HEAT CAPACITY AND MAGNETIZATION OF QUANTUM DOTS IN HIGH MAGNETIC FIELDS

S. J. Lee[a], G. Ihm[b], J. H. Oh[c], and K. J. Chang[c]
[a]Department of Physics, Korea Military Academy, Seoul 139-799, Korea
[b]Department of Physics, Chungnam National University, Taejon 305-764, Korea
[c]Department of Physics, Korea Advanced Institute of Science and Technology, Taejon 305-338, Korea

Abstract We investigate the electronic structure of quantum dots with two electrons in high magnetic fields and find that the combined effect of electron-electron interactions and hybrid-magnetoelectric quantization governs the whole energy spectrum. The spin transitions in the ground state are found to strongly depend on the shape of the quantum dot and clearly appear in heat capacity and magnetization at sufficiently low temperature.

The introduction of magnetic field to low dimensional systems shows many interesting properties because of the coupling between comparable electric and magnetic confinements (the hybrid-magnetoelectric effects). It is well known that the hybrid-magnetoelectric effects can be resolved by the equilibrium properties such as the magnetization[1-4] and heat capacity[5,6] in low dimensional systems under the magnetic field. For a quantum dot, since the effects of electron-electron interaction on the electronic structure can not be detected by optical measurements, the heat capacity and magnetization are proposed as a sensitive probe to see the interaction effects.[4]

In this work we investigate the electronic energy levels, magnetization, and heat capacity of a spheroidal (or three-dimensional) quantum dot which has comparable confinements in all the three directions. For various shapes of the spheroidal quantum dot, the combined effect of electron-electron interactions and and hybrid-magnetoelectric quantization are examined as a function of magnetic field and compared with those of a two-dimensional quantum dot, so called, quantum-dot disk.[7]

Our model for a spheroidal quantum dot is characterized by two different frequencies, the confinement frequency ω_p on the $x-y$ plane and ω_v along the z-direction. Ignoring the Zeeman spin-splittings, the Hamiltonian of two electrons is separated into two terms representing the center-of-mass and relative motions, respectively. The eigenenergies of the center-of-mass is the same as those of a single electron in the spheroidal quantum dot. Since the Hamiltonian of the relative motion can not be solved analytically, the method of the matrix diagonalization is used for its eigen energies.

Fig. 1-(a) shows the calculated many-body energy levels as a function of magnetic field for a disk-like quantum dot where $\hbar\omega_p = 3$ meV, $\hbar\omega_v = 30$ meV, and the magnetic field is

applied along the z-direction ($\theta = 0^o$). As the magnetic field increases, the ground state of the disk-like dot in Fig. 1-(a) has a sequence of azimuthal angular momentum l values ($l = 0, -1, -2, \ldots$) with the other quantum numbers zero. Since the total spin of the two electrons is a singlet (triplet) for even (odd) l values, the ground-state transitions entail an alternating sequence of the singlet and triplet states. These spin transitions are likely to occur at lower magnetic fields as the lateral size of a quantum-dot disk increases because the relative contribution of the electron-electron interaction to the confinement energy also increases.[7] Figs. 1-(b) and -(c) show the energies of a football-like quantum dots ($\hbar\omega_p = 3$ meV, $\hbar\omega_v = 1.5$ meV) at $\theta = 0°$ and $55°$, respectively, and a notable difference is found in the ground state. There are no spin transitions for $\theta = 0°$ over the magnetic fields considered, while a spin transition is found for $\theta = 55°$ at a magnetic field between 2 and 3 Tesla. We find that an important factor in determining the ground-state transition is the competition between the hybrid-magnetoelectric effect and the electron-electron interaction.[8]

From the calculated eigenvalues, the equilibrium thermodynamic quantities, heat capacity and magnetization, are calculated and plotted as a function of magnetic field in Figs. 2 and 3, respectively. At sufficiently low temperatures, the thermodynamic quantities are mainly contributed from the two lowest energy levels with the chemical potential lying in the middle of these two states. In this case, C_v can be approximated as;

