Generalization of geometry-induced effect noted by Takagi and Tanzawa

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The formulation on a particle motion in n-demensional curved manifold $\mathbf{M}_n$ embedded in p-dimensional Euclidean space $\mathbf{R}_p$ is summarized, and the geometry-induced gauge structure is explained. Next we examine the scalar field theory with a soliton solution, and point out that in spite of the infinite degrees of freedom such a field theory has the same mathematical structure as a particle motion in $\mathbf{M}_n \subset \mathbf{R}_p$, and our formalism affords a clearer view of understanding physical contents of such a field theory.

1.Introduction Quantum theory on a curved manifold has been investigated from various points of view [1,2,3]. Quantum treatment of soliton such as Skyrmion provides a typical example of quantum theory on a curved manifold [4]. We consider the motion of a particle on an n-dimensional curved manifold $\mathbf{M}_n$ embedded in a p-dimensional Euclidean space $\mathbf{R}_p$, and the particle motion is thought to be confined by some confining potential. Then a correction term with the order $\hbar^2$ in the effective Hamiltonian on $\mathbf{M}_n$ appears as a quantum effect due to a particle motion in the directions perpendicular to $\mathbf{M}_n$ [2]. Such an effect is dropped from the beginning when we apply simply Dirac method for constrainted dynamical systems.

Two years ago, Takagi and Tanzawa [5] have pointed out that , for a particle motion in a thin tube (in $\mathbf{R}_3$) forming a closed loop, in the effective Hamiltonian on $\mathbf{M}_1$ there appears an effective vector potential , which depends on the geometry of $\mathbf{M}_1$, and that there exists a complete analogy with Aharonov-Bohm effect, called a geometry-induced AB effect. One of the present authors (K.F.) and N.Ogawa [6] genaralized this result to the case of a particle motion in a thin neighborhood of $\mathbf{M}_n$ embedded in $\mathbf{R}_p$. The aim of the present report is to apply this formalism to a scalar field theory which allows a classical solution, and to examine the correspondence to the field theory of the extended object given by Sakita and others [7].

In the following, we first summarize the formalism in case of a particle motion on $\mathbf{M}_n \subset \mathbf{R}_p$ (in Sec.2), and extend it to the case of a scalar field theory with a classical solution(in Sec.3). The last section is devoted to discussions and summary of remaining tasks.

2. Particle motion in a thin layer along M_n embedded in R_n

2-1. Basic relations As in [6], a set of coordinates $\{X^A; A = 1,\cdots,p\}$ of a point in a thin-layer neighbourhood of $\mathbf{M}_n$ is expressed as

$$X^A(q^\beta) = x^A(q^b) + \sum_{U=n+1}^{p} q^U N_U{}^A(q^b), \quad s = 1,\cdots,p; \tag{2.1}$$

$\{q^\beta, \beta = 1,\cdots,n,n+1,\cdots,p\}$ consists of two parts; the first part $\{q^b, b = 1,\cdots,n\}$ is a set of curvilinear coordinates on $\mathbf{M}_n$ and the remaining part is $\{q^U, U = n+1,\cdots,p\}$.

$N_U{}^A(q^b)$ is a unit normal vector to $\mathbf{M}_n$ at a point $x^A(q^b)$. For simplicity we omit $\sum_{U=n+1}^{p}$ for a dummy index U and write *e.g.* $x^A(q^b)$ as $x^A(q)$. The metric tensor in $\mathbf{R}_n$, written as $\tilde{G}_{\alpha\beta}(q,q^U)$ is given by

$$\tilde{G}_{\alpha\beta} = \tilde{B}_\alpha{}^A \eta_{AD} \tilde{B}_\beta{}^A \quad \text{with} \quad \tilde{B}_\beta{}^A(q,q^U) = \frac{\partial X^A(q,q^U)}{\partial q^\beta}, \tag{2.2a}$$

and the metric tensor on $\mathbf{M}_n$ as

$$g_{ab}(q) = B_a{}^A \eta_{AD} B_b{}^D \quad \text{with} \quad B_b{}^A(q) = \frac{\partial x^A(q)}{\partial q^b}. \tag{2.2b}$$

Since $B_b{}^A$ is tangent to $\mathbf{M}_n$, we have $B_b{}^A \eta_{AD} N_V{}^D = 0$. Note that we have $\tilde{B}_\beta{}^A = (\tilde{B}_b{}^A, \tilde{B}_V{}^A) = (B_b{}^A + \partial_b N_W{}^A \cdot q^W, N_V{}^A)$.

