量子情報から量子テレポーテーションへ

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要旨

シャノン以降，情報伝送の分野は飛躍的な進歩をし続けて，現在では，シャノンの情報通信理論は，数学，物理学，経済学，生命科学といった様々な分野に関わっている．量子力学を基礎にした情報理論，すなわち，量子情報理論も，近年の科学技術において不可欠であり，特に，光通信，量子コンピュータ，量子カオスの研究の基礎を与えている．

本論文では，量子情報理論の数理的基礎について論じ，それらを量子力学系のカオスと量子コンピュータの研究に適用する．

最初に，量子情報理論について振り返り，力学系のカオスを測る指標である「カオス尺度」を構成するために量子情報理論がどのように適用されるかを説明する．さらに，量子テレポーテーション過程を量子チャネルによって厳密に記述し，Boson Fock空間におけるコヒーレント状態を用いて，新たな量子テレポーテーション過程を記述し，その実現可能性を議論する．本研究は，以下の論文を主にベースとしている．

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FROM QUANTUM INFORMATION TO QUANTUM TELEPORTATION

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1. Introduction

After Shannon, great development has been made in communication of information. Shannon's information theory is now linked to several different fields such as mathematics, physics, economics, life science. In the same vein, the information theory on quantum mechanics, so-called quantum information theory, is an indiespensable foundation in recent science and technology, in particular, to study optical communication, quantum computer and quantum chaos[23, 27, 25].

In this report, we discuss the fundamentals of quantum information[1, 17, 19, 27] and apply them to study the chaos of quantum dynamical systems[25, 13] and quantum computer[31, 6, 26, 29].

We first review the fundamentals of quantum information, and we explain how to apply them to construct a "chaos degree" measuring the chaos of dynamics[21, 25]. Further we discuss the quantum teleportation process[2, 14, 9, 10] in a mathematical vein of quantum channel.

2. Fundamentals in Quantum Information

Fundamental mathematical concepts in quantum information theory are quantum entropy describing the amount of information and quantum channel describing the dynamics associated with information communication.

The concept of channel is not only useful in information theory but also a convenient mathematical tool to treat several physical dynamics in a unified way[17].

In classical systems, an input (or initial) system is described by the set of all random variables $\mathcal{A} = M(\Omega)$ and its state space $\mathfrak{S} = P(\Omega)$ and an output (or final) system is by $M(\overline{\Omega})$ and $P(\overline{\Omega})$.

A quantum input system is described by the set $\mathcal{A} = B(\mathcal{H})$ of all bounded linear operators on $\mathcal{H}$ and $\mathfrak{S}$ is the set $T(\mathcal{H})$ of all density operators on $\mathcal{H}$. An output system is $\overline{\mathcal{A}} = B(\overline{\mathcal{H}})$ and $\overline{\mathfrak{S}} = T(\overline{\mathcal{H}})$.

More general quantum system is described by a C*-algebra $\mathcal{A}$ and its state space $\mathfrak{S}$.

In any case, a channel is a mapping from $\mathfrak{S}$ to $\overline{\mathfrak{S}}$. Almost all physical transformations are described by this mapping.

DEFINITION 2.1. *:Let Λ^* be a channel from $\mathfrak{S}$ to $\overline{\mathfrak{S}}$.*

(1) Λ^* *is linear if* $\Lambda^*(\lambda\varphi+(1-\lambda)\psi)=\lambda\Lambda^*\varphi+(1-\lambda)\Lambda^*\psi$ *holds for all* $\varphi,\psi\in\mathfrak{S}$ and any $\lambda\in[0,1]$.

(2) Λ^* *is completely positive (C.P.) if* Λ^* *is linear and its dual* $\Lambda:\overline{\mathcal{A}}\rightarrow\mathcal{A}$ *satisfies*

$$\sum_{i,j=1}^{n} A_i^*\Lambda(\overline{A_i^*}\overline{A_j})A_j\geq 0$$

for any $n\in\mathbf{N}$ *and any* $\{\overline{A}_i\}\subset\overline{\mathcal{A}}$, $\{A_i\}\subset\mathcal{A}$.

