TEMPERATURE DEPENDENT DIFFUSION CONSTANT AND MOBILITY OF POSITIVE ION IN BULK LIQUID HELIUM NEAR ABSOLUTE ZERO TEMPERATURES

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Abstract

Using the rate of momentum transfer through the scatterings among quasiparticles and phonons by ion, diffusion constant $D(T)$ and the mobility of positive ion $\mu_+(T)$ are evaluated near absolute zero temperatures in bulk liquid ^{4}He. The diffusion constant $D(T)$ has T^{-7} dependence while the mobility of positive ion varies with T^{-4}, which agrees with the experimental results.

Since Landau's phenomenological theory of the superfluid[1] the properties of liquid helium at low frequencies, low momenta and low temperatures can be explained by Landau's two fluid theory[2]. Among many properties of liquid ^{4}He Landau and Khalatnikov[3] first investigated the kinetic coefficients that are also researched by others[4]. Recently Behringer and Meyer[5] investigated diffusive phenomena in the normal and superfluid phases in liquid ^{3}He-^{4}He mixtures and Um et al[6] analyzed the quasi diffusion relation between phonons and rotons in liquid ^{4}He.

After Atkins[7] proposed the original model of the mobility for charged carriers, it has been one of the interesting topics in physics of systems. The movement of the charged carriers is limited by collisions of thermally excited phonons at very low temperatures and rotons relatively high temperatures in bulk liquid ^{4}He.

The diffusion constant and mobility are closely related to the elementary excitation spectrum of liquid ^{4}He. Recently we have derived the temperature-dependent excitation spectrum, which exhibits the anomalous phonon-like at low momenta and roton-like at large momenta in the ring-diagram approximation :

$$\varepsilon_{ph}(q,T) = C_0 q[A(T) + \gamma(T)q^2 - \delta(T)q^3 + \cdots], \tag{1}$$

$$\varepsilon_{rot} = \Delta + \frac{(P-P_0)^2}{2\mu_r}, \tag{2}$$

where q and P are phonon and roton momentum, C_0 the sound velocity at absolute zero temperature. Δ, P_0 and μ_r are the roton parameters and the temperature-dependent coefficient in Eq.(1) are all positive constant which can be determined from the potential parameters. Here we have taken the soft potential with Lennard-Jones type tails.

In this paper we evaluate the diffusion constant and mobility as a function of temperature near absolute zero temperatures in bulk liquid ^{4}He. In the case of bulk liquid ^{4}He there are many experimental and theoretical reports, but we will focus our attention on two papers of Schwarz[9] and Hannahs-Williams[10].

From the definition and relation between the diffusion constant D and mobility μ may be expressed as

$$\vec{V}_D = \mu \vec{F} \quad D = \mu k_B T, \tag{3}$$

where $\vec{V}_D$ is the mean drift velocity of the particle in an external field $\vec{F}$ and $\vec{F}$ is the force exterted on the particle by the field.

We first consider a roton moving slowly with the mean drift velocity $\vec{V}_D$ in the stationary phonon field. In this scatterings we assume that a phonon is scattered from an incident momentum $\vec{q}$ to a final momentum $\vec{q'}$ with transition probability $\Gamma_{\vec{V}_D}(\vec{q}-\vec{q'})$ per unit time[11] is given as

$$\frac{d\vec{P}_{ph}}{dt} = \sum_{\vec{q},\vec{q'}} \hbar(\vec{q}-\vec{q'})n(\varepsilon_{\vec{q}})\{1+n(\varepsilon_{\vec{q'}})\}\Gamma_{\vec{V}_D}(\vec{q}-\vec{q'}), \tag{4}$$

where $n(\varepsilon_{\vec{q}})$ is the equilibrium distribution function of phonon. In case of phonon to be in equilibrium with moving roton, $n(\varepsilon_{\vec{q}})$ becomes $\bar{n}(\varepsilon_{\vec{q}}) = n(\varepsilon_{\vec{q}} - \hbar\vec{q}\cdot\vec{V}_D)$, and dP_{ph}/dt will vanish identically if the phonon were in thermal equilibrium with moving roton. Thus rewriting Eq.(4), we get

$$\frac{d\vec{P}_{ph}}{dt} = \sum_{\vec{q},\vec{q'}} \hbar(\vec{q}-\vec{q'})[n(\varepsilon_{\vec{q}})\{1+n(\varepsilon_{\vec{q'}})\} - \bar{n}(\varepsilon_{\vec{q}})\{1+\bar{n}(\varepsilon_{\vec{q'}})\}]\Gamma_{\vec{V}_D}(\vec{q}-\vec{q'}). \tag{5}$$