The fundamental equations for $B_b{}^A$ and $N_V{}^A$ are

$$\partial_a B_b{}^A = \Gamma^d{}_{ab} B_d{}^A + H_{Wab}\eta^{WU} N_U{}^A, \tag{2.3a}$$

$$\partial_a N_V{}^A = -H_{Va}{}^d B_d{}^A - T_{VW,a}\eta^{WU} N_U{}^A, \tag{2.3b}$$

where $\Gamma^d{}_{ab}$ is Christoffel symbol constructed in terms of g_{bd} and $H_{Wab} = H_{Wba} = H_{Wb}{}^e g_{ea}$, $T_{VW,a} = -T_{WV,a}$.

Concrete forms of $\tilde{G}_{\alpha\beta}$ and its inverse $\tilde{G}^{\alpha\beta}$ are given as follows:

$$[\tilde{G}_{\alpha\beta}] = \begin{bmatrix} \tilde{G}_{ab} & \tilde{G}_{aU} \\ \tilde{G}_{Vb} & \tilde{G}_{VU} \end{bmatrix} \tag{2.4a}$$

with $\tilde{G}_{ab} = \lambda_{ab} + \tilde{G}_{aX}\eta^{XY}\tilde{G}_{bY}$, $\lambda_{ab} = g_{ab} - 2H_{Wab}q^W + q^X q^Y H_{Xa}{}^e H_{Yeb}$,

$\tilde{G}_{aU} = \tilde{G}_{Ua} = T_{UX,a}q^X$, $\tilde{G}_{VU} = \eta_{VU}$;

$$[\tilde{G}^{\alpha\beta}] = \begin{bmatrix} \tilde{G}^{ab} & \tilde{G}^{aU} \\ \tilde{G}^{Vb} & \tilde{G}^{VU} \end{bmatrix} \tag{2.4b}$$

with $\tilde{G}^{ab} = \lambda^{ab}$, $\lambda^{ab}\lambda_{bd} = \delta^a{}_d$, $\tilde{G}^{aU} = \tilde{G}^{Ua} = -\lambda^{ad}T_{XW,d}q^W\eta^{XU}$,

$\tilde{G}^{VU} = \eta^{VU} + \eta^{VX}T_{XW,b}q^W\lambda^{bd}T_{YZ,d}q^Z\eta^{YU}$.

From the condition $\partial_a B_b{}^A = \partial_b B_a{}^A$, one obtains the curvature tensor on $\mathbf{M}_n$ given as

$$R_{ab,cd} = \eta_{AB}(H^A{}_{ac}H^B{}_{bd} - H^A{}_{ad}H^B{}_{bc}) \tag{2.5a}$$

with $H^A{}_{ab} = H_{Vab}\eta^{VW}N_W{}^A$ (Euler-Schouten tensor);

hence, the tensor $R_{bc} = g^{ad}R_{ab,cd} = \eta_{AB}(H^A{}_b{}^d H^B{}_{dc} - H^A{}_a{}^a H^B{}_{bc})$, and scalar curvature

$$R = g^{bc}R_{bc} = \eta_{AB}(H^A{}_b{}^d H^B{}_d{}^b - H^A{}_a{}^a H^B{}_b{}^b). \tag{2.5b}$$

From $\partial_a\partial_b N_V{}^A - \partial_b\partial_a N_V{}^A = 0$, one obtains

$$R_{da,VW}\eta^{VW} = \eta^{VW}(-H_{Vd}{}^b H_{Wba} + H_{Va}{}^b H_{Wbd}), \tag{2.5c}$$

where $R_{da,VW} \equiv -\partial_d T_{VW,a} + \partial_a T_{VW,d} + T_{XV,d}\eta^{XY}T_{YW,a} - T_{XV,a}\eta^{XY}T_{YW,d}$. (2.5d)