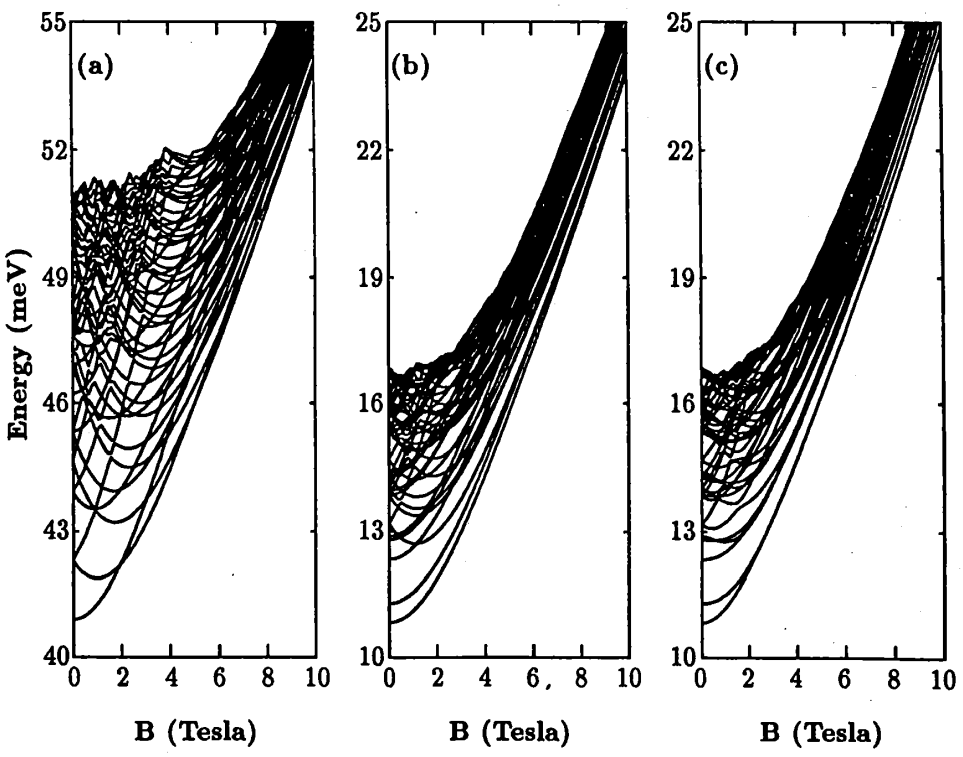

Fig. 1. The energy dispersions are plotted as a function of magnetic field for the quantum dots with keeping $\hbar\omega_p = 3$ meV; (a) $\hbar\omega_v = 30$ meV and $\theta = 0°$, (b) $\hbar\omega_v = 1.5$ meV and $\theta = 0°$, and (c) $\hbar\omega_v = 1.5$ meV and $\theta = 55°$.

$$C_v \simeq \frac{\exp(z')z'^2}{[1+\exp(z')]^2} + \frac{\exp(-z')z'^2}{[1+\exp(-z')]^2}, \tag{1}$$

where $z' = \Delta E/2k_BT$ and ΔE is the energy differece between two lowest energy levels. Then, C_v has a maximum at $\Delta E/k_BT = 4.8$ and becomes zero when $\Delta E = 0$ or $\Delta E \to \infty$. Since ΔE oscillates as a function of magnetic field, C_v also varies with the magnetic field shown in Fig. 2. Thus, the ground-state transition ($\Delta E = 0$) is signaled by the minimum in heat capacity, $C_v \simeq 0$, accompanied by double peaks at sufficiently low temperatures. We find a strongly oscillating pattern in C_v of the disk-like dot [Fig. 2-(a)], while C_v of the football-like dot [Fig. 2-(b)] varies smoothly with magnetic field because of the absence of the ground-state transitions. Fig. 2-(c) shows the variation of C_v with magnetic field for the football-like dot at a tilt angle $\theta = 55°$. We find a ground-state transition at a magnetic field of $2 < B < 3$ Tesla, and the energy spectra are almost degenerate for magnetic fields above B_c.

Next, we consider the magnetization M and the results are shown in Fig. 3. Since $M = -\partial E/\partial B$ at $T = 0$ K, the magnetization at very low temperatures is closely related to the slope of the ground state energy. Similarly to the heat capacity, we find a considerable difference between the two quantum dots at $\theta = 0°$. The magnetization of the disk-like dot [Fig. 3-(a)] has sharp jumps at the ground-state transition points, while the football-like dot [Fig. 3-(b)] shows no peaks. At $T = 0$ K, a sharp discontinuity in magnetization is expected at the transition point because the slope of the ground state energy is discontinuous. However, at finite but very low temperatures, as shown in the inset of Fig. 3-(a), the sharp peaks are broadened and its maximum is shifted to higher magnetic field as temperature increases. The amount of the shift is approximately proportional to temperature. In Fig. 3-(c), the magnetizations, both the x- and z-components, in the football-like dot are shown in the presence of a tilted magnetic field, i.e., at $\theta = 55°$. We find that M_z exhibits a smooth monotonic diamagnetism while M_x shows a discontinuous jump at $B = 2.7$ Tesla. Thus, as in case of heat capacity, this result indicates again the ground-state transition, which is caused by the enhanced hybrid-magnetoelectric effect due to the x-component of magnetic field.