Most of channels appeared in physical processes are the C.P. channels[19]. For instance, the open system dynamics can be written as follows: If a system Σ_1 interacts with an external system Σ_2 described by another Hilbert space $\mathcal{K}$ and the initial states of Σ_1 and Σ_2 are ρ and σ, respectively, then the combined state θ_t of Σ_1 and Σ_2 at time t after the interaction between two systems is given by $\theta_t\equiv U_t(\rho\otimes\sigma)U_t^*$,where $U_t=\exp(-itH)$ with the total Hamiltonian H of Σ_1 and Σ_2. A channel is obtained by taking the partial trace w.r.t. $\mathcal{K}$ such as $\rho\rightarrow\Lambda_t^*\rho\equiv tr_{\mathcal{K}}\theta_t$.

The quantum entropy was introduced and developed to study some physical problems such as irreversible behavior, symmetry breaking, and it is an expression of the amount of information carried by a state. Here we review three fundamental quantum entropies which are important to study the information transmission processes.

Consider an quantum system described by a density operator on a Hilbert space $\mathcal{H}$. The entropy of a state ρ was introduced by von Neumann [16, 27] as

$$S(\rho)=-tr\rho\log\rho.$$

The second entropy is the relative emtropy for two states $\rho,\sigma\in\mathfrak{S}(\mathcal{H})$, which is defined by

$$S(\rho,\sigma)=\begin{cases} tr\rho\,(\log\rho-\log\sigma) & (\rho\ll\sigma)\\ +\infty & (\text{otherwise})\end{cases},$$

where $\rho\ll\sigma$ means that $tr\sigma A=0\Rightarrow tr\rho A=0$ for any $A\geq 0$ [32].

Let $\Lambda^* : \mathfrak{S}(\mathcal{H}) \to \overline{\mathfrak{S}}(\overline{\mathcal{H}})$ be a channel and define the compound state by $\theta_E = \sum_k p_k E_k \otimes \Lambda^* E_k$ with a Schatten decomposition $\sum_k p_k E_k$ of ρ, which expresses the correlation between the initial state ρ and the final state $\Lambda^*\rho$ [17, 19]. Another useful entropy is the mutual entropy [17] for a state $\rho \in \mathfrak{S}(\mathcal{H})$ and a channel Λ^* , which is given by

$$\begin{aligned} I(\rho;\Lambda^*) &= \sup\{S(\theta_E, \rho\otimes\Lambda^*\rho); E = \{E_k\}\} \\ &= \sup\left\{\sum_k p_k S(\Lambda^* E_k, \Lambda^*\rho); E = \{E_k\}\right\}, \end{aligned}$$

where the supremum is taken over all Schatten decompositions. The mutual entropy expresses the amount of information trasmitted from the initial state ρ to the final state $\Lambda^*\rho$, so that it satisfies the fundamental inequality of Shannon type [30, 17]:

$$0 \le I(\rho;\Lambda^*) \le \min\{S(\rho), S(\Lambda^*\rho)\}.$$

In Shannon's communication theory in classical systems, ρ is a probability distribution $p = (p_k)$ and Λ^* is a transition probability $(t_{i,j})$, so that the Schatten decomposition of ρ is unique and the compound state of ρ and its output $\overline{\rho}$ $(\equiv \overline{p} = (\overline{p}_i))$ is the joint distribution $r = (r_{i,j})$ with $r_{i,j} \equiv t_{i,j}p_j$. Then the above entropies become the Shannon entropy and mutual entropy, respectively;

$$S(p) = -\sum_k p_k \log p_k,\ \ I(p;\Lambda^*) = \sum_{i,j} r_{i,j} \log \frac{r_{i,j}}{p_j \overline{p}_i}.$$

Such quantum entropies have been used to define the capacity of channel [24, 28]and to study quantum communication processes [19].