Using the extrinsic mean curvature H defined by $H \equiv [\eta_{ab}H^A{}_b{}^b H^B{}_d{}^d]^{1/2}/n$, one obtains

$$R = H^A{}_b{}^d\eta_{AB}H^B{}_d{}^b - n^2H^2. \tag{2.6}$$

2-2. Canonical quantization and form of kinetic energy We examine the form of kinetic energy

$$\tilde{K} = \frac{1}{2}\dot{X}^A \eta_{AB} \dot{X}^B, \qquad \dot{X}^A = \frac{dX^A}{d\tau}, \tag{2.7}$$

expressed in terms of q^β-variables. In order to perform the quantum-mechanical calculations from the outset, we adopt the procedure which is consistent for the transformation from Euclidean coordinates to curvilinear ones as in the present case. We assume

$$[q^\beta, \dot{q}^\delta] = i\hbar f^{\beta\delta}(q^\gamma), \qquad [q^\beta, q^\delta] = 0, \tag{2.8a}$$

where $f^{\beta\delta}$ is a function of only (q^γ). When we require the cannonical commutation relations

$$[q^\alpha, p_\beta] = i\hbar \delta^\alpha{}_\beta, \qquad [p_\alpha, p_\beta] = 0 \tag{2.8b}$$

$$\text{for} \qquad p_\beta \equiv \frac{\partial \tilde{K}}{\partial \dot{q}^\beta} = \frac{1}{2}\{\tilde{G}_{\beta\delta}, \dot{q}^\delta\} \equiv < \tilde{G}_{\beta\delta}, \dot{q}^\delta >, \tag{2.8c}$$

we obtain $f^{\alpha\beta}\tilde{G}_{\beta\delta} = \delta^\alpha{}_\delta$, *i.e.* $f^{\alpha\beta}$ is the inverse of $\tilde{G}_{\beta\delta}$.

Now we rewrite $\tilde{K}$ (2.7) in the covariant form

$$\tilde{K} = \frac{1}{2}\tilde{G}^{-1/4} p_\alpha \tilde{G}^{-1/2} \tilde{G}^{\alpha\beta} p_\beta \tilde{G}^{-1/4}, \qquad \tilde{G} = |\det \tilde{G}_{\alpha\beta}|. \tag{2.9a}$$

We obtain by noting $\tilde{G} = |\det \lambda_{ab} \cdot \det \eta_{VW}| = |\det \lambda_{ab}| \equiv \lambda$,

$$\tilde{K} = \frac{1}{2}\lambda^{-1/4}\Pi_a \lambda^{1/2}\lambda^{ab}\Pi_b \lambda^{-1/4} + \frac{1}{2}\lambda^{-1/4} p_V \lambda^{1/2} \eta^{VW} p_W \lambda^{-1/4}. \tag{2.9b}$$

Here, Π_a is defined by

$$\Pi_a \equiv p_a + \frac{1}{2} T_{VW,a} L^{VW}, \tag{2.9c}$$

$$L^{WX} \equiv q^W \eta^{XV} p_V - q^X \eta^{WV} p_V = \eta^{XV} p_V q^W - \eta^{WV} p_V q^X. \tag{2.9d}$$

L^{WX} satisfies the commutation relation

$$[L^{VX}, L^{WY}] = i\hbar(\eta^{VW} L^{XY} + \eta^{XY} L^{VW} - \eta^{VY} L^{XW} - \eta^{XW} L^{VY}). \tag{2.9e}$$

In the thin layer approximation $|H_{Uab} q^U| \ll 1 \text{and} |T_{UV,b} q^U| \ll 1$ [5,6], one obtains

$$\tilde{K} \quad \overset{thin\ layer}{\longrightarrow} \quad K^* = K + \frac{1}{2} p_V \eta^{VW} p_W + \Delta V^*, \tag{2.10a}$$

$$\text{where} \quad K = \frac{1}{2} g^{-1/4} \Pi_a g^{1/2} g^{ab} \Pi_b g^{-1/4}, \quad \Delta V^* = \frac{\hbar^2}{2}[-\frac{R}{2} - \frac{1}{4} n^2 H^2]. \tag{2.10b}$$

ΔV^* comes from the last term in (2.9b). It may be worthy of noting that we have

$$\Delta V^* = -\frac{1}{2}[\tilde{Y}(q, q^V) - Y(q)]|_{thin\ layer}; \tag{2.11a}$$

$$\text{where} \ -\frac{1}{2}\tilde{Y}(q, q^V) = \hbar^2[\frac{1}{4}\partial_\delta(\tilde{G}^{\delta\beta}\tilde{\Gamma}_\beta + \frac{1}{8}\tilde{G}^{\delta\beta}\tilde{\Gamma}_\delta\tilde{\Gamma}_\beta)], \tilde{\Gamma}_\alpha \equiv \tilde{\Gamma}^\beta_{\alpha\beta} = \tilde{G}^{\gamma\beta}\partial_\alpha\tilde{G}_{\gamma\beta}/2.$$

Y(q) is the quantity constructed in terms of g_{ab} corresponding to $\tilde{Y}$;

$$-\frac{1}{2}Y/\hbar^2 = \frac{1}{4}\partial_b(g^{bd}\Gamma_d) + \frac{1}{8}g^{bd}\Gamma_b\Gamma_d = \frac{1}{8}[R + g^{ab}\Gamma_a{}^e{}_d\Gamma^d{}_{be}]. \tag{2.11b}$$

Using (2.11b), we can rewrite (2.10b) as

$$K(2.10b) = \frac{1}{2}\Pi_a g^{ab}\Pi_b - \frac{1}{2}Y(q). \tag{2.11c}$$

2-3. Commutator $[\Pi_a, \Pi_b]$ Utilizing some relations given in 2-1, we obtain

$$[\Pi_b, \Pi_d] = \frac{i\hbar}{2} R_{bd,VW} L^{VW}. \tag{2.12a}$$

This is analogous to a charged particle moving in magnetic field $\vec{H}$, in which we have

$$[\Pi_j, \Pi_k] = i\hbar\frac{e}{c}F_{jk}; \quad \Pi_j = p_j - \frac{e}{c}A_j; \quad j,k = 1,2,3; \quad F_{jk} = \frac{1}{2}\epsilon_{jkl}H^l. \tag{2.12b}$$

We see that the field $T_{VW,a} = N_V{}^A\partial_a N_W{}^B\eta_{AB}$ plays a role of gauge potential. The gauge property including the non-Abelian one is seen as follows: When the total Hamiltonian $\tilde{H} = \tilde{K}(2.9b) + V$ has the part of potential which confines the particle motion to $\mathbf{M}_n$, and is invariant under rotation of the set of $\{N_U{}^A,\ U = n+1, \cdots, p\}$ such as

$$N'_V{}^A(q) = N_W{}^A(q)\Lambda^W{}_V(q), \qquad \eta_{XY}\Lambda^X{}_W\Lambda^Y{}_V = \eta_{WV}, \tag{2.13a}$$

we obtain

$$T_{WV,b} \longrightarrow T'_{WV,b} = (\Lambda^{-1})_W{}^X T_{XY,b}\Lambda^Y{}_V + (\Lambda^{-1})_{WX}\partial_b\Lambda^X{}_V. \tag{2.13b}$$

$T_{WV,b}$ cannot be eliminated globally. In case of a tube embedded in $\mathbf{R}_3$[5], Π_b reduces to

$$\Pi_1 = p_1 + T_{23}L^{23} \equiv p - \omega L. \tag{2.14a}$$

It is pointed out in Ref.[5] due to multivaluedness of triangular function

$$\int_0^l \omega(q)dq = \int_0^l \tau(q)dq \pmod{2\pi} \tag{2.14b}$$

is obtained, where l is the length of center line of the tube; τ is the torsion appearing in Frenet-Seret equation in $\mathbf{R}_3$.

3. Application to field theory of extended object

3-1. Purpose In this section we extend the formalism given in Secion 2 to field theory. For simplicity, we examine the scalar filed theory which allows a soliton solution. We consider the Lagrangian expressed as

$$L = -\frac{1}{2}\partial_\mu \phi^A(x)\eta_{AB}\partial^\mu \phi^B(x) - V(\phi(x)), \tag{3.1}$$

where (x^μ) is a space-time coordinate; its metric is $\eta_{\mu\nu}$ with $\mathrm{diag}(\eta_{\mu\nu}) = (- + + \cdots)$; the upper index A of ϕ^A denotes the internal degrees of freedom.

The field operator $\phi^A(\vec{x}, x^0)$ is assumed to be expanded as [7]

$$\phi^B(\vec{x}, x^0) = \phi_0^B(\vec{x}, q^b) + \sum_U \phi_U{}^B(\vec{x}, q^b) q^U, \tag{3.2a}$$

where $\{q^b,\ b = 1, \cdots, n\}$ denotes a set of collective coordinates representing the center of mass coordinates of the classical soliton, the orientation on the internal space and so on; ϕ_0^B is the soliton solution satisfying

$$-\frac{\partial}{\partial \vec{x}}\frac{\partial}{\partial \vec{x}}\phi_0^B(\vec{x}, q^b) + \frac{\partial V}{\partial \phi_0^A(\vec{x}, q^b)}\eta^{AB} = 0. \tag{3.2b}$$

$\partial \phi_0^B(\vec{x}, q)/\partial q^b$ satisfies

$$[-\vec{\nabla}^2 \delta_D{}^B + \frac{\partial^2 V}{\partial \phi_0^D(\vec{x}, q)\partial \phi_0^A(\vec{x}, q)}\eta^{AB}]\partial_b \phi_0^D(\vec{x}, q) = 0; \tag{3.2c}$$

$\phi_U{}^D(\vec{x}, q)$'s are non-zero-mode solutions corresponding to (3.2b), and are normalized to

$$\int d\vec{x} \phi_V{}^B(\vec{x}, q)\eta_{BD}\phi_W{}^D(\vec{x}, q) = \eta_{VW}, \quad \int d\vec{x} \partial_b \phi_0^B(\vec{x}, q)\eta_{BD}\phi_W{}^D(\vec{x}, q) = 0. \tag{3.2d}$$

(Hereafter we write a function of q^b's, $f(q^b)$, as f(q) for simplicity.)

The expression (3.2a) with the properties (3.2d) is completely analogous with (2.1). With the aim of making clear the analogy, we write (3.2a) and (3.2d) as

$$\phi^{Bx}(x^0) = \phi_0^{Bx}(q) + N_U{}^{Bx}(q) \cdot q^U, \tag{3.3a}$$

$$N_V{}^{Bx}(q)\eta_{Bx,Dy}N_U{}^{Dy}(q) = \eta_{VW}, \qquad B_b{}^{Bx}(q)\eta_{Bx,Dy}N_U{}^{Dx}(q) = 0, \tag{3.3b}$$

where $\eta_{Bx,Dy} = \eta_{BD}\delta(\vec{x} - \vec{y})$; $B_b^{Bx}(q) = \partial_b \phi_0^{Bx}(q)$. The following subsections is devoted to investigate the role of this analogy in constructing the field theory of extended object.

3-2. Fundamental relations and the metric Various relations given in the previous section remain to hold if the index A(representing the vector property in $\mathbf{R}_p$) is changed to Ax; from (2.3a) and (2.3b) one obtains

$$\partial_a B_b{}^{Ax}(q) = \Gamma^d{}_{ab} B_d{}^{Ax} + H_{Wab}\eta^{WU} N_U{}^{Ax}, \tag{3.4a}$$

$$\partial_a N_V{}^{Ax}(q) = -B_b{}^{Ax} H_{Va}{}^b - T_{VW,a}\eta^{WU} N_U{}^{Ax}. \tag{3.4b}$$

As to $\tilde{B}^{Ax}_\beta \equiv \partial\phi^{Ax}/\partial q^\beta$, we have

$$\tilde{B}^{Ax}_b(q) = B_b{}^{Ax}(q) + \partial_b N_V{}^{Ax}\cdot q^V, \qquad \tilde{B}^{Ax}_V(q) = N_V{}^{Ax}(q); \tag{3.5a}$$

and the small interval

$$ds^2 = d\phi^{Bx}\eta_{Bx,Dy}d\phi^{Dy} = \tilde{G}_{\alpha\beta}(q,q^V)dq^\alpha dq^\beta \qquad \text{with} \quad \tilde{G}_{\alpha\beta} = \tilde{B}^A_\alpha \eta_{AB}\tilde{B}^B_\beta. \tag{3.5b}$$

3-3. Canonical quantization and Hamiltonian form From Lagrangian (3.1), one obtains the momentum operator p_{Ax}, conjugate to ϕ^{Ax}, defined by

$$p_{Ax} \equiv \frac{\partial L}{\partial\partial\phi^{Ax}/\partial x^0} = \frac{\partial\phi^{Dy}}{\partial x^0}\eta_{Dy,Ax} \equiv \dot{\phi}^{Dy}\eta_{Dy,Ax}. \tag{3.6a}$$

Utilizing

$$\dot{\phi}^{Bx} = (\tilde{B}^{Bx}_\beta\cdot\dot{q}^\beta + \dot{q}^\beta\cdot\tilde{B}^{Bx}_\beta)/2 \equiv < \tilde{B}^{Bx}_\beta, \dot{q}^\beta >, \tag{3.6b}$$

and following the quantization procedure described in 2-2, one obtains

$$p_\beta \equiv \frac{\partial L}{\partial\dot{q}^\beta} = < \tilde{G}_{\beta\alpha}, \dot{q}^\alpha >, \tag{3.6c}$$

$$p_{Ax} = < \eta_{Ax,By}\tilde{B}^{By}_\beta, \dot{q}^\beta > = < \eta_{Ax,By}\tilde{B}^{By}_\beta\tilde{G}^{\beta\delta}, p_\delta > . \tag{3.6d}$$

From the commutation relations (2.8b), we obtain the equal-time ones as

$$[\phi^{Bx},\phi^{Dy}] = 0, \quad [\phi^{Bx}, p_{Dy}] = i\hbar\delta^{Bx}{}_{Dy}, \quad [p_{Bx}, p_{Dy}] = 0. \tag{3.6e}$$

Hamiltonian $H[\phi,p]$ is defined by

$$H[\phi,p] \equiv \frac{1}{2}\{p_{Ax},\dot{\phi}^{Ax}\} - \int d\vec{x}L, \tag{3.7a}$$

which is expressed as

$$H[\phi,p] = \frac{1}{2}p_{Ax}\eta^{Ax,By}p_{By} + \frac{1}{2}(\vec{\nabla}\phi)^{Ax}\eta_{Ax,By}(\vec{\nabla}\phi)^{By} + \int V(\phi)d\vec{x}. \tag{3.7b}$$

This is expressed in terms of (q^α, p_β) variables as

$$H[\phi,p] = \tilde{K}(2.9b) + \int V(\phi)d\vec{x}, \tag{3.7c}$$

$$\tilde{K}(2.9b) = \frac{1}{2}\Pi_a \lambda^{ab} \Pi_b + 1/2 p_V \eta^{VW} p_W - \frac{1}{2}\tilde{Y}(q, q^U). \tag{3.7d}$$

Here, the last term $-\frac{1}{2}\tilde{Y}$ is given by (2.11a) and can be rewritten as

$$-\frac{1}{2}\tilde{Y} = \frac{\hbar^2}{8}[\tilde{R} + \tilde{\Gamma}^{\delta}_{\alpha\gamma}\tilde{G}^{\alpha\beta}\tilde{\Gamma}^{\gamma}_{\beta\delta}]; \tag{3.7e}$$

in the present case, $\tilde{R} = 0$. We will examine the correspondence of the above expressions to those derived by Gervais and others [7] in the next subsection.

3-4. Momentum operators From (3.6c) and (3.6d), one obtains

$$p_\beta = < \tilde{B}^{Ax}_{\beta}, p_{Ax} >, \quad i.e. \quad p_U = < N_U{}^{Ax}, p_{Ax} >, \quad p_b = < \tilde{B}^{Bx}_{b}, p_{Bx} > . \tag{3.8}$$

We define

$$\Pi_{Bx} \equiv \eta_{Bx,Dy} N_V{}^{Dy} \eta^{VW} p_W. \tag{3.9a}$$

By employing the second relation of (3.8), one obtains

$$\Pi_{Bx} = < \eta_{Bx,Dy} N_V{}^{Dy} \eta^{VW} N_W{}^{Ez}, p_{Ez} > . \tag{3.9b}$$

When we define $p_{0,Bx} \equiv p_{Bx} - \Pi_{Bx}$, we obtain

$$p_{0,Bx} = < \delta^{Ez}_{Bx} - \eta_{Bx,Dy} N_V{}^{Dy} \eta^{VW} N_W{}^{Ez}, p_{Ez} > = < \eta_{Bx,Dy} B_b{}^{Dy} g^{bd} B_d{}^{Ez}, p_{Ez} >; \tag{3.9c}$$

$$N_V{}^{Bx} \cdot \Pi_{Bx} = p_V, \qquad B_b{}^{Bx} \cdot \Pi_{Bx} = 0 \tag{3.9d}$$

$$< N_V{}^{Bx}, p_{0,Bx} > = 0, \quad < B_b{}^{Bx}, p_{0,Bx} > = < B_b{}^{Bx}, p_{Bx} > . \tag{3.9e}$$

Therefore, we write ϕ^{Bx} and p_{Bx} as

$$\phi^{Bx} = \phi_0^{Bx} + \chi^{Bx}, \quad p_{Bx} = p_{0,Bx} + \Pi_{Bx}, \tag{3.10a}$$

with $\chi^{Bx} = N_V{}^{Bx} q^V$, $\Pi_{Bx} = (3.9a)$, from which we have

$$< \Pi_{Bx}, \partial_b \phi^{Bx} > = -\frac{1}{2} T_{WV,b} L^{WV}. \tag{3.10b}$$

Thus we see Π_a in (3.7d) is expressed as

$$\Pi_a = p_a - < \Pi_{Bx}, \partial_a \phi^{Bx} >, \tag{3.10c}$$

which is the same as given by Gervais and others [7].

Next we examine the form of

$$< p_{Bx}, \dot{\phi}^{Bx} >= \int d\vec{x}L - H[\phi, p](3.7c) \tag{3.11a}$$

in the c-number theory. We obtain

$$p_{Bx}\dot{\phi}^{Bx} = p_\beta \dot{q}^\beta = p_{0,Bx}\dot{\phi}^{Bx} + \Pi_{Bx}\dot{\chi}^{Bx}, \tag{3.11b}$$

$$p_{0,Bx}\dot{\phi}^{Bx} = p_d\dot{q}^d - p_V T_{XW,d} q^W \eta^{XV} \dot{q}^d = \Pi_d \dot{q}^d. \tag{3.11c}$$

4. Discussions and conclusion In sec.3, we examine the scalar field theory which allows a soliton solution with the aim of establishing the correspondence between such a field theory and the formalism of one-particle motion on a curved manifold $\mathbf{M}_n$ embedded in $\mathbf{R}_p$. Our formulation seems helpful to see general mathematical structure of the field theory with soliton and fluctuation around it. A noteworthy point in the present formalism is that there appears a geometry-induced gauge structure which corresponds to that giving rise to the geometry-induced Aharonov-Bohm effect for a particle moving in a thin-tube in $\mathbf{R}_3$.

One of the future tasks is to investigate about what physical effects such a geometry-induced structure brings. It may be interesting to examine the possiblity of inducing the gauge field as an independent dynamical degree of freedom after compactifying q^U-space. From such a point of view, the recent works presented by Kikkawa and others [8] seems interesting, on which we shall discuss elsewhere.

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