The quantity in the square bracket are obviously order of $\vec{V}_D$ and thus expanding the terms in the first order $\vec{V}_D$ and neglecting Doppler correction $\vec{q}$ and replacing $\Gamma_{\vec{V}_D}$ by its value Γ_0 for a stationary roton, we may obtain the first order contribution to dP_{ph}/dt. To evaluate this contribution we introduce the direction vector $\hat{n}$ described by angle ψ_n, i.e., $\hat{q}' = \hat{n}$ with respect to the direction $\hat{q}$ and then for intergration over $\hat{n}$, we insert $\delta(\hat{n}-\hat{q}') = \delta(\psi_n - \psi_{q'})$ instead of $\hat{q}' = \hat{n}$ into the first order contribution dP_{ph}/dt. In this expression we may find that the total transition probability per unit time for a phonon with incident momentum $\vec{q}$ to scatter elastically to a final momentum $\vec{q'}$. It is exactly equivalent to the product of the differential equation $d\sigma/d\psi_n$ and the incident flux $\mid d\varepsilon_q/dq \mid /V\hbar$, where V is the volume of the system. Then the total momentum transfer rate can be expressed as

$$\frac{d\vec{P}_{ph}}{dt} = \frac{\hbar}{6\pi^2}\vec{V}_D \int_0^\infty dq q^4 \left\{-\frac{\partial n(\varepsilon_{\vec{q}})}{\partial \varepsilon_{\vec{q}}}\right\} \left|\frac{\partial \varepsilon_{\vec{q}}}{\partial q}\right| \sigma_{th}(q). \tag{6}$$

where $\sigma_{th}(q)$ is the momentum transfer cross section.

Comparing of Eq.(6) with Eq.(3) and applying Eqs(1)-(2) to Eq.(6) together with the distribution function and the differential cross section of phonons by rotons in bulk liquid ^{4}He[6] we obtain the diffusion constant $D(T)$:

$$D(T) = \frac{24\pi^3\rho_0^2\hbar^7C_0^{10}}{8!\zeta(8)P_0^2}\alpha(k_BT)^{-7}\left[1+\frac{3\gamma(T)10!\zeta(10)}{C_0^2A(T)8!\zeta(8)}k_BT^2 - OT^3 + \cdots\right]. \tag{7}$$

where α is constant related with C_0 and roton parameters[3].

Now we consider the motion of positive ion instead of roton in the phonon field. The ionic recoil is generally complicated kinematic, but for liquid ^{4}He it is not relatively important. In case of thermal phonon the typical phonon wave number $k_BT/\hbar C_0$ varies linearly with temperature, where $C_0 \simeq 2.4\times10^2 m/s$ and from the dimensionless ratio $< E_{rec} > /k_BT$, we may estimate the characteristic phonon temperature $T_{ph} = m_4C_0^2/k_B \simeq 28K$, where $< E_{rec} >$ is the spread of ionic recoil energy and m_4 is ^{4}He atomic mass. It is clear that $< E_{rec} >_{ph} \ll k_BT$, therefore we may consider that ion-phonon scattering is always elastic.

By the definition of the mobility we may reexpress $\vec{V}_D = \mu(E)\vec{E}$ ($\vec{V}_D = \mu\vec{F}$). Equating the force on the ion of Eq.(6) to an applied electric field $e\vec{E}$ we get the mobility of positive ion :

$$\frac{e}{\mu_+} = \frac{\hbar}{6\pi^2}\int dqq^4\left\{-\frac{\partial n(\varepsilon_{\vec{q}})}{\partial\varepsilon_{\vec{q}}}\right\}\left|\frac{\partial\varepsilon_{\vec{q}}}{\partial q}\right|\sigma_{th}(q). \tag{8}$$

