifold. As a result, we obtained $\eta = (\alpha - 1)/2$. This assertion is valid for the case that the property of response does not depend on the distribution of reseptors.

3, The property of the manifold describing the model

The structure of the statistical manifold is determined by probarbility distribution functions and the $\alpha$-connection. In the present model the structure of the statistical manifold determines the respose property of the complex system. A specialized case $\alpha = 1$ , we have $\eta = 0$ and $a << b$ . This case means that the response property is sensitive from definition(1). Also note that $\alpha = 1$ means that the manifold is flat, if $p(x, \theta)$ is exponential family.

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