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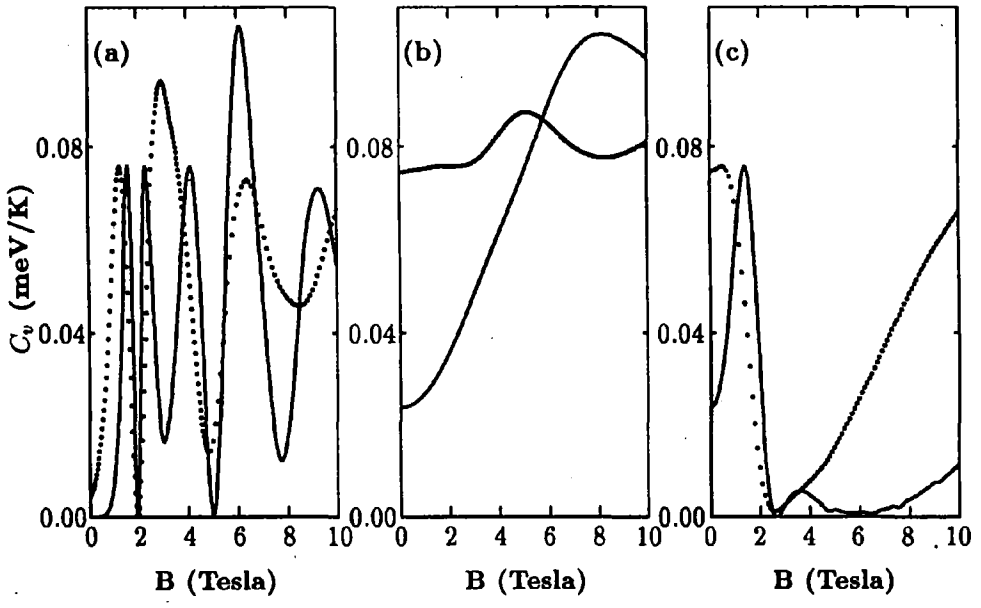

Fig. 2. Heat capacities vs. magnetic field at $T = 0.5$ K (solid line) and $T = 1$ K (dotted line) for the quantum dots shown in Fig. 1.

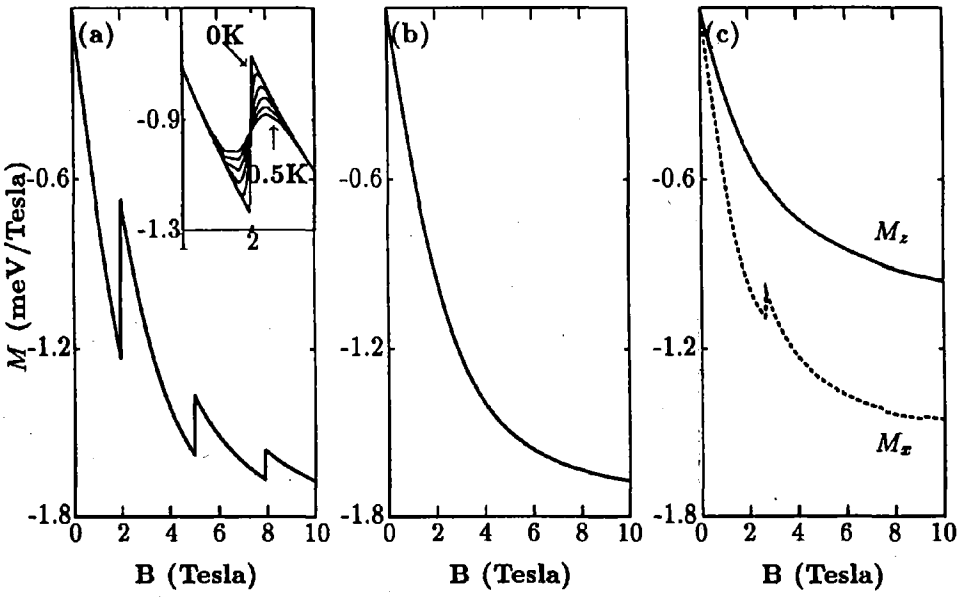

Fig. 3. Magnetizations vs. magnetic field at $T = 0$ K for the quantum dots shown in Fig. 1. In (c), dotted and solid lines denote M_x and M_z, respectively. The inset in (a) enlarges the temperature dependence of magnetization near the ground-state transition point.