A simple model for the dynamics of vegetated dunes is proposed. Using the model, formation processes of transverse dunes, parabolic dunes and elongated parabolic dunes are simulated according to environmental factors. The results have qualitative correspondence to the observation by Hack[1] in which he showed the relation between the wind condition, available sand, the ratio of surface covered by plants, and the dominant type of dunes observed in each arid (or semiarid) area.

The model is a 2D lattice model[2], wherein two field variables,i)the local height \( h(i,j,n) \) of sand bed, and ii)the local density of vegetation \( c(i,j,n) \), are allocated at each site of the horizontally extending 200 × 200 lattice. Here \( \{i,j\} \) and \( n \) indicate, respectively, horizontal position and time step. Wind is constantly blowing in positive-i direction, and each \( (i,j) \) site covers a sufficiently wider area than that occupied by individual grains. The above field variables are set to interact each other through the suppression factor \( a(i,j,n) \) as explained below.

I. For the evolution of \( c(i,j,n) \), a set of simple rules are adopted. In the case where the height of sand surface remains unchanged with time, wherein plants will grow thick up to the saturation density without being cut away by the drastic deflation of ground or being buried by the rapid accumulation of sands, \( c(i,j,n) \) is allowed to increase linearly until the maximum value \( c_{\text{max}} \). On the other hand, if the temporal change of surface height is too fast, the growth rate of plants is suppressed, then \( c(i,j,n) \) decreases down to the minimum limit \( c_{\text{min}} \). To reflect the above situation, we use a discrete dynamics which is expressed by a sectional linear map;

\[
\begin{align*}
    c(i,j,n+1) &= A(c(i,j,n) - b(i,j,n)) + c_{\text{min}} & (b \leq c \leq (c_{\text{max}} - c_{\text{min}})/A + b) \\
    c(i,j,n+1) &= c_{\text{max}} & (b < c < (c_{\text{max}} - c_{\text{min}})/A + b) \\
    c(i,j,n+1) &= c_{\text{min}} & (c < b)
\end{align*}
\]

here \( b = b(i,j,n) \equiv |h(i,j,n) - h(i,j,n-1)|, c = c(i,j,n) \) and \( A \) is a positive constant to determine the growth rate of plants.

II. For the evolution of \( h(i,j,n) \), discretized conservation law of \( h(i,j,n) \),

\[
\begin{align*}
    h(i,j,n+1) - h(i,j,n) &= Q_{\text{in}}(i,j,n) - Q_{\text{out}}(i,j,n)
\end{align*}
\]

holds, where \( Q_{\text{in}}(i,j,n) \) is the total mass of sand coming into site \( (i,j) \) at time step \( n \), while \( Q_{\text{out}}(i,j,n) \) is the same quality leaving from \( (i,j) \) at \( n \). Both of the saltation flux and the creep flux contribute to \( Q_{\text{in}}(i,j,n) \) and \( Q_{\text{out}}(i,j,n) \). Specifically the saltation flux caused by the grains leaving from \( (i,j) \) at \( n \) is expressed by,

\[
q_{\text{sal}}(i,j,n) = q_0(tanh(\nabla h(i,j,n)) + 1.0)(tanh(\nabla h(i,j,n)) + 1.0 + \alpha).
\]

where \( \nabla h(i,j,n) \equiv h(i,j,n) - h(i-1,j,n) \) and \( \alpha \) is a constant to determine the bed-load in the windward slope of dunes. The above reflects the qualitative nature of wind, also the resulting saltation flux around dunes which is intensive in the windward particularly around the crest, whereas almost no flux in the lee side[3]. On the other hand, the flux by creep \( q_{\text{creep}}(i,j,n) \) is set proportional to the local gradient of sand surface. The crucial effect caused by permitting the growth of plants is such
Figure 1: Snapshots of simulated dunes under various pairs of control parameters, wind force \( q_a \) and sand supply at the source \( h_{\text{source}} \). The left part in each figure shows the spatial distribution of vegetation density \( c(i,j,n) \). The darker tone indicates the more densely vegetated place, whereas white parts indicate the areas with bare sand surfaces. The right part in each figure shows the surface height distribution \( h(i,j,n) \), where the darker position means the higher surface. Sand is supplied from the most windward 2 rows. Steady wind is blowing from the left to the right. (a) \( q_a = 3.0, h_{\text{source}} = 1.3 \). When the wind force is too weak no distinguishable dune is formed. (b) \( q_o = 7.0, h_{\text{source}} = 2.0 \). Under rather strong wind with sufficient amount of sand supply transverse dunes prevail while small parabolic dunes are seen just lee of the sand source. The latter will soon grow up to the former. (c) \( q_a = 5.0, h_{\text{source}} = 1.6 \). Parabolic dunes, the arms of which extend in the windward direction, are formed under mildly blowing wind with intermediate amount of sand at the source. (d) \( q_o = 7.0, h_{\text{source}} = 1.3 \). Elongated Parabolic dunes are formed under similar conditions with (c) except for smaller amount of sand at the source.

that the sand transport sharply decreases when the cover ratio of sand surface by plants exceeds a critical value \( c_{cr} \). To realize the situation in a simple expression, the suppression factor \( a(i,j,n) \) is introduced like,

\[
a(i,j,n) = 1 + \frac{1}{2}(\tanh(c_{cr} - c) - 1).
\]

(4)

With the suppression factor, the saltation/creep flux is forced to decrease as \( q(i,j,n) = a(i,j,n)^2 q'(i,j,n) \) where \( q(i,j,n) \) and \( q'(i,j,n) \) are, respectively, the local flux by saltation/creep with vegetation and without vegetation.

Intending to compare the simulation outputs with the diagram by Hack, two kind of quantities are chosen as the control parameters; One is i)the amount of sand at the source. Namely, the average height, \( \langle h_{\text{source}} \rangle \), of the sand surface at the source sites along the windward boundary. ii)the wind strength which should be a monotonically increasing function of saltation flux. Specifically, the variable \( q_0 \) in eq.(2) is adopted as the index of wind strength.

The results are shown in fig.1 and fig.2. They have qualitative correspondence to the real counterpart, and the simplicity of the algorithm and the consequent easiness of the handling of this model provide us with wide applicability for the investigation of the complex interplay between vegetation and dunes.

Figure 2: Phase diagram to show the dominant types of dunes under various pairs of control parameters. The alphabets in the diagram indicate the conditions corresponding to respective snapshots in fig.1. Symbols \( \Delta \) and \( \Delta x \) indicate, respectively, parabolic dunes, and, barchan(or transverse) dunes, whereas \( \times \) mean the conditions for no dune formation. The \( \Delta \)s accompanied by * mean the conditions for the development of rather thin parabolic dunes with long arms. At the condition with the symbol \( \square \), many irregular mounds are formed which are not clearly categorized as particular type of dunes.

References