3. Chaos Degree

In the context of information dynamics, a chaos degree associated with a dynamics in classical systems was introduced in [22]. It has been applied to several dynamical maps such logistic map, Baker's transformation and Tinkerbel map with succesful explainations of their chaotic characters[23, 13]. This chaos degree has several merits compared with usual measures such as Lyapunov exponent.

Here we discuss the quantum version of the classical chaos degree, which is defined by quantum entropies in Section 2, and we call the quantum chaos degree the entropic quantum chaos degree. In order to contain both classical and quantum cases, we define the entropic chaos degree in C*-algebraic terninology. This setting will not be used in the sequel application, but for mathematical completeness we first discuss the C*-algebraic setting.

Let $(\mathcal{A}, \mathfrak{S})$ be an input C* system and $(\overline{\mathcal{A}}, \overline{\mathfrak{S}})$ be an output C* system; namely, $\mathcal{A}$ is a C* algebra with unit I and $\mathfrak{S}$ is the set of all states on $\mathcal{A}$. We assume $\overline{\mathcal{A}} = \mathcal{A}$ for simplicity. For a weak* compact convex subset $\mathcal{S}$ (called the reference space) of $\mathfrak{S}$, take a state φ from the set $\mathcal{S}$ and let

$$\varphi = \int_{\mathcal{S}} \omega d\mu_{\varphi}$$

be an extremal orthogonal decomposition of φ in $\mathcal{S}$, which describes the degree of mixture of φ in the reference space $\mathcal{S}$. The measure μ_{φ} is not uniquely determined unless $\mathcal{S}$ is the Schoque simplex, so that the set of all such measures is denoted by $M_{\varphi}(\mathcal{S})$. The entropic chaos degree with respect to $\varphi \in \mathcal{S}$ and a channel Λ^* is defined by

$$D^{\mathcal{S}}(\varphi; \Lambda^*) \equiv \inf \left\{ \int_{\mathcal{S}} S^{\mathcal{S}}(\Lambda^* \varphi)\, d\mu_{\varphi}; \mu_{\varphi} \in M_{\varphi}(\mathcal{S}) \right\} \quad (3.1)$$

where $S^{\mathcal{S}}(\Lambda^*\varphi)$ is the mixing entropy of a state φ in the reference space $\mathcal{S}$ [24]. When $\mathcal{S} = \mathfrak{S}$, $D^{\mathcal{S}}(\varphi; \Lambda^*)$ is simply written as$D(\varphi; \Lambda^*)$. This $D^{\mathcal{S}}(\varphi; \Lambda^*)$ contains the classical chaos degree and the quantum one.

The classical entropic chaos degree is the case that $\mathcal{A}$ is abelian and φ is the probability disribution of an orbit generated by a dynamics (channel) Λ^*; $\varphi = \sum_k p_k \delta_k$, where δ_k is the delta measure such as $\delta_k(j) \equiv \begin{cases} 1 & (k=j) \\ 0 & (k \neq j) \end{cases}$. Then the claasical entropic chaos degree is

$$D_c(\varphi; \Lambda^*) = \sum_k p_k S(\Lambda^* \delta_k).$$

We explain the entropic chaos degree of a quantum system described by a density operator. Let $\digamma^*$ be a channel sending a state to a state and $\rho^{(n_0)}$ be an intial state. After time n_0, the state is $\digamma^{*n}\rho^{(n_0)}$, whose Schatten decomposition is denoted by $\sum_k \lambda_k^{(n_0)} E_k^{(n_0)}$. Then we define a channel Λ_m^* on $\otimes_1^m \mathcal{H}$ by

$$\Lambda_m^* \sigma = \digamma^* \sigma \otimes \cdots \otimes \digamma^{*m} \sigma, \; \sigma \in \mathfrak{S}(\mathcal{H}),$$

from which the entropic chaos degree (3.1) for the channels $\digamma^*$ and Λ_m^* are written as

$$D_q\left(\rho^{(n_0)};\mathcal{F}^*\right) = \inf\left\{\sum_k \lambda_k^{(n_0)} S\left(\mathcal{F}^* E_k^{(n_0)}\right);\left\{E_k^{(n_0)}\right\}\right\},$$

$$D_q\left(\rho^{(n_0)};\Lambda_m^*\right) = \inf\left\{\frac{1}{m}\sum_k \lambda_k^{(n_0)} S\left(\Lambda_m^* E_k^{(n_0)}\right);\left\{E_k^{(n_0)}\right\}\right\},$$

where the infimum is taken over all Schatten decompositions of $\mathcal{F}^{*n_0}\rho$.

We can judge whether the dynamics $\mathcal{F}^*$ causes a chaos or not by the value of D as

$$\begin{aligned} D &> 0 \text{ and not constant} \Longleftrightarrow \text{chaotic},\\ D &= \text{constant} \Longleftrightarrow \text{quasi-chaotic},\\ D &= 0 \Longleftrightarrow \text{stable}. \end{aligned}$$

The classical version of this degree was applied to study the chaotic behaviors of several nonlinear dynamics [13, 20]. The quantum entropic chaos degree is applied to the analysis of quantum spin system[12] and quantum Baker's type transformation[15], and we could measure the chaos of these systems. The information theoretical meaning of this degree was explained in [22, 23].

4. Quantum Chaos Degree

In this section, we apply it to study the appearance of chaos in quantum spin systems[12]. The chaos degree defined in the previous section has following properties.

THEOREM 4.1. *For any $\rho=\sum_k \lambda_k E_k \in \mathfrak{S}(\mathcal{H})$, $\mathcal{F}^*:\mathfrak{S}(\mathcal{H})\to\mathfrak{S}(\mathcal{H})$, and $\Lambda_m^*:\mathfrak{S}(\otimes_1^m\mathcal{H})\to\mathfrak{S}(\otimes_1^m\mathcal{H})$, the following statements hold.*

(1) Let U_t be a unitary operator satisfying $U_t=\exp(itH)$ for any $t\in\mathbf{R}$.
If $\mathcal{F}^*\rho = AdU_t(\rho)\equiv U_t\rho U_t^*$, then $D_q(\rho;\mathcal{F}^*)=D_q(\rho;\Lambda_m^*)=0$.
(2) Let σ be a fixed state on $\mathcal{H}$. If $\mathcal{F}^*\rho=\sigma$, then $D_q(\rho;\mathcal{F}^*) = D_q(\rho;\Lambda_m^*)=S(\sigma)$.
(3) Let λ be a fixed positive real number. If $\mathcal{F}^*\rho=e^{-\lambda}\rho+\left(1-e^{-\lambda}\right)\sigma$, then $\lim_{\lambda\to\infty}D_q(\rho;\mathcal{F}^*)=\lim_{\lambda\to\infty}D_q(\rho;\Lambda_m^*)=S(\sigma)$.
(4) Let $\{P_n\}$ be the positive operated measure and $\mathcal{F}^*\rho=\sum_k P_k\rho P_k$. Then $D_q(\rho;\mathcal{F}^{*j}) = D_q(\rho;\Lambda_m^*)$ = constant for any $j,m\in\mathbf{N}$, that is, $\mathcal{F}^*$ is quasi-chaotic. If $[P_k,\rho]=0$, then $D_q(\rho;\mathcal{F}^{*j}) = D_q(\rho;\Lambda_m^*)=0$, that is, $\mathcal{F}^*$ is stable.

The proof of this theorem is given in [12]. We will apply the quantum entropic chaos degree to spin 1/2 system. See [12] again for the details.

Let $\vec{X} = (x_1, x_2, x_3)$ be a vector in R^3 satisfying $\left\|\vec{X}\right\| = \sqrt{\sum_{i=1}^3 x_i^2} \leq 1$ and I be the identity 2×2 matrix. Any state ρ in a spin 1/2 system is expressed as

$$\rho = \frac{1}{2}\left(I + \vec{\sigma} \cdot \vec{X}\right),$$

where $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the Pauli spin matrix vector;

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Let f be a non-linear map from $\mathbf{R}^3$ to $\mathbf{R}^3$ satisfying $\left\|f\left(\vec{X}\right)\right\| \leq 1$ for any $\vec{X} \in \mathbf{R}^3$ with $\left\|\vec{X}\right\| \leq 1$. A channel $\mathcal{F}^*$ is defined by

$$\mathcal{F}^*\rho = \frac{1}{2}\left(I + \vec{\sigma} \cdot f\left(\vec{X}\right)\right)$$

for any state ρ.

We now define Baker's type map and see whether this map produces the chaos. For any vector $\vec{X} = (x_1, x_2, x_3)$ on $\mathbf{R}^3$, we consider the following map $f : \mathbf{R}^3 \to \mathbf{R}^3$:

$$f(x_1, x_2, x_3) = \begin{cases} f_1(x_1, x_2, x_3) & \left(-\frac{1}{\sqrt{2}} \leq x_1 < 0\right) \\ f_2(x_1, x_2, x_3) & \left(0 \leq x_1 \leq \frac{1}{\sqrt{2}}\right) \end{cases}$$

where,

$$\begin{aligned} f_1(x_1, x_2, x_3) &= \left(2a\left(x_1 + \frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}, \frac{1}{2}a\left(x_2 + \frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}, 0\right) \\ f_2(x_1, x_2, x_3) &= \left(2a\left(x_1 + \frac{1}{\sqrt{2}}\right) - \sqrt{2}a - \frac{1}{\sqrt{2}}, \frac{1}{2}a\left(x_2 + \frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}a - \frac{1}{\sqrt{2}}, 0\right) \end{aligned}$$

Whenever $\frac{1}{\sqrt{2}} \leq |x_1| \leq 1$ $\left(resp. \frac{1}{\sqrt{2}} \leq |x_2| \leq 1\right)$, we replace x_1 with $0\,(resp. x_2 = 0)$.

The entropic chaos degree $D\left(\rho^{(n_0)}; \mathcal{F}^*\right)$ and $D\left(\rho^{(n_0)}; \Lambda_m^*\right)$ can be computed [12], and the result of $D\left(\rho^{(n_0)}; \Lambda_m^*\right)$ is shown in Fig. 4.1 for an initial value $\vec{X} = (0.3, 0.3, 0.3)$. We took 740 different a's between 0 and 1 with $m = 1000, n_0 = 2000$.

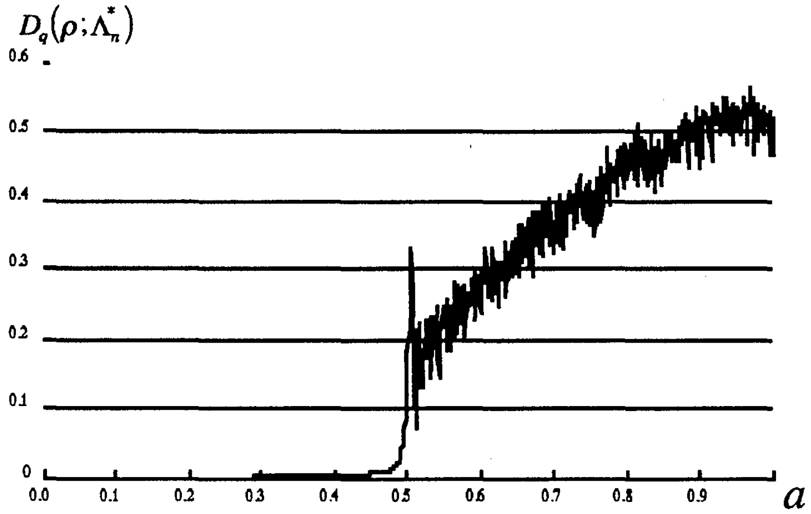

Figure 4.1: The change of $D(\rho^{(n)};\Lambda_m^*)$ w.r.t. a

The result shows that the quantum dynamica constructed by Baker's type transformation is stable in $0 \leq a \leq 0.5$ and chaotic in $0.5 < a \leq 1.0$. Though there are several approaches to study chaotic behaviors of quantum systems, we used a new quantity to measure the degree of chaos for a quantum system. Our chaos degree has the following merits:(1) once the channel Λ^*, describing the dynamics of a quantum system, is given, it is easy to compute this degree numerically; (2) the argorithm computing the degree is easily set for any quantum state.

5. Quantum Teleportation

Quantum teleportation has been introduced by Benett et al. [3] and discussed by a number of authors in the framework of the singlet state [4]. Recently, a rigorous formulation of the teleportation problem of arbitrary quantum states by means of quantum channel was given in [13] based on the general channel theoretical formulation of the quantum information theory. Here we report some results by rigrous studies of the teleportation processes in [2, 9, 10].

The following is a generalization of the channel theoretical approach to the teleportation problem proposed by [13]:

Step 0:: A girl named Alice has an unknown quantum state ρ on (a N–dimensional) Hilbert space $\mathcal{H}_1$ and she was asked to teleport it to a boy named Bob.

Step 1:: For this purpose, we need two other Hilbert spaces $\mathcal{H}_2$ and $\mathcal{H}_3$, $\mathcal{H}_2$ is attached to Alice and $\mathcal{H}_3$ is attached to Bob. Prearrange a so-called entangled state σ on $\mathcal{H}_2 \otimes \mathcal{H}_3$ having certain correlations and prepare an ensemble of the combined system in the state $\rho \otimes \sigma$ on $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$.

Step 2:: One then fixes a family of mutually orthogonal projections $(F_{nm})_{n,m=1}^{N}$ on the Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$ corresponding to an observable $F := \sum\limits_{n,m} z_{n,m} F_{n,m}$, and for a fixed one pair of indices n, m, Alice performs a first kind incomplete measurement, involving only the $\mathcal{H}_1 \otimes \mathcal{H}_2$ part of the system in the state $\rho \otimes \sigma$, which filters the value z_{nm}, that is, after measurement on the given ensemble $\rho \otimes \sigma$ of identically prepared systems, only those where F shows the value z_{nm} are allowed to pass. According to the von Neumann rule, after Alice's measurement, the state becomes

$$\rho_{nm}^{(123)} := \frac{(F_{nm} \otimes \mathbf{1})\rho \otimes \sigma (F_{nm} \otimes \mathbf{1})}{\mathrm{tr}_{123}(F_{nm} \otimes \mathbf{1})\rho \otimes \sigma (F_{nm} \otimes \mathbf{1})}$$

where tr_{123} is the full trace on the Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$.

Step 3:: Bob is informed which measurement was done by Alice. This is equivalent to transmit the information that the eigenvalue z_{nm} was detected. This information is transmitted from Alice to Bob without disturbance and by means of classical tools.

Step 4:: Making only partial measurements on the third part on the system in the state $\rho_{nm}^{(123)}$ means that Bob will control a state $\Lambda_{nm}(\rho)$ on $\mathcal{H}_3$ given by the partial trace on $\mathcal{H}_1 \otimes \mathcal{H}_2$ of the state $\rho_{nm}^{(123)}$ (after Alice's measurement)

$$\begin{aligned} \Lambda_{nm}(\rho) &= \mathrm{tr}_{12}\, \rho_{nm}^{(123)} \\ &= \mathrm{tr}_{12} \frac{(F_{nm} \otimes \mathbf{1})\rho \otimes \sigma (F_{nm} \otimes \mathbf{1})}{\mathrm{tr}_{123}(F_{nm} \otimes \mathbf{1})\rho \otimes \sigma (F_{nm} \otimes \mathbf{1})} \end{aligned}$$

Thus the whole teleportation scheme given by the family (F_{nm}) and the entangled state σ can be characterized by the family (Λ_{nm}) of channels from the set of states on $\mathcal{H}_1$ into the set of states on $\mathcal{H}_3$ and the family (p_{nm}) given by

$$p_{nm}(\rho) := \mathrm{tr}_{123}(F_{nm} \otimes \mathbf{1})\rho \otimes \sigma (F_{nm} \otimes \mathbf{1})$$

of the probabilities that Alice's measurement according to the observable F will show the value z_{nm}.

The teleportation scheme works perfectly with respect to a certain class $\mathfrak{S}$ of states ρ on $\mathcal{H}_1$ if the following conditions are fulfilled.

(E1): For each n, m there exists a unitary operator $u_{nm} : \mathcal{H}_1 \to \mathcal{H}_3$ such that

$$\Lambda_{nm}(\rho) = u_{nm}\, \rho\, u_{nm}^* \quad (\rho \in \mathfrak{S})$$

(E2):

$$\sum_{nm} p_{nm}(\rho) = 1 \quad (\rho \in \mathfrak{S})$$

: (E1) means that Bob can reconstruct the original state ρ by unitary keys $\{u_{nm}\}$ provided to him.

: (E2) means that Bob will succeed to find a proper key with certainty.

Such a teleportation process can be classified into two cases [2], where we discussed to find the solutions of the teleportation in each case and the conditions of the uniqueness of unitary key. Along the paper [2], we here report the results constructing more realistic teleportation models.

In the papers [3, 4], the authors used EPR spin pair to construct a teleportation model. In order to have a more handy model, we here use coherent states to construct a model. One of the main points for such a construction is how to prepare the entangled state. The EPR entangled state used in [3] can be identified with the splitting of a one particle state, so that the teleportation model of Bennett et al. can be described in terms of Fock spaces and splittings, which makes us possible to work the whole teleportation process in general beam splitting scheme. Moreover to work with beams having a fixed number of particles seems to be not realistic, especially in the case of large distance between Alice and Bob, because we have to take into account that the beams will lose particles (or energy). For that reason one should use a class of beams being insensitive to this loss of particles. That and other arguments lead to superpositions of coherent beams.

In this report, we discuss the construction, given in [9], of a teleportation model being perfect in the sense of conditions (E1) and (E2), where we take the Boson Fock space $\Gamma(L^2(G)) := \mathcal{H}_1 = \mathcal{H}_2 = \mathcal{H}_3$ with a certain class ρ of states on this Fock space. Then we consider a teleportation model where the entangled state σ is given by the splitting of a superposition of certain coherent states. Unfortunately this model doesn't work perfectly, that is, neither (E2) nor (E1) hold. However this model is more realistic than the perfect model, and we show that this model provides a nice approximation to be perfect. To estimate the difference between the perfect teleportation and non-perfect teleportation, we add a further step in the teleportation scheme:

Step 5:: Bob will perform a measurement on his part of the system according to the projection

$$F_+ := \mathbf{1} - |\exp(0) >< \exp(0)|$$

where $|\exp(0) >< \exp(0)|$ denotes the vacuum state (the coherent state with density 0).

Then our new teleportation channels (we denote it again by Λ_{nm}) have the form

$$\Lambda_{nm}(\rho) := \mathrm{tr}_{12} \frac{(F_{nm} \otimes F_+)\rho \otimes \sigma(F_{nm} \otimes F_+)}{\mathrm{tr}_{123}(F_{nm} \otimes F_+)\rho \otimes \sigma(F_{nm} \otimes F_+)}$$

and the corresponding probabilities are

$$p_{nm}(\rho) := \mathrm{tr}_{123}(F_{nm} \otimes F_+)\, \rho \otimes \sigma(F_{nm} \otimes F_+)$$

For this teleportation scheme, (E1) is fulfilled. Furthermore we get

$$\sum_{nm} p_{nm}(\rho) = \frac{(1 - e^{-\frac{d}{2}})^2}{1 + (N-1)e^{-d}} \quad (\to 1 \ (d \to +\infty))$$

Here N denotes the dimension of the Hilbert space and d is the expectation value of the total number of particles (or energy) of the beam, so that in the case of high density (or energy) "$d \to +\infty$" of the beam the model works perfectly. Mathematical details of the above results are given in [9].

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