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Generalization of geometry-induced effect
noted by Takagi and Tanzawa

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The formulation on a particle motion in n-dimensional curved manifold \( M_n \) embedded in p-dimensional Euclidean space \( \mathbb{R}^p \) is summarized, and the geometry-induced gauge structure is explained. Next we examine the scalar field theory with a soliton solution, and point out that in spite of the infinite degrees of freedom such a field theory has the same mathematical structure as a particle motion in \( M_n \subset \mathbb{R}^p \), and our formalism affords a clearer view of understanding physical contents of such a field theory.

1. Introduction

Quantum theory on a curved manifold has been investigated from various points of view [1,2,3]. Quantum treatment of soliton such as Skyrmion provides a typical example of quantum theory on a curved manifold [4]. We consider the motion of a particle on an n-dimensional curved manifold \( M_n \) embedded in a p-dimensional Euclidean space \( \mathbb{R}^p \), and the particle motion is thought to be confined by some confining potential. Then a correction term with the order \( \hbar^2 \) in the effective Hamiltonian on \( M_n \) appears as a quantum effect due to a particle motion in the directions perpendicular to \( M_n \) [2]. Such an effect is dropped from the beginning when we apply simply Dirac method for constrained dynamical systems.

Two years ago, Takagi and Tanzawa [5] have pointed out that, for a particle motion
in a thin tube (in $\mathbb{R}^3$) forming a closed loop, in the effective Hamiltonian on $M_1$ there appears an effective vector potential, which depends on the geometry of $M_1$, and that there exists a complete analogy with Aharonov-Bohm effect, called a geometry-induced AB effect. One of the present authors (K.F.) and N.Ogawa [6] genaralized this result to the case of a particle motion in a thin neighborhood of $M_n$ embedded in $\mathbb{R}_p$. The aim of the present report is to apply this formalism to a scalar field theory which allows a classical solution, and to examine the correspondence to the field theory of the extended object given by Sakita and others [7].

In the following, we first summarize the formalism in case of a particle motion on $M_n \subset \mathbb{R}_p$ (in Sec.2), and extend it to the case of a scalar field theory with a classical solution(in Sec.3). The last section is devoted to discussions and summary of remaining tasks.

2. Particle motion in a thin layer along $M_n$ embedded in $\mathbb{R}_n$

2-1. Basic relations

As in [6], a set of coordinates $\{X^A; A = 1, \ldots, p\}$ of a point in a thin-layer neighbourhood of $M_n$ is expressed as

$$X^A(q^\beta) = x^A(q^b) + \sum_{U=n+1}^p q^U N_U^A(q^b), \quad s = 1, \ldots, p; \quad (2.1)$$

$\{q^\beta, \beta = 1, \ldots, n, n+1, \ldots, p\}$ consists of two parts; the first part $\{q^b, b = 1, \ldots, n\}$ is a set of curvilinear coordinates on $M_n$ and the remaining part is $\{q^U, U = n+1, \ldots, p\}$.

$N_U^A(q^b)$ is a unit normal vector to $M_n$ at a point $x^A(q^b)$. For simplicity we omit $\sum_{U=n+1}^p$ for a dummy index $U$ and write e.g. $x^A(q^b)$ as $x^A(q)$. The metric tensor in $\mathbb{R}_n$, written as $\bar{G}_{\alpha\beta}(q, q^U)$ is given by

$$\bar{G}_{\alpha\beta} = \bar{B}_\alpha^A \eta_{AD} \bar{B}_\beta^A \quad \text{with} \quad \bar{B}_\beta^A(q, q^U) = \frac{\partial x^A(q, q^U)}{\partial q^\beta}, \quad (2.2a)$$

and the metric tensor on $M_n$ as

$$g_{ab}(q) = B_a^A \eta_{AD} B_b^D \quad \text{with} \quad B_b^A(q) = \frac{\partial x^A(q)}{\partial q^b}. \quad (2.2b)$$

Since $B_b^A$ is tangent to $M_n$, we have $B_b^A \eta_{AD} N_V^D = 0$. Note that we have $\bar{B}_\beta^A = (\bar{B}_\beta^A, \bar{B}_V^A) = (B_b^A + \partial_b N_V^A \cdot q^W, N_V^A)$. 

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The fundamental equations for $B_b^A$ and $N_v^A$ are

\begin{align}
\partial_a B_b^A &= \Gamma^d_{ab} B_d^A + H_{Wab}\eta^{WU} N_u^A, \\
\partial_a N_v^A &= -H_{va}^d B_d^A - T_{VW,a}\eta^{WU} N_u^A,
\end{align}

(2.3a) (2.3b)

where $\Gamma^d_{ab}$ is Christoffel symbol constructed in terms of $g_{bd}$ and $H_{Wab} = H_{Wba} = H_{WB}^e g_{ea}$, $T_{VW,a} = -T_{WV,a}$.

Concrete forms of $\tilde{G}_{\alpha\beta}$ and its inverse $\tilde{G}^{\alpha\beta}$ are given as follows:

\[
[\tilde{G}_{\alpha\beta}] = \begin{bmatrix}
\tilde{G}_{ab} & \tilde{G}_{aU} \\
\tilde{G}_{Vb} & \tilde{G}_{VV}
\end{bmatrix}
\]

(2.4a)

with

\[
\tilde{G}_{ab} = \lambda_{ab} + \tilde{G}_{aX}\eta^{XY} \tilde{G}_{bY}, \quad \lambda_{ab} = g_{ab} - 2H_{Wab}q^W + q^X q^Y H_{xe}^a H_{ye}^b,
\]

\[
\tilde{G}_{aU} = \tilde{G}_{ua} = T_{UX,a} q^X, \quad \tilde{G}_{VV} = \eta_{UU};
\]

\[
[\tilde{G}^{\alpha\beta}] = \begin{bmatrix}
\tilde{G}^{ab} & \tilde{G}^{aU} \\
\tilde{G}^{Vb} & \tilde{G}^{VV}
\end{bmatrix}
\]

(2.4b)

with

\[
\tilde{G}^{ab} = \lambda^{ab}, \quad \lambda^{ab} = \delta^{ad}, \quad \tilde{G}^{aU} = \tilde{G}^{ua} = -\lambda^{ad} T_{XW,d} q^W \eta^{XU},
\]

\[
\tilde{G}^{UU} = \eta^{UU} + \eta^{VX} T_{XW,b} q^W \lambda^{bd} T_{YZ,d} q^Z \eta^{YU}.
\]

From the condition $\partial_a B_b^A = \partial_b B_a^A$, one obtains the curvature tensor on $M_n$ given as

\[
R_{a,b,c,d} = \eta_{AB}(H^{A}_{ac} H^{B}_{bd} - H^{A}_{ad} H^{B}_{bc})
\]

(2.5a)

with $H^{A}_{ab} = H_{Vab}\eta^{VW} N_w^A$ (Euler-Schouten tensor);

hence, the tensor $R_{bc}$ = $g^{ad} R_{ab,cd} = \eta_{AB}(H^A_{b} H^{B}_{dc} - H^A_{a} H^{B}_{bc})$, and scalar curvature

\[
R = g^{bc} R_{bc} = \eta_{AB}(H^A_{a} H^{B}_{d} - H^A_{a} H^{B}_{b}).
\]

(2.5b)

From $\partial_a \partial_b N_v^A - \partial_b \partial_a N_v^A = 0$, one obtains

\[
R_{da,VW}\eta^{VW} = \eta^{VW}(-H_{Va}^d H_{Wba} + H_{Va}^b H_{Wbd}),
\]

(2.5c)

where

\[
R_{da,VW} \equiv -\partial_d T_{VW,a} + \partial_a T_{VW,d} + T_{XV,d}\eta^{XY} T_{YW,a} - T_{XV,a}\eta^{XY} T_{YW,d}.
\]

(2.5d)

Using the extrinsic mean curvature $H$ defined by $H \equiv [\eta_{ab} H_{A}^b H_{B}^d]^1/2 / n$, one obtains

\[
R = H_{A}^d \eta_{AB} H_{B}^b - n^2 H^2.
\]

(2.6)

2-2. Canonical quantization and form of kinetic energy

We examine the form of
kinetic energy

$$\tilde{K} = \frac{1}{2} \dot{X}^A \eta_{AB} X^B, \quad \dot{X}^A = \frac{dX^A}{dr},$$  \hspace{1cm} (2.7)

expressed in terms of \(q^\beta\)-variables. In order to perform the quantum-mechanical calculations from the outset, we adopt the procedure which is consistent for the transformation from Euclidean coordinates to curvilinear ones as in the present case. We assume

$$[q^\beta, \dot{q}^\delta] = i\hbar f^{\beta\delta}(q^\gamma), \quad [q^\beta, q^\delta] = 0,$$  \hspace{1cm} (2.8a)

where \(f^{\beta\delta}\) is a function of only \((q^\gamma)\). When we require the cannonical commutation relations

$$[q^\alpha, p_\beta] = i\hbar \delta^\alpha_\beta, \quad [p_\alpha, p_\beta] = 0$$  \hspace{1cm} (2.8b)

for \(p_\beta \equiv \frac{\partial \tilde{K}}{\partial q^\beta} = \frac{1}{2} \{\tilde{G}_{\beta\delta}, q^\delta\} \equiv <\tilde{G}_{\beta\delta}, q^\delta>,$$

we obtain \(f^{\alpha\beta} \tilde{G}_{\beta\delta} = \delta^\alpha_\delta\), i.e. \(f^{\alpha\beta}\) is the inverse of \(\tilde{G}_{\beta\delta}\).

Now we rewrite \(\tilde{K}\) (2.7) in the covariant form

$$\tilde{K} = \frac{1}{2} \tilde{G}^{-1/4} p_\alpha \tilde{G}^{-1/2} \tilde{G}^{\alpha\beta} p_\beta \tilde{G}^{-1/4}, \quad \tilde{G} = |\det \tilde{G}_{a\beta}|.$$

We obtain by noting \(\tilde{G} = |\det \lambda_{a\beta} \cdot \det \eta_{VW}| = |\det \lambda_{a\beta}| \equiv \lambda,$$

$$\tilde{K} = \frac{1}{2} \lambda^{-1/4} \Pi_a \lambda^{1/2} \lambda^{ab} \Pi_b \lambda^{-1/4} + \frac{1}{2} \lambda^{-1/4} p_V \lambda^{1/2} \eta^{VW} p_W \lambda^{-1/4}.$$  \hspace{1cm} (2.9b)

Here, \(\Pi_a\) is defined by

$$\Pi_a \equiv p_a + \frac{1}{2} T_{VW,a} L^{VW},$$  \hspace{1cm} (2.9c)

$$L^{WX} \equiv q^W \eta^{XV} p_V - q^X \eta^{WV} p_V = \eta^{XV} p_V q^W - \eta^{WV} p_V q^X.$$  \hspace{1cm} (2.9d)

\(L^{WX}\) satisfies the commutation relation

$$[L^{VX}, L^{WY}] = i\hbar (\eta^{VW} L^{XY} + \eta^{XY} L^{VW} - \eta^{YY} L^{XW} - \eta^{XW} L^{YY}).$$  \hspace{1cm} (2.9e)

In the thin layer approximation \(|H_{Ua} q^U| \ll 1\) and \(|T_{UV,a} q^U| \ll 1\) [5,6], one obtains

$$\tilde{K} \xrightarrow{\text{thin layer}} K^* = K + \frac{1}{2} p_V \eta^{VW} p_W + \Delta V^*.$$  \hspace{1cm} (2.10a)
where \( K = \frac{1}{2} g^{-1/4} \Pi_a g^{1/2} g^{ab} \Pi_b g^{-1/4} \), \( \Delta V^* = \frac{\hbar^2}{2} [ -\frac{R}{2} - \frac{1}{4} n^2 H^2 ] \). (2.10b)

\( \Delta V^* \) comes from the last term in (2.9b). It may be worthy of noting that we have

\[
\Delta V^* = -\frac{1}{2} [ \tilde{Y}(q, q^* V) - Y(q) ] \text{thin layer};
\]

where \(-\frac{1}{2} \tilde{Y}(q, q^* V) = \hbar^2 [\frac{1}{4} \partial_b (\mathcal{G}^{\delta \beta} \bar{r}_b + \frac{1}{8} \mathcal{G}^{\delta \beta} \bar{r}_a \bar{r}_b \bar{r}_c \bar{r}_d)] \], \( \bar{r}_a \equiv \bar{r}_{a \beta} = \bar{G}_{\gamma \beta} \partial_{a \gamma} \mathcal{G}_{\gamma \beta} / 2 \).

\( Y(q) \) is the quantity constructed in terms of \( g_{ab} \) corresponding to \( \tilde{V} \);

\[
-\frac{1}{2} Y/\hbar^2 = \frac{1}{4} \partial_b (g^{bd} \Gamma_d) + \frac{1}{8} g^{bd} \Gamma_b \Gamma_d = \frac{1}{8} [R + g^{ab} \Gamma_a e_d \Gamma_{bd}].
\]

Using (2.11b), we can rewrite (2.10b) as

\[
K(2.10b) = \frac{1}{2} \Pi_a g^{ab} \Pi_b - \frac{1}{2} Y(q).
\]

2-3. Commutator \([\Pi_a, \Pi_b] \)

Utilizing some relations given in 2-1, we obtain

\[
[\Pi_b, \Pi_d] = \frac{i \hbar}{2} R_{bd,v} L_{v}.
\]

This is analogous to a charged particle moving in magnetic field \( \vec{H} \), in which we have

\[
[\Pi_j, \Pi_k] = i \hbar \frac{e}{c} F_{jk}; \quad \Pi_j = p_j - \frac{e}{c} \vec{A}_j; \quad j, k = 1, 2, 3; \quad F_{jk} = \frac{1}{2} \epsilon_{jkl} \vec{H}_l.
\]

We see that the field \( T_{Vv,a} = N V^A \partial_a N W^B \eta_{AB} \) plays a role of gauge potential. The gauge property including the non-Abelian one is seen as follows: When the total Hamiltonian \( \tilde{H} = \tilde{K}(2.9b) + V \) has the part of potential which confines the particle motion to \( M_n \), and is invariant under rotation of the set of \( \{ N_U^A, U = n + 1, \cdots, p \} \) such as

\[
N' V^A(q) = N W^A(q) \Lambda^W V(q), \quad \eta_{XY} \Lambda^X \Lambda^Y V = \eta_{WV},
\]

we obtain

\[
T_{Vv,b} \rightarrow T'_{Vv,b} = (\Lambda^{-1})_W^X T_{XY,b} \Lambda^Y V + (\Lambda^{-1})_W X \partial_b \Lambda^X V.
\]

\( T_{Vv,b} \) cannot be eliminated globally. In case of a tube embedded in \( \mathbb{R}_3[5] \), \( \Pi_b \) reduces to

\[
\Pi_1 = p_1 + T_{23} L_{23} \equiv p - \omega L.
\]

It is pointed out in Ref.[5] due to multivaluedness of triangular function

\[
\int_0^l \omega(q) dq = \int_0^l \tau(q) dq \quad (\text{mod } 2\pi)
\]

is obtained, where \( l \) is the length of center line of the tube; \( \tau \) is the torsion appearing in Frenet-Seret equation in \( \mathbb{R}_3 \).
3. Application to field theory of extended object

3-1. Purpose

In this section we extend the formalism given in Section 2 to field theory. For simplicity, we examine the scalar field theory which allows a soliton solution. We consider the Lagrangian expressed as

\begin{equation}
L = -\frac{1}{2} \partial_\mu \phi^A(x) \eta_{AB} \partial^\mu \phi^B(x) - V(\phi(x)),
\end{equation}

where \((x^\mu)\) is a space-time coordinate; its metric is \(\eta_{\mu\nu}\) with \(\text{diag}(\eta_{\mu\nu}) = (- + + \cdots)\); the upper index \(A\) of \(\phi^A\) denotes the internal degrees of freedom.

The field operator \(\phi^A(\vec{x}, x^0)\) is assumed to be expanded as [7]

\begin{equation}
\phi^B(\vec{x}, x^0) = \phi^B_0(\vec{x}, q^b) + \sum_U \phi^B_U(\vec{x}, q^b)q^U,
\end{equation}

where \(\{q^b, b = 1, \ldots, n\}\) denotes a set of collective coordinates representing the center of mass coordinates of the classical soliton, the orientation on the internal space and so on; \(\phi^B_0\) is the soliton solution satisfying

\begin{equation}
-\frac{\partial}{\partial \vec{x}} \frac{\partial}{\partial \vec{x}} \phi^B_0(\vec{x}, q^b) + \frac{\partial V}{\partial \phi^A_0(\vec{x}, q^b)} \eta^{AB} = 0.
\end{equation}

\(\partial \phi^B_0(\vec{x}, q)/\partial q^b\) satisfies

\begin{equation}
[-\nabla^2 - \partial^2 \delta^B_D + \nabla^2 \frac{\partial^2 V}{\partial \phi^B_0(\vec{x}, q) \partial \phi^A_0(\vec{x}, q)} \eta^{AB}] \partial_b \phi^D_0(\vec{x}, q) = 0;
\end{equation}

\(\phi^D_U(\vec{x}, q)\)'s are non-zero-mode solutions corresponding to (3.2b), and are normalized to

\begin{equation}
\int d\vec{x} \phi^B_0(\vec{x}, q) \eta_{BD} \phi^D(\vec{x}, q) = \eta_{NV}, \quad \int d\vec{x} \partial_b \phi^B_0(\vec{x}, q) \eta_{BD} \phi^D(\vec{x}, q) = 0.
\end{equation}

(Hereafter we write a function of \(q^b\)'s, \(f(q^b)\), as \(f(q)\) for simplicity.)

The expression (3.2a) with the properties (3.2d) is completely analogous with (2.1). With the aim of making clear the analogy, we write (3.2a) and (3.2d) as

\begin{equation}
\phi^B_0(\vec{x}) = \phi^B_0(\vec{x}) + N_U B^D(x) q^U,
\end{equation}

\(N_V B^D q^B_0(\vec{x}) \eta_{BD} D^V(\eta_{NV}, \quad B^D B^D(\eta_{BD} \delta(\vec{x} - \vec{y}); B^D_B(\eta_0 B^D(q)) = 0,
\end{equation}

where \(\eta_{BD} = \eta_{BD} \delta(\vec{x} - \vec{y}); B^D_B(\eta_0 B^D(q)\). The following subsections is devoted to investigate the role of this analogy in constructing the field theory of extended object.
3-2. Fundamental relations and the metric

Various relations given in the previous section remain to hold if the index $A$ (representing the vector property in $\mathbb{R}^p$) is changed to $Ax$; from (2.3a) and (2.3b) one obtains

$$
\partial_a B^A_{\beta x}(q) = \Gamma^d_{ab} B^A_d(q) + H_{Wa} \eta^{WU} N_U^{Ax},
$$

$$
\partial_a N^{Ax}_{\beta x}(q) = -B^A_{\beta x} H_{Va} b - T_{VW,a} \eta^{WU} N_U^{Ax}.
$$

As to $\dot{\phi}^Ax_i = \delta^A_{\alpha} x^i$, we have

$$
\dot{B}^A_{\beta x}(q) = B^A_{\beta x}(q) + \partial_b N^{Ax}_{\beta x} \cdot q^V,
$$

$$
\ddot{B}^A_{\beta x}(q) = N^{Ax}_{\beta x}(q);
$$

and the small interval

$$
ds^2 = d\phi^{Bz} \eta_{Bz, Dy} d\phi^{Dy} = \tilde{C}_{\alpha \beta}(q, q^V) dq^\alpha dq^\beta
$$

with $\tilde{C}_{\alpha \beta} = B^A_{\alpha A} \eta_{AB} B^B_{\beta}$. (3.5b)

3-3. Canonical quantization and Hamiltonian form

From Lagrangian (3.1), one obtains the momentum operator $p_{Ax}$, conjugate to $\phi^{Ax}$, defined by

$$
p_{Ax} = \frac{\partial L}{\partial \dot{\phi}^{Ax} / \partial x^0} = \frac{\partial \phi^{By}}{\partial x^0} \eta_{Dy, Ax} = \phi^{Dy} \eta_{Dy, Ax}.
$$

Utilizing

$$
\dot{\phi}^{Bz} = (\dot{B}_{Bz}^{Bz} \cdot q^\beta + \dot{q}^\beta \cdot \dot{B}_{Bz}^{Bz}) / 2 \equiv < \dot{B}_{Bz}^{Bz}, q^\beta >,
$$

and following the quantization procedure described in 2-2, one obtains

$$
p_{Bz} = \frac{\partial L}{\partial q^\beta} = < \tilde{C}_{\beta \alpha}, \dot{q}^\alpha >,
$$

$$
p_{Ax} = \eta_{Ax, Bz} \dot{B}_{Bz}^{By}, q^\beta >= < \eta_{Ax, Bz} \dot{B}_{Bz}^{By} \tilde{C}^{\beta \delta}, p_{\delta} >.
$$

From the commutation relations (2.8b), we obtain the equal-time ones as

$$
[\phi^{Bz}, \phi^{By}] = 0, \quad [\phi^{Bz}, p_{Dy}] = i \hbar \delta^{Bz, Dy}, \quad [p_{Bz}, p_{Dy}] = 0.
$$

Hamiltonian $H[\phi, p]$ is defined by

$$
H[\phi, p] \equiv \frac{1}{2} \{ p_{Ax}, \phi^{Ax} \} - \int d\bar{z} L,
$$

which is expressed as

$$
H[\phi, p] = \frac{1}{2} p_{Ax} \eta^{Ax, By} p_{By} + \frac{1}{2} (\nabla \phi)^{Ax} \eta_{Ax, By} (\nabla \phi)^{By} + \int V(\phi) d\bar{z}.
$$

This is expressed in terms of $(q^\alpha, p_{\beta})$ variables as

$$
H[\phi, p] = \tilde{K}(2.9b) + \int V(\phi) d\bar{z},
$$
Here, the last term $-\frac{1}{2}\tilde{Y}$ is given by (2.11a) and can be rewritten as
\[ -\frac{1}{2}\tilde{Y} = \frac{\hbar^2}{8}[\tilde{R} + \Gamma^\alpha_{\alpha\gamma}G^\alpha_{\beta\gamma}] ; \] (3.7e)
in the present case, $\tilde{R} = 0$. We will examine the correspondence of the above expressions to those derived by Gervais and others [7] in the next subsection.

3.4. Momentum operators

From (3.6c) and (3.6d), one obtains
\[ p_\beta = \langle \hat{B}_\beta^A, p_{Ax} \rangle, \quad \text{i.e.} \quad p_U = \langle N_U^A, p_{Ax} \rangle, \quad p_b = \langle \hat{B}_b^B, p_{Bx} \rangle . \] (3.8)

We define
\[ \Pi_{Bx} \equiv \eta_{Bx,Dy}N_V^Dy_{W}^V p_{W} . \] (3.9a)

By employing the second relation of (3.8), one obtains
\[ \Pi_{Bx} = \langle \eta_{Bx,Dy}N_V^Dy_{W}^V N_W^Ez, p_{Ez} \rangle . \] (3.9b)

When we define $p_{0,Bx} \equiv p_{Bx} - \Pi_{Bx}$, we obtain
\[ p_{0,Bx} = \langle \delta_{Bz}^Ez - \eta_{Bx,Dy}N_V^Dy_{W}^V N_W^Ez, p_{Ez} \rangle = \langle \eta_{Bx,Dy}B_b^B, p_{Bx} \rangle . \] (3.9c)

\[ N_V^Bz \cdot \Pi_{Bx} = p_V, \quad B_b^B \cdot \Pi_{Bx} = 0 \] (3.9d)
\[ < N_V^Bz, p_{0,Bx} > = 0, \quad < B_b^B, p_{0,Bx} > = < B_b^B, p_{Bx} > . \] (3.9e)

Therefore, we write $\phi^{Bz}$ and $p_{Bx}$ as
\[ \phi^{Bz} = \phi_0^{Bz} + \chi^{Bz}, \quad p_{Bx} = p_{0,Bx} + \Pi_{Bx} , \] (3.10a)
with $\chi^{Bz} = N_V^{Bz}q^V$, $\Pi_{Bz} = (3.9a)$, from which we have
\[ < \Pi_{Bz}, \partial_b \phi^{Bz} > = -\frac{1}{2}T_{WV,b}L^{WV} . \] (3.10b)

Thus we see $\Pi_a$ in (3.7d) is expressed as
\[ \Pi_a = p_a - < \Pi_{Bz}, \partial_a \phi^{Bz} > , \] (3.10c)
which is the same as given by Gervais and others [7].
Next we examine the form of

\[
\langle p_{Bx}, \dot{Bx} \rangle = \int d\vec{x} L - H[\phi, p](3.7c)
\]

in the c-number theory. We obtain

\[
\begin{align*}
\dot{Bx} = \dot{p} = p_{Bx} Bx^p + \Pi_{Bx} Bz, \\
p_{Bx} = p_{Bx} = \partial\phi_{Bx} + \Pi_{Bx} Bz,
\end{align*}
\]

**4. Discussions and conclusion**

In sec.3, we examine the scalar field theory which allows a soliton solution with the aim of establishing the correspondence between such a field theory and the formalism of one-particle motion on a curved manifold \( M_n \) embedded in \( \mathbb{R}^p \). Our formulation seems helpful to see general mathematical structure of the field theory with soliton and fluctuation around it. A noteworthy point in the present formalism is that there appears a geometry-induced gauge structure which corresponds to that giving rise to the geometry-induced Aharonov-Bohm effect for a particle moving in a thin-tube in \( \mathbb{R}^3 \).

One of the future tasks is to investigate about what physical effects such a geometry-induced structure brings. It may be interesting to examine the possibility of inducing the gauge field as an independent dynamical degree of freedom after compactifying \( q^U \)-space. From such a point of view, the recent works presented by Kikkawa and others [8] seems interesting, on which we shall discuss elsewhere.