To simplify the calculations, we introduce the thermal averaged transport cross section[13]

$$\bar{\sigma}_{th} = \int_0^\infty dqq^4\left[-\frac{\partial n(\varepsilon_{\vec{q}})}{\partial\varepsilon_{\vec{q}}}\right]\left|\frac{\partial\varepsilon_{\vec{q}}}{\partial q}\right|\sigma_{th}(q)/\int_0^\infty dqq^4\left[-\frac{\partial n(\varepsilon_{\vec{q}})}{\partial\varepsilon_{\vec{q}}}\right]\left|\frac{\partial\varepsilon_{\vec{q}}}{\partial q}\right|. \tag{9}$$

Substituting Eq.(9) into Eq.(8) and applying the anomalous phonon spectrum(Eq.(1)) together with the distribution function for phonons to Eq.(8) we obtain the mobility of positive ion in phonon-limited region in bulk liquid ^{4}He :

$$\frac{e}{\mu_+} = \frac{4!\zeta(4)}{6\pi^2\hbar^3C_0^4}\bar{\sigma}_{ph}A(T)k_B^4T^4\left[1+3\frac{6!\zeta(6)}{4!\zeta(4)}\frac{\gamma(T)}{C_0^2A(T)}k_B^2T^2 - OT^3+\cdots\right], \tag{10}$$

where $\bar{\sigma}_{ph}$ represents to the thermal average cross section in Eq.(9).

To analyze the temperature variation of $D(T)$ and $\mu_+(T)$, we have taken the parameters which are determined from the analysis of the thermal conductivity and first viscosity[8] and the results obtained from Khalatnikov[14]. The parameters are listed in Table 1.

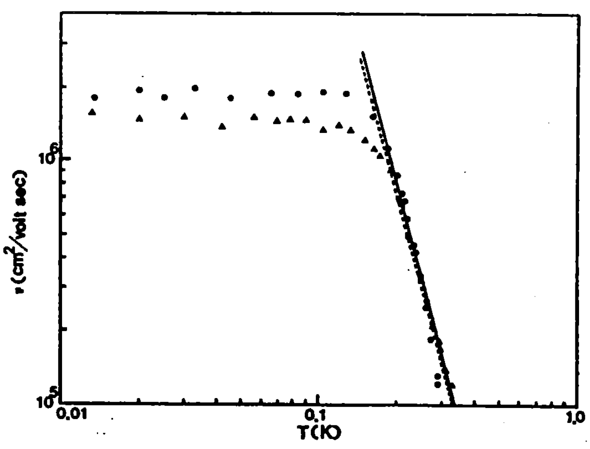
Table 1. Theoretical parameters in Bulk liquid ^{4}He

ρ	$C_0(m/s)$	$\Delta/k_B(K)$	$P_0(\AA^{-1})$	μ_r
$2.18\times10^{-2}\AA^{-3}$	2.8.2	8.61	1.93	$0.153m_4$

As temperature decreases the diffusion constant $D(T)$ increases rapidly with variation of T^{-7}. It is purely due to the phonon since the roton decreases exponentially near zero temperatures. In kinetic theory of gases the diffusion constant is expressed as the order of magnitude $D \propto \vec{V}(n\sigma_t)^{-1}$, where $\vec{V}$ is the mean thermal velocity of the light particles, n the particle density and σ_t transport cross section. Near absolute zero temperatures $\vec{V} \simeq C_0$, $n \sim n_{ph} \propto T^4$, $pC_0 \propto k_BT$ and the transport cross section is given as $\sigma_t(T) = T^4$. Therefore we obtain $D(T) \propto T^{-7}$ which is in good agreement with Eq.(7).

In the phonon limited regions the mobility of positive ion in bulk liquid ^{4}He is given in Eq.(10). The mobility has the temperature variation with T^{-4} with extra terms. Hannahs and Williams[10] measured the mobility of positive ion from obtaining the quality factor Q at the depth under the surface of approximately $300\AA$ of liquid ^{4}He surface by measurement of the plasmon resonance width over temperature ranges from $14mK$ to $400mK$ and also used the measured effective mass of ion to be $(30\pm1)m_4$ at $T = 0K$. In Fig. 1 the solid line is our theoretical curve, while the dotted line is their result close to T^{-4} variation. In fitting data we have taken the effective mass $m^* \simeq 40m_4$ as Baym et al did. In Eq.(10) our result show T^{-4} variation. As decreasing temperatures the mobility does not show the temperature dependence for $T < 100mK$, because the mobility is related closely to the ripplon scattering at the surface and increases very slowly.

Schwarz[9] calculated theoretically the mobility of positive ion below $1K$ under the hydrodynamic approximation with electrostrictional variations. This result for phonon limited mobility is found to be in excellent agreement with experiment in the assumption of a solid-liquid square surface energy of order $\sigma_{ls} \sim 0.1ergcm^{-2}$ for core parameters. This result provides $e/\mu(T)$ within 10% of the measured values. In Figure 2 the solid curves are our theoretical curve for the varies $\bar{\sigma}_{ph}$, while the dotted curves are Schwarz's results for the various surface energy σ_{ls}.



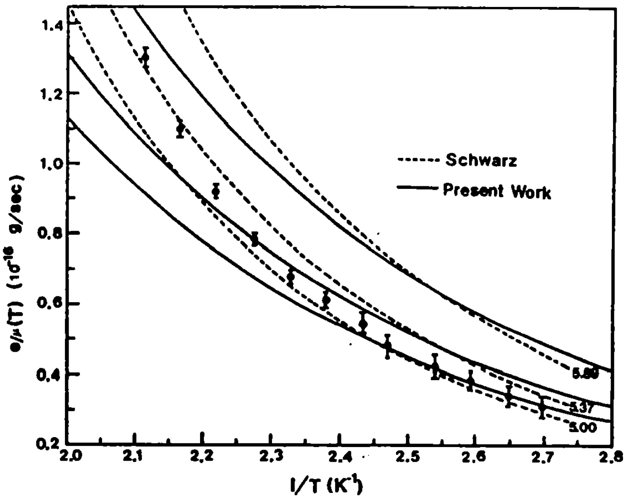


Fig. 1. Mobility as a function of temperature. Fig. 2. $e/\mu(T)$ versus inverse temperature.

Here, we have chosen $\bar{\sigma}_{ph} = \pi a_+^2$ in the phonon limited region where a_+ is the average radius of ions. For $a_+ = 5.00$ our curve fits the experimental curve, which correspond to core parameter 0.15, in excellant agreement except roton limited region ($T^{-1} <\sim 2.35K^{-1}$).

In conclusion we remark that the diffusion constant $D(T)$ varies with T^{-7} and the mobility $\mu_+(T)$ of positive ion has T^{-4} dependence near absolute zero temperatures in bulk liquid ^{4}He.

References

[1] L.D.Landau, J. Phys. U.S.S.R. 11, 91(1947).
[2] P.C.Hohenberg and P.C.Martin, Ann. Phys.(N.Y) 34, 291(1965).
[3] L.D.Landau and I.M.Khalatnikov, Zh. Eksp. Teor. Fiz. 19, 637,709(1949).
[4] T.R.Kirkpatrick and J.R.Dorfman, J. Low Temp. Phys. 58, 339(1985); C.I.Um, C.W.Jun, W.H.Kahng and T.F.George, Phys. Rev. B38, 8838(1988).
[5] R.P.Behringer and H.Meyer, J. Low Temp. Phys. 46, 407, 435(1982).
[6] C.I.Um, C.W.Jun, H.J.Shin and T.F.George, J. Low Temp. Phys. 78, 51(1990).
[7] K.R.Atkins, Phys. Rev. 116, 1339(1959).
[8] C.I.Um, S.K.Yoo, S.Y.Lee, S.T.Nam, T.F.George and L.N.Pandey, Physica B203, 151(1994); C.I.Um, S.K.Yoo and S.Y.Lee, J. Korean Phys. Soc. 25, 339(1992).
[9] K.W.Schwarz, Phys. Rev. A6, 1958(1972).
[10] S.Hannahs and G.Williams, Phys. Rev. B42, 7907(1990).
[11] G.Baym, R.G.Barrera and C.J.Pethick, Phys. Rev. Lett. 22, 20(1969).
[12] A.Isihara, S.T.Choh, W.H.Kahng and C.I.Um, Physica B100, 74(1980).
[13] W.G.Rayfield and R.Reif, Phys. Rev. 136, A1194(1964).
[14] I.M.Khalatnikov and Y.A.Matveev. Zh. Eksp. Teor. Fiz. 51, 99(1983).