Fully-developed turbulence: a Langevin description of the energy cascade

京都大学・理学部　　Philippe Marcq

高レイノルズ数の乱流ジェットの実験データを加法的連続的確率過程として解析する。通常の時間変数のかわりにスケール変数を用い、エネルギー散逸場の分布関数のスケール依存性が線形ランジュバン方程式によってよく記述されることを示す。この時、ノイズはマルコフ的で非ガウス的特徴を持つ。さらに、実験データとよく一致するエネルギー散逸の分散のスケール依存性が得られる。

The existence of fluctuations of the energy dissipation field is one well-accepted signature of the small-scale intermittency of fully-developed turbulent flows [(1)]. The purpose of this work is to characterize these fluctations in a quantitative manner, thanks to a detailed analysis of experimental data.

Our velocity signal was measured in a low temperature jet flow at high Reynolds number ($Re = 20000$, $R_\lambda = 341$) by hot-wire anemometry (see, e.g., [(2)] for the specifics of the experimental set-up). Taylor's frozen turbulence hypothesis is used to convert time series obtained at one point of the flow into the spatial dependence of the velocity field $v(x)$. The one-dimensional surrogate of the dissipation field $\epsilon_r(x)$ is conventionally defined as:

$$\epsilon_r(x) = \frac{15\nu}{r} \int_{x-r/2}^{x+r/2} \left(\frac{dv}{dx'}\right)^2 dx', \tag{1}$$

where x and r respectively denote physical space and scale, and ν is the kinematic viscosity.

Our main result is that the energy cascade is well-described by the following *linear* Langevin equation:

$$\frac{dY}{dl}(l) = \gamma(l)\, Y(l) + \sqrt{2D(l)}\, \xi(l), \tag{2}$$

where the stochastic variable $Y(l)$ is defined as $Y(l) = \ln \epsilon_r - \langle \ln \epsilon_r \rangle$ at scale $l = \ln(L/r)$ (L denotes the flow's integral scale). The drift and diffusion coefficients $\gamma(l)$ and $D(l)$ of the Ornstein-Uhlenbeck process (2) are given by:

$$\begin{aligned} \gamma(l) &= \gamma_0 - \gamma_1\, l, \\ D(l) &= D_0\, e^{2\delta l}, \end{aligned} \tag{3}$$

where γ_0, γ_1, D_0 and δ are (measurable) positive constants. The interested reader is refered to [(3)] for details of our measurement protocol.

We find that the random force $\xi(l)$ is approximately Markov — i.e. its autocorrelation scale $\tau(l)$ is always much smaller than $1/\gamma(l)$ — but also non-Gaussian — the skewness of

$P(\xi, l)$ is negative. This last point is consistent with the well-known existence of a weak deviation of the statistics of ϵ_r from a log-normal distribution.

The Langevin equation (2-3) is exactly solvable, and yields the following expression of the variance of $Y(l)$:

$$\langle Y(l)^2 \rangle = e^{2\gamma_0 l - 2\gamma_1 l^2} \left(\langle Y(0)^2 \rangle + 2D_0 \int_0^l e^{2(\delta-\gamma_0)l' + 2\gamma_1 l'^2} \mathrm{d}l' \right) \tag{4}$$

where we assumed that the random force $\xi(l)$ is δ-correlated ($\langle \xi(l)\xi(l') \rangle = \delta(l - l')$) and that the initial width is $\langle Y(0)^2 \rangle$ at large scale $l = 0$ (e.g. $r = L$). Fig. 1 shows that Eq. (4) is in excellent agreement with experimental data.

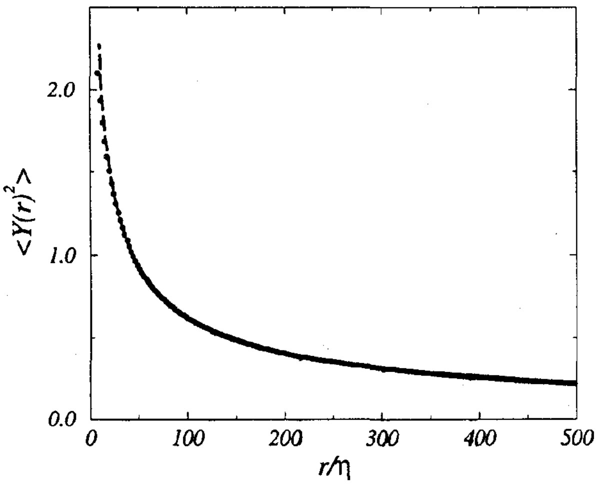

図 1: Variance of $Y(r)$. The prediction obtained from Eq. (4) (dashed line, for the numerical values $\gamma_0 = 0.32$, $\gamma_1 = 0.025$, $D_0 = 0.01$, $\delta = 0.40$, estimated independently) is indistinguishable from experimental data (dots) in the range of scales $10\eta \leq r \leq L = 500\eta$, where η is Kolmogorov's dissipation scale.

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